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AN ALGORITHMIC APPROACH OF SOLVING FUZZY LINEAR SYSTEM USING FOURIER MOTZKIN ELIMINATION METHOD

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Abstract

In this paper, we first change the fuzzy linear programming problem into the fuzzy linear system of equations. Then a Fourier Motzkin elimination method is discussed to solve the above converted fuzzy linear system of equations.

1. Introduction

A method discovered by Fourier [7] in 1826 for manipulating linear inequalities can be adapted to solve LP models. The theoretical insight given by this method is demonstrated as well as its clear geometrical interpretation. It has been rediscovered a number of times by different authors: Motzkin [11] (the name Fourier Motzkin algorithm is often used for this method) Dantzig and Cottle [9] and Kuhn [10] (some authors also referred to this method as Kuhn-Fourier algorithm).

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The Fourier's method is used for solving a system of linear constraints of the form $a_1x_1 + \cdots + a_nx_n \succ b$ on the set of real numbers (or more generally on an ordered field) where $\succ$ is $>, \geq$ or $=$ and $a_1, a_2, ..., a_n, b$ are real numbers.

The Fourier Motzkin elimination consists in successive elimination of the unknowns. Each step transforms the constraints system s_n with the unknowns $x_1, x_2, ..., x_n$ to a new system s_{n-1} in which one of the unknowns, say x_n does not occur anymore: x_n has been eliminated.

The concept of fuzzy numbers and arithmetic operations with these numbers was first introduced and investigated by Zadeh [20], etc. One of the major applications of fuzzy number arithmetic is treating linear systems and their parameters that are all partially respected by fuzzy number. Friedman et al. [8] introduced a general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system. After that, in the literature of fuzzy linear system of equations, various methods are proposed to solve these systems [1, 3-5, 13-17].

In this paper, all the variables are considered as fuzzy variables. Applying Fourier Motzkin elimination algorithm in the fuzzy linear programming problem and obtaining the values of the fuzzy variables.

This paper is organized as follows: Section 2 introduces some preliminary definitions. Fourier Motzkin elimination method in fuzzy linear systems is described in Section 3 and also Fourier Motzkin elimination algorithm for solving fuzzy linear systems is given. Finally, in Section 4, the effectiveness of the proposed method is illustrated by means of an example.

2. Preliminaries

2.1. Fuzzy set

A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$.

In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval $[0, 1]$ called *membership function*.

2.2. Fuzzy number

A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number:

(i) $\widetilde{A}$ must be a normal fuzzy set;

(ii) ${}^{\alpha}\widetilde{A}$ must be a closed interval for every $\alpha \in [0, 1]$; and

(iii) the support of $\widetilde{A}$ must be bounded.

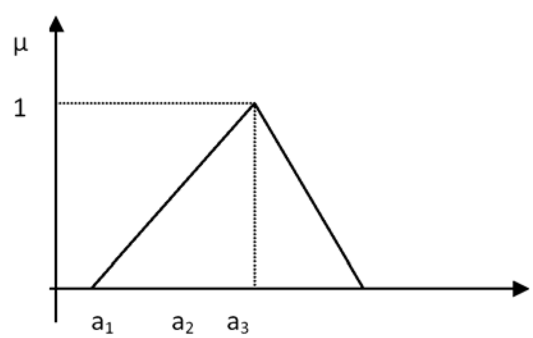
2.3. Triangular fuzzy number

It is a fuzzy number represented with three points as follows: $\widetilde{A} = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and holds the following conditions:

(i) a_1 to a_2 is an increasing function;

(ii) a_2 to a_3 is a decreasing function; and

(iii) $a_1 \le a_2 \le a_3$.



Triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$.

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3, \\ 0 & \text{for } x > a_3. \end{cases}$$

2.4. Operation of triangular fuzzy number using function principle

Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$. Then

(i) **Addition**: $\widetilde{A} + \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii) **Subtraction**: $\widetilde{A} - \widetilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) **Multiplication**:

$$\widetilde{A} \times \widetilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)).$$

(iv) **Division**:

$$\widetilde{A}/\widetilde{B} = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)).$$

2.5. Fuzzy linear system of equations

Consider the $m \times n$ fuzzy linear system of equations [11]:

$$\left.\begin{array}{c} a_{11} \times \widetilde{x}_{11} + a_{12} \times \widetilde{x}_{12} + \cdots + a_{1n} \times \widetilde{x}_{1n} = \widetilde{b}_1 \\ a_{21} \times \widetilde{x}_{21} + a_{22} \times \widetilde{x}_{22} + \cdots + a_{2n} \times \widetilde{x}_{2n} = \widetilde{b}_2 \\ \vdots \\ a_{m1} \times \widetilde{x}_{m1} + a_{m2} \times \widetilde{x}_{m2} + \cdots + a_{mn} \times \widetilde{x}_{mn} = \widetilde{b}_m \end{array}\right\}.$$

The matrix form of the above equations is $A \times \widetilde{x} = \widetilde{b}$, where the coefficient matrix A is (a_{ij}), where $i = 1$ to m and $j = 1$ to n, $\widetilde{x}$ is a fuzzy variable and $\widetilde{b}$ is also a fuzzy variable.

2.6. Graded mean integration method

The graded mean integration method [6] is used to defuzzify the triangular fuzzy number. The representation of triangular fuzzy number is $\widetilde{A} = (a_1, a_2, a_3)$ and its defuzzified value is obtained by $A = \frac{a_1 + 4a_2 + a_3}{6}.$

3. Fourier Motzkin Elimination Method in Fuzzy Linear Systems

We briefly review the Fourier-Motzkin elimination method in fuzzy linear systems. Consider a fuzzy linear system $A\widetilde{x} \le \widetilde{b},$ where $A \in R^{m,n},$ $\widetilde{x}, \widetilde{b}$ are fuzzy variables and let $I := \{1, 2, ..., m\}.$ The fuzzy linear systems in the following form:

$$\begin{aligned} a_{11}\widetilde{x}_1 + a_{12}\widetilde{x}_2 + \cdots + a_{1n}\widetilde{x}_n &\le \widetilde{b}_1 \\ a_{21}\widetilde{x}_1 + a_{22}\widetilde{x}_2 + \cdots + a_{2n}\widetilde{x}_n &\le \widetilde{b}_2 \\ &\vdots \\ a_{m1}\widetilde{x}_1 + a_{m2}\widetilde{x}_2 + \cdots + a_{mn}\widetilde{x}_n &\le \widetilde{b}_m. \end{aligned} \tag{3.1}$$

Eliminate $\widetilde{x}_1$ from the system (3.1) for each I where $a_{i1} \ne 0,$ we multiply the ith inequality by $1/|a_{i1}|.$ This gives an equivalent system:

$$\begin{aligned} &\widetilde{x}_1 + a'_{i2}\widetilde{x}_2 + \cdots + a'_{in}\widetilde{x}_n \le \widetilde{b}'_i \quad (i \in I^+) \\ &a_{i2}\widetilde{x}_2 + \cdots + a_{in}\widetilde{x}_n \le \widetilde{b}_i \quad (i \in I^0) \\ &-\widetilde{x}_1 + a'_{i2}\widetilde{x}_2 + \cdots + a'_{in}\widetilde{x}_n \le \widetilde{b}'_i \quad (i \in I^-), \end{aligned} \tag{3.2}$$

where $I^+ = \{i : a_{i1} > 0\},$ $I^0 = \{i : a_{i1} < 0\},$ $I^- = \{i : a_{i1} < 0\},$ $a'_{ij} = a_{ij}/|a_{i1}|$ and $\widetilde{b}'_i = \widetilde{b}_i/|a_{i1}|.$

Thus, the row index set $I = \{1, 2, ..., m\}$ is partitioned into subsets $I^+,$

I^0 and I^-, some of which may be empty. It follows that $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$ is a solution of the original system (3.1) if and only if $\tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n$ satisfy

$$\sum_{j=2}^{n} a'_{kj}\tilde{x}_j - \tilde{b}'_k \leq \tilde{b}'_i - \sum_{j=2}^{n} a'_{ij}\tilde{x}_j \quad (i \in I^+,\, k \in I^-),$$

$$\sum_{j=2}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i \quad (i \in I^0) \tag{3.3}$$

and $\tilde{x}_1$ satisfies

$$\max_{k \in I^-}\left(\sum_{j=2}^{n} a'_{kj}\tilde{x}_j - \tilde{b}'_k\right) \leq \tilde{x}_1 \leq \min_{i \in I^+}\left(\tilde{b}'_i - \sum_{j=2}^{n} a'_{ij}\tilde{x}_j\right). \tag{3.4}$$

If I^- (resp. I^+) is empty, then the first set of constraints in (3.3) vanishes and the maximum (resp. minimum) in (3.4) is interpreted as ∞ (resp. $-\infty$). If I^0 is empty and either I^- or I^+ is empty too, then we terminate: the general solution of $A\tilde{x} \leq \tilde{b}$ is obtained by choosing $\tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n$ arbitrarily and choosing $\tilde{x}_1$ according to (3.4).

The constraint (3.4) says that $\tilde{x}_1$ lies in a certain interval which is determined by $\tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n$. The polyhedron defined by (3.3) is the projection of p along the $\tilde{x}_1$-axis, i.e., into the space of the variables $\tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n$. One may then proceed similarly and eliminate $\tilde{x}_2$, $\tilde{x}_3$, etc., eventually one obtains a system $l \leq \tilde{x}_n \leq u$.

If $l > u$, then one concludes that $A\tilde{x} \leq \tilde{b}$ has no solution, otherwise one may choose $\tilde{x}_n \in [l, u]$, and then choose $\tilde{x}_{n-1}$ in an interval which depends on $\tilde{x}_n$, this back substitution procedure produces a solution $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$ to $A\tilde{x} \leq \tilde{b}$. Moreover, every solution of $A\tilde{x} \leq \tilde{b}$ may be produced in this way (if the system is inconsistent, then this might possibly be discovered at an early stage and one terminates).

The number of constraints may grow exponentially fast as fuzzy variables are eliminated using Fourier-Motzkin elimination. Actually, a main problem in practice is that the number of inequalities becomes "too large" during the elimination process, even when redundant inequalities are removed. It is therefore of interest to know situations where the projected linear systems are not very large or, at least, have some interesting structure.

Theorem 3.1. *Consider the fuzzy system of linear equations of form* $A\tilde{x} \leq \tilde{b}$, *where A has m rows and n columns.*

Now the system can be written in the following three classes of form:

$$\begin{aligned} &D(\tilde{x}) : a_i\tilde{x}' \leq \tilde{b}_i && i = 1, \ldots, m_1, \\ &E(\tilde{x}) : -\tilde{x}_1 + a_j\tilde{x}' \leq \tilde{b}_j && j = m_1 + 1, \ldots, m_2, \\ &F(\tilde{x}) : \tilde{x}_1 + a_k\tilde{x}' \leq \tilde{b}_k && k = m_2 + 1, \ldots, m, \end{aligned}$$

where $\tilde{x}$ *is* $[\tilde{x}_2, \tilde{x}_3 \cdots \tilde{x}_n]^T$, *i.e., the same set of variables without* $\tilde{x}_1$.

Each constraint expressed in the form: $\tilde{x}_1 \leq ()(F(\tilde{x})), \tilde{x}_1 \geq ()(E(\tilde{x}))$ *and the constraints which do not have* $\tilde{x}_1$ *in the* $(D(\tilde{x}))$.

Now consider the system defined below by

$$\begin{aligned} &D(\tilde{X}) : a_i\tilde{x}' \leq \tilde{b}_i && i = 1, \ldots, m_1, \\ &a_j\tilde{x}' - \tilde{b}_j \leq \tilde{b}_k - a_k\tilde{x}' && j = m_1 + 1, \ldots, m_2, \;\; k = m_2 + 1, \ldots, m. \end{aligned}$$

The system (3.1) *has a solution iff system* (3.2) *has a solution*.

Proof. Let us say that the fuzzy linear system (3.1) has a solution, i.e., we have fuzzy vector $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n]$ satisfying system (3.1).

The value of $\tilde{x}_1$ chosen has to satisfy

$$\tilde{x}_1 \geq -\tilde{b}_j + a_j\tilde{x}', \quad \forall j = m_1 + 1, \ldots, m_2,$$

$$\tilde{x}_1 \leq b_k - a_k\tilde{x}', \quad \forall k = m_2 + 1, \ldots, m.$$

Hence fuzzy system (3.2) is trivially satisfied.

Conversely, if system (3.2) is satisfied, then system (3.1) is also satisfied.

Consider a solution $\tilde{x}' = [\tilde{x}_2, \tilde{x}_3, ..., \tilde{x}_n]$ to system (3.2). Let

$$l = \max(a_j \tilde{x}' - \tilde{b}_j,\ j = m_1 + 1, ..., m_2)$$

and

$$u = \min(\tilde{b}_k - a_k \tilde{x}', k = m_2 + 1, ..., m).$$

If $l > u$, then one of the constraints of system (3.2) has been violated. So $l \leq u$, an assignment of $\tilde{x}_1$ to any value in the range $[l, u]$ trivially satisfies system (3.1).

This elimination procedure clearly gives an algorithm for deciding feasibility of a linear system of inequalities. First, eliminate $\tilde{x}_1$ then $\tilde{x}_2$ and so on till you have only $\tilde{x}_n$. If $\tilde{x}_n$ occurs as a feasible range $[a, b]$, $a < b$, then the system is feasible. Otherwise, we get zero. $\tilde{x} \leq -1$, which is a contradiction.

3.2. Fourier Motzkin elimination algorithm

Step 1. Formulate the fuzzy linear programming problem from the given problem.

Step 2. Then change the objective function as inequality and join it with the constraints. Now we get the fuzzy linear system of equation.

(For max problem, use '≤' and for min problem '≥' inequality).

Step 3. Now change all the inequalities of the fuzzy system into '≥' for minimization problem and '≤' for maximization problem.

Step 4. Now we are going to eliminate one by one in the $(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$.

(i) Divide each equation by its modulus value of $\tilde{x}_1$ coefficient for all the equations.

(ii) Now we have three classes of $\tilde{x}_1$ coefficient, i.e., '−1' or '+1'or '0' in the fuzzy system of linear equations.

(iii) Adding or subtracting any two classes of equations to eliminate $\tilde{x}_1$.

Step 5. Repeat Step 4 until all the 'n' fuzzy variables are eliminated.

Step 6. After eliminating all the 'n' fuzzy variables, we get the $\widetilde{Z}$ values and substitute the $\widetilde{Z}$ in above, we get the values of fuzzy variables in back to back substitution.

4. A Numerical Example

Problem. A firm produces two products. These products are processed on two different machines. The time required to manufacture one unit of each of the two products and the daily capacity of the two machines are in fuzzy as given in the table below.

Machine	Time required for products		Machine capacity (in fuzzy)
	Product 1	Product 2	
M1	3	5	(14, 15, 16)
M2	5	2	(9, 10, 11)

It is required to determine the total number of units to be manufactured for each product. The profits per unit for products 1 and 2 are 5 and 3, respectively.

Solution

Formulation of the problem

Maximize $\widetilde{z} = 5\widetilde{x}_1 + 3\widetilde{x}_2$

Subject to the constraints

$$3\widetilde{x}_1 + 5\widetilde{x}_2 \leq (14, 15, 16)$$

$$5\widetilde{x}_1 + 2\widetilde{x}_2 \leq (9, 10, 11)$$

and the non-negative constraints

$$\widetilde{x}_1 \geq (0, 0, 0)$$

$$\widetilde{x}_2 \geq (0, 0, 0).$$

First, we have to include the objective function in the constraints to form a fuzzy linear system of equations. For maximization problem, change the equal '=' in the objective as '≤' and (for minimization problem, change '≥') join it with all the constraints

$$\left.\begin{array}{l} \tilde{z} \le 5\tilde{x}_1 + 3\tilde{x}_2 \\ 3\tilde{x}_1 + 5\tilde{x}_2 \le (14, 15, 16) \\ 5\tilde{x}_1 + 2\tilde{x}_2 \le (9, 10, 11) \\ \tilde{x}_1 \ge (0, 0, 0) \\ \tilde{x}_2 \ge (0, 0, 0) \end{array}\right\}. \tag{4.1}$$

Equation (4.1) is a fuzzy linear system of equations. Now change all the inequalities in the system as '≤' for maximization (and ≥ for minimization)

$$\left.\begin{array}{l} -5\tilde{x}_1 - 3\tilde{x}_2 + \tilde{z} \le 0 \\ 3\tilde{x}_1 + 5\tilde{x}_2 \le (14, 15, 16) \\ 5\tilde{x}_1 + 2\tilde{x}_2 \le (9, 10, 11) \\ -\tilde{x}_1 \le (0, 0, 0) \\ -\tilde{x}_2 \le (0, 0, 0) \end{array}\right\}. \tag{4.2}$$

Now we are going to eliminate $\tilde{x}_1$, and dividing each coefficient of the system (4.2) by its coefficient of $\tilde{x}_1$, we have

$$\left.\begin{array}{l} -\tilde{x}_1 - 0.6\tilde{x}_2 + 0.2\tilde{z} \le 0 \\ \tilde{x}_1 + 1.66\tilde{x}_2 \le (4.66, 5, 5.33) \\ \tilde{x}_1 + 0.4\tilde{x}_2 \le (1.8, 2, 2.2) \\ -\tilde{x}_1 \le (0, 0, 0) \\ -\tilde{x}_2 \le (0, 0, 0) \end{array}\right\}. \tag{4.3}$$

Now we have three classes of equations in the fuzzy linear system (4.3) we get the coefficient of $\tilde{x}_1$ in the first class equation is '−1', in the second class equation '+1' and in the third class equation is '0'.

Now adding the first class equations with the second class equations to eliminate $\widetilde{x}_1$,

$$\left.\begin{array}{l} 1.06\widetilde{x}_2 + 0.2\widetilde{z} \le (4.66,\ 5,\ 5.33) \\ -0.2\widetilde{x}_2 + 0.2\widetilde{z} \le (1.8,\ 2,\ 2.2) \\ 1.66\widetilde{x}_2 \le (4.66,\ 5,\ 5.33) \\ 0.4\widetilde{x}_2 \le (1.8,\ 2,\ 2.2) \\ -\widetilde{x}_2 \le (0,\ 0,\ 0) \end{array}\right\}. \tag{4.4}$$

Now eliminate $\widetilde{x}_2$ using the same procedure

$$\left.\begin{array}{l} \widetilde{x}_2 + 0.189\widetilde{z} \le (3.77,\ 4.72,\ 5.03) \\ -\widetilde{x}_2 + \widetilde{z} \le (9,\ 10,\ 11) \\ \widetilde{x}_2 \le (2.81,\ 3.01,\ 3.21) \\ \widetilde{x}_2 \le (4.5,\ 5,\ 5.5) \\ -\widetilde{x}_2 \le (0,\ 0,\ 0) \end{array}\right\}. \tag{4.5}$$

Now adding first class equations with second class equations

$$\left.\begin{array}{l} 0.189\widetilde{z} \le (12.77,\ 14.72,\ 16.03) \\ \widetilde{z} \le (11.81,\ 13.01,\ 14.21) \\ \widetilde{z} \le (13.5,\ 15,\ 16.5) \\ 0.189\widetilde{z} \le (3.77,\ 4.72,\ 5.03) \\ 0 \le (2.81,\ 3.01,\ 3.21) \\ 0 \le (4.5,\ 5,\ 5.5) \end{array}\right\}. \tag{4.6}$$

There is no possibility to eliminate $\widetilde{z}$ in (4.6) so stop the process.

From the above equation (4.6), we have

$$\widetilde{z} \le (10.74,\ 12.38,\ 13.48)$$

$$\widetilde{z} \le (11.81,\ 13.01,\ 14.21)$$

$$\widetilde{z} \le (13.5,\ 15,\ 16.5)$$

$$\widetilde{z} \le (19.95,\ 24.97,\ 26.61).$$

Now choosing the minimum value for $\tilde{z}$ to satisfy all the above conditions. So $\tilde{z} = (10.74, 12.38, 13.48)$.

Substitute $\tilde{z}$ in (4.5)

$$\tilde{x}_2 + 0.189(10.74, 12.38, 13.48) \leq (3.77, 4.72, 5.03)$$

$$-\tilde{x}_2 + (10.74, 12.38, 13.48) \leq (9, 10, 11)$$

$$\tilde{x}_2 \leq (2.81, 3.01, 3.21)$$

$$\tilde{x}_2 \leq (4.5, 5, 5.5)$$

$$-\tilde{x}_2 \leq (0, 0, 0),$$

we get

$$\tilde{x}_2 \leq (3.77, 4.72, 5.03) - (2.03, 2.34, 2.55)$$

$$-\tilde{x}_2 \leq (9, 10, 11) - (10.74, 12.38, 13.48)$$

$$\tilde{x}_2 \leq (2.81, 3.01, 3.21)$$

$$\tilde{x}_2 \leq (4.5, 5, 5.5)$$

$$-\tilde{x}_2 \leq (0, 0, 0),$$

then

$$\tilde{x}_2 \leq (1.27, 2.38, 3)$$

$$\tilde{x}_2 \geq (0.26, 2.38, 4.48)$$

$$\tilde{x}_2 \leq (2.81, 3.01, 3.21)$$

$$\tilde{x}_2 \leq (4.5, 5, 5.5)$$

$$\tilde{x}_2 \geq (0, 0, 0)$$

$$\therefore (0.26, 2.38, 4.48) \leq \tilde{x}_2 \leq (1.27, 2.38, 3).$$

The defuzzified value of $\tilde{x}_2$ on both sides is nearly 2.3, therefore, select any one

$$\therefore \tilde{x}_2 = (1.27,\ 2.38,\ 3).$$

Substitute $\tilde{x}_2$ and $\tilde{Z}$ in (4.3).

$$-\tilde{x}_1 - 0.6(1.27,\ 2.38,\ 3) + 0.2(10.74,\ 12.38,\ 13.48) \le 0$$

$$\tilde{x}_1 + 1.66(1.27,\ 2.38,\ 3) \le (4.66,\ 5,\ 5.33)$$

$$\tilde{x}_1 + 0.4(1.27,\ 2.38,\ 3) \le (1.8,\ 2,\ 2.2)$$

$$-\tilde{x}_1 \le (0,\ 0,\ 0)$$

$$-(1.27,\ 2.38,\ 3) \le (0,\ 0,\ 0).$$

We get

$$-\tilde{x}_1 - (0.762,\ 1.428,\ 1.8) + (2.148,\ 2.476,\ 2.696) \le 0$$

$$\tilde{x}_1 + (2.11,\ 3.95,\ 4.98) \le (4.66,\ 5,\ 5.33)$$

$$\tilde{x}_1 + (0.510,\ 0.95,\ 1.2) \le (1.8,\ 2,\ 2.2)$$

$$-\tilde{x}_1 \le (0,\ 0,\ 0)$$

$$-(1.27,\ 2.38,\ 3) \le (0,\ 0,\ 0).$$

Then

$$\tilde{x}_1 \ge (0.35,\ 1.05,\ 1.934)$$

$$\tilde{x}_1 \le (-.32,\ 1.05,\ 3.22)$$

$$\tilde{x}_1 \le (0.6,\ 1.05,\ 1.69)$$

$$\tilde{x}_1 \ge 0.$$

From the above equations, we get

$$(0.35,\ 1.05,\ 1.934) \le \tilde{x}_1 \le (0.6,\ 1.05,\ 1.69).$$

The defuzzified value of $\tilde{x}_1$ on both the inequalities are same.

We take

$$\tilde{x}_1 = (0.6, 1.05, 1.69) = 1.08, \quad \tilde{x}_2 = (1.27, 2.38, 3) = 2.3,$$

$$\tilde{Z} = (10.74, 12.38, 13.48) = 12.3.$$

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