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THE EFFECTS OF HEAT SOURCE AND RADIATION ON UNSTEADY MHD FREE CONVECTIVE FLUID FLOW EMBEDDED IN A POROUS MEDIUM WITH TIME-DEPENDENT SUCTION

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Abstract

This paper investigates the effects of heat source and radiation on unsteady two dimensional, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy

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effects. A time-dependent is assumed and the radiative flux is described using the differential approximation for radiation. Asymptotic series expansion about a small parameter ε, is performed to obtain the flow fields. The velocity of the porous plate increases exponentially with time and a variable suction velocity is applied normal to the plate. The governing equations of the flow field are solved using perturbation technique and the expressions are obtained for velocity, temperature and concentration fields. The effects of various parameters such as P_r, S_c, M, Q, k, N_r, k_r, A, G_m and G_r are discussed and derived through graphs and tables.

Nomenclature

u, v	Velocity components
T	Dimensionless temperature
q'_r	Radiative flux vector
x, y	Cartesian coordinates
t	Time
g	Acceleration due to gravity
β	Coefficient of volume expansion due to temperature
β_*	Coefficient of volume expansion due to concentration
ρ	Density
c_p	Specific heat at constant pressure
ν	Kinematic viscosity
M	Magnetic parameter
κ	Thermal conductivity
U_0	Mean velocity
S_c	Schmidt parameter

P_r Prandtl number

k_r^2 Chemical reaction rate constant

ε Small reference parameter << 1

G_r Thermal Grashof number

G_m Mass Grashof number

A Suction parameter

n Constant exponential index

D Molar diffusivity

N_r Thermal radiation parameter

k Permeability of the porous medium

Q Heat source parameter

Subscripts

w Condition at the wall

∞ Free stream conditions primes denote dimensional quantities

Introduction

The problem of fluid flow in an electro-magnetic field has been studied for its importance in geophysics, astrophysics, metallurgy and aerodynamic extrusion of plastic sheets and other engineering processes such as in chemical engineering, petroleum engineering, composite and ceramic engineering and heat exchangers. In the last five years, many investigations dealing with the heat flow and mass transfer over a vertical porous plate with variable suction heat generation/absorption or hall current have been reported. Some of the works are Raptis [1] who investigated the flow of a micro-polar fluid past a continuously moving plate in the presence of a radiation. Azzam [2] presented radiation effects on the MHD mixed free-fixed convective flow past a semi-infinite moving vertical plate for high temperature differences. Chamkha [3] studied thermal radiation and buoyancy effects on hydro magnetic flow over an accelerating permeable surface with heat source or sink. Chen [4] studied heat and mass transfer effects with variable wall temperature and concentration. Cookey et al. [5] investigated the influence of viscous dissipation and radiation on unsteady free-convective flow past an infinite heated vertical plate in a porous medium with time-dependent suction. Ogulu et al. [6] analyzed unsteady MHD free convective flow of a compressible fluid past moving a vertical plate in the presence of radiative heat transfer. Again, Ogulu and Mbeledogu [7] studied heat and mass transfer of unsteady MHD natural convective flow of a rotating fluid past of a vertical porous flat plate in the presence of radiative heat and mass transfer. Prakash and Ogulu [8] investigated MHD free convection and mass transfer flow of a micro-polar thermally radiating and reacting fluid with time dependent suction. Sunitha et al. [9] studied radiation and mass transfer effects on MHD free convective flow past an impulsively started isothermal vertical plate with viscous dissipation. Recently, Shanker et al. [10] presented a numerical solution for radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption using Galerkin finite element method and Makinde and Sibanda [11] studied MHD mixed convection flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. Again, Makinde and Olanrewaju [12] investigated unsteady mixed convection with Soret and Dufour effects past a vertical porous plate moving through a binary mixture of a chemically reacting fluid.

The present analysis aims to study the effects thermal radiation, time-dependent suction and chemical reaction on two-dimensional MHD free convective Boussinesq fluid flow over a semi-infinite vertical plate moving exponentially with time in the presence of heat source under the influence of applied transverse magnetic field normal to the flow has been studied. The problem is governed by the system of coupled partial differential equations. And employing a perturbation technique the solutions are obtained.

Mathematical Analysis

Let us consider a problem of unsteady two-dimensional, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects. A variable time-dependent suction velocity $v' = -V_0(1 + \varepsilon A e^{n't'})$ is considered normal to the flow. The plate is taken along x'-axis in vertical upward direction against to the gravitational field. And y'-axis is taken normal to the flow in the direction of applied transverse magnetic field. Further, due to semi-infinite plane surface assumption the flow variables are the functions of y' and t' only. Then under the usual Boussinesq's approximation the flow is governed by the following set of equations.

Continuity equation:

$$\frac{\partial v'}{\partial y'} = 0. \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{k'} u'. \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \nu \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} + Q_0 (T - T_\infty). \tag{3}$$

Concentration equation:

$$\frac{\partial C}{\partial t'} + v' \frac{\partial C}{\partial y'} = \nu \frac{\partial^2 C}{\partial y'^2} - k_r'^2 C. \tag{4}$$

By using Rosselant approximation as in Raptis, Ogulu and Makinde, we can write the radiative flux q_r as:

$$q_r = \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y'}. \tag{5}$$

All the variables are defined in the nomenclature. It is assumed that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in a Taylor series about the free stream temperature T_∞ so that after rejecting higher order terms:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

The equation of energy after submission of equation's (5) and (6) can now be written as

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y'^2}. \tag{7}$$

From equation (1) we have $\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = f(t)$. Here we can see that the suction velocity is a function of time t' only. So, it is assumed in the form Cookey et al. [5];

$$v' = f(t) = -V_0 \left(1 + \varepsilon A e^{n't'}\right), \tag{8}$$

where A is the suction parameter and $\varepsilon A \ll 1$. Here the minus sign indicates that the suction is towards the plate. Now, for convenience, let us introduce the following non-dimensional parameters in equation's (2), (4) and (7) to get dimensionless form.

$$u = \frac{u'}{U_0}, \ y = \frac{U_0 y'}{\nu}, \ t = \frac{U_0^2 t'}{\nu}, \ P_r = \frac{\rho c_p \nu}{k},$$

$$S_c = \frac{\nu}{D}, \ \theta = \frac{T - T_\infty}{T - T_w}, \ \phi = \frac{C - C_\infty}{C - C_w},$$

$$G_r = \frac{g\beta\nu(T - T_\infty)}{U_0^3}, \ G_m = \frac{g\beta^*\nu(C - C_\infty)}{U_0^3}, \ n = \frac{\nu n'}{U_0^2}, \ k = \frac{k' U_0^2}{\nu^2},$$

$$Q = \frac{Q_0 \nu}{U_0^2}, \ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \ k_r^2 = \frac{k_r'^2 \nu}{U_0^2}, \ N_r = \frac{16\sigma^* T_\infty^3}{3k^* k}. \tag{9}$$

On introducing equation (9) into equations (2), (4) and (7), we obtain the following governing equations in dimensionless form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - \left(M + \frac{1}{k}\right)u, \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} = \left(\frac{1 + N_r}{P_r}\right)\frac{\partial^2 \theta}{\partial y^2} + Q\theta, \tag{11}$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} - k_r^2\varphi. \tag{12}$$

The corresponding initial and boundary conditions are

$$u = 1 + \varepsilon e^{nt}, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt} \quad \text{on} \quad y = 0$$
$$u \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{on} \quad y \to \infty. \tag{13}$$

Method of Solution

From equations (10)-(12) are coupled non-linear partial differential equations whose solutions in closed-form are difficult to obtain, is possible. To solve, these coupled non linear partial differential equations, we assume, the following Soundalgekar [13], that the unsteady flow is superimposed on the mean study flow, so that in the neighborhood of the plate, we have:

$$u = u_0(y) + \varepsilon e^{nt}u_1(y) + o(\varepsilon^2)$$
$$\theta = \theta_0(y) + \varepsilon e^{nt}\theta_1(y) + o(\varepsilon^2)$$
$$\phi = \phi_0(y) + \varepsilon e^{nt}\phi_1(y) + o(\varepsilon^2). \tag{14}$$

We now substitute equation (14) equations (10)-(12) equating harmonic and non-harmonic terms neglecting higher order terms in ε, we obtain:

$$u_0'' + u_0' - M'u_0 = -G_r\theta_0 - G_m\phi_0, \tag{15}$$

$$u_1'' + u_1' - (M' + n)u_1 = -G_r\theta_1 - G_m\phi_1 - Au_0', \tag{16}$$

$$\theta_0'' + h\theta_0' + Qh\theta_0 = 0, \tag{17}$$

$$\theta_1'' + h\theta_1' + (Q - n)h\theta_1 = -Ah\theta_0', \tag{18}$$

$$\phi_0'' + S_c\phi_0' - k_r^2 S_c\phi_0 = 0, \tag{19}$$

$$\phi_1'' + S_c\phi_1' - S_c(n + k_r^2)\phi_1 = -AS_c\phi_0', \tag{20}$$

where $M' = M + \frac{1}{k}$, $h = \frac{P_r}{1 + N_r}$ and primes indicate differentiation with respect to y.

The boundary conditions are:

$$u_0 = 1,\ u_1 = 1,\ \theta_0 = 1,\ \theta_1 = 1,\ \phi_0 = 1,\ \phi_1 = 1 \quad \text{on} \quad y = 0$$

$$u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0,\ \phi_0 \to 0,\ \phi_1 \to 0 \quad \text{as} \quad y \to \infty. \tag{21}$$

Solving equations (15)-(20) subject to the boundary conditions equation (21) we get:

$$u_0(y) = B_7 e^{-m_5 y} - B_5 e^{-m_3 y} - B_6 e^{-m_1 y}, \tag{22}$$

$$u_1(y) = (1 + B_8 + B_9 + B_{10} + B_{11} - B_{12})e^{-m_6 y}$$

$$- B_8 e^{-m_1 y} - B_9 e^{-m_2 y} - B_{10} e^{-m_3 y} - B_{11} e^{-m_4 y} + B_{12} e^{-m_5 y}, \tag{23}$$

$$\theta_0(y) = e^{-m_3 y}, \tag{24}$$

$$\theta_1(y) = B_3 e^{-m_3 y} + B_4 e^{-m_4 y}, \tag{25}$$

$$\phi_0(y) = e^{-m_1 y}, \tag{26}$$

$$\phi_1(y) = B_1 e^{-m_1 y} + B_2 e^{-m_2 y}, \tag{27}$$

where

$$m_1 = \frac{S_c + \sqrt{S_c^2 + 4k_r^2 S_c}}{2}, \quad m_2 = \frac{S_c + \sqrt{S_c^2 + 4S_c(n + k_r^2)}}{2},$$

$$m_3 = \frac{h + \sqrt{h^2 - 4Qh}}{2}, \quad m_4 = \frac{h + \sqrt{h^2 + 4h(n - Q)}}{2},$$

$$m_5 = \frac{1 + \sqrt{4M' + 1}}{2}, \quad m_6 = \frac{1 + \sqrt{1 + 4(M' + n)}}{2},$$

$$B_1 = \frac{Am_1S_c}{m_1^2 - (n + m_1 + k_r^2)S_c}, \quad B_2 = 1 - \frac{Am_1S_c}{m_1^2 - (n + m_1 + k_r^2)S_c},$$

$$B_3 = \frac{Am_3h}{m_3^2 + (Q - n - m_3)h}, \quad B_4 = 1 - \frac{Am_3h}{m_3^2 + (Q - n - m_3)h},$$

$$B_5 = \frac{G_r}{m_3^2 - m_3 - M}, \quad B_6 = \frac{G_m}{m_1^2 - m_1 - M},$$

$$B_7 = 1 + \frac{G_r}{m_3^2 - m_3 - M} + \frac{G_m}{m_1^2 - m_1 - M}, \quad B_8 = \frac{G_mB_1 + Am_1B_6}{m_1^2 - m_1 - (M' + n)},$$

$$B_9 = \frac{G_mB_2}{m_2^2 - m_2 - (M' + n)}, \quad B_{10} = \frac{G_rB_3 + Am_3B_5}{m_3^2 - m_3 - (M' + n)},$$

$$B_{11} = \frac{G_rB_4}{m_4^2 - m_4 - (M' + n)}, \quad B_{12} = \frac{Am_5B_7}{m_5^2 - m_5 - (M' + n)}.$$

Such that the velocity, temperature and concentration distributions can be expressed as:

$$\begin{aligned} u(y, t) = {} & (B_7e^{-m_5y} - B_5e^{-m_3y} - B_6e^{-m_1y}) \\ & + \varepsilon[(1 + B_8 + B_9 + B_{10} + B_{11} - B_{12})e^{-m_6y} \\ & - B_8e^{-m_1y} - B_9e^{-m_2y} - B_{10}e^{-m_3y} - B_{11}e^{-m_4y} + B_{12}e^{-m_5y}]e^{nt} \end{aligned} \quad (28)$$

$$\theta(y, t) = e^{-m_3y} + \varepsilon(B_3e^{-m_3y} + B_4e^{-m_4y})e^{nt} \quad (29)$$

$$\phi(y, t) = e^{-m_1y} + \varepsilon(B_1e^{-m_1y} + B_2e^{-m_2y})e^{nt}. \quad (30)$$

Skin-friction

Skin-friction coefficient (τ) at the plate is given by

$$\tau = \left[\frac{du}{dy}\right]_{y=0}. \tag{31}$$

Using equations (28) and (31), we obtain skin-friction as follows:

$$\begin{aligned}\tau = {} & (m_1 B_6 + m_3 B_5 - m_5 B_7) \\ & + \varepsilon e^{nt}[(m_1 B_8 + m_2 B_9 + m_3 B_{10} + m_4 B_{11} - m_5 B_{12}) \\ & - m_6(1 + B_8 + B_9 + B_{10} + B_{11} - B_{12})].\end{aligned} \tag{32}$$

Nusselt number

The rate of heat transfer coefficient (Nu) at the plate is given by

$$Nu = \left[\frac{d\theta}{dy}\right]_{y=0}. \tag{33}$$

Using equations (29) and (33), Nusselt number (Nu) is derived as:

$$Nu = -m_3 + \varepsilon(-m_3 B_3 - m_4 B_4)e^{nt}. \tag{34}$$

Sherwood number

The rate of mass transfer coefficient (Sh) at the plate is given by

$$Sh = \left[\frac{d\phi}{dy}\right]_{y=0}. \tag{35}$$

From equations (30) and (35), we obtain Sherwood number (Sh) as follows:

$$Sh = -m_1 + \varepsilon(-m_1 B_1 - m_2 B_2)e^{nt}. \tag{36}$$

Results and Discussion

In order to get a physical insight into the problem, some numerical computations are carried out for the non-dimensional velocity u, temperature θ, concentration ϕ, skin-friction, Nusselt number and Sherwood number in terms of the parameters P_r, S_c, M, Q, k, N_r, k_r, A, G_m and G_r, respectively. The values of Prandtl number are chosen such that for Air $(P_r = 0.71)$, electrolytic solution $(P_r = 1.00)$, water $(P_r = 7.0)$ and water at 4°C $(P_r = 11.4)$. The values of Schmidt number are chosen so that for hydrogen $(S_c = 0.22)$, water-vapour $(S_c = 0.60)$, Ammonia $(S_c = 0.78)$, methanol $(S_c = 1.0)$ and propyl benzene $(S_c = 2.62)$.

Figure 1-3 display the effects of Prandtl number (P_r), thermal radiation parameter (N_r) and heat source parameter (Q) on temperature distribution respectively. Increase in the Prandtl number (P_r) is observed to lead to decrease in temperature boundary layer while increase in the thermal radiation parameter (N_r) or heat source parameter (Q) results in an increase in the thermal boundary layer. Figures 4 and 5 depict the effect of the S_c (Schmidt number) and k_r chemical reaction rate constant on the species concentration. It is observed that an increase in Schmidt number S_c or chemical reaction rate constant k_r decrease in concentration and concentration boundary layer. Again, from Figures 1 and 4 we observe that suction parameter (A) has negligible effect on temperature and concentration fields.

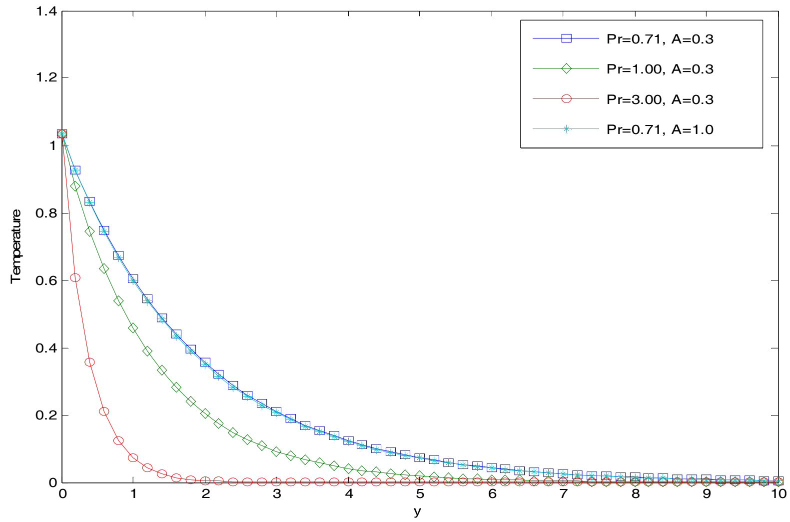

Figure 1. Temperature profiles for different P_r.

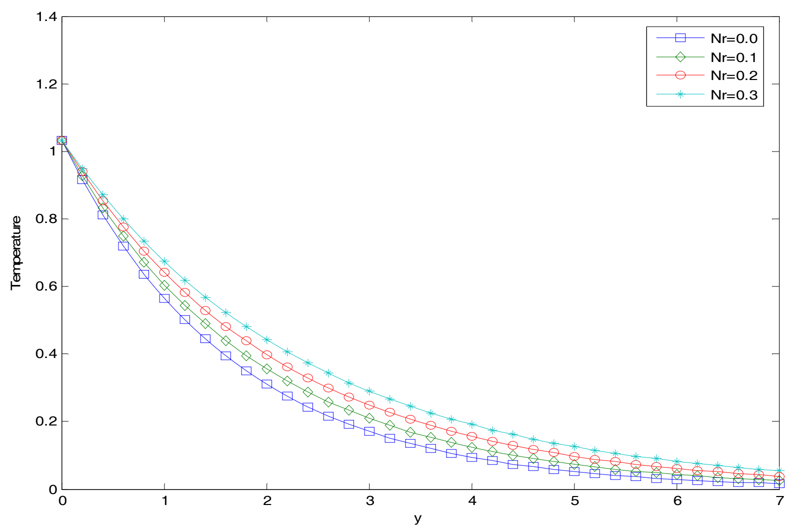

Figure 2. Temperature profiles for different N_r.

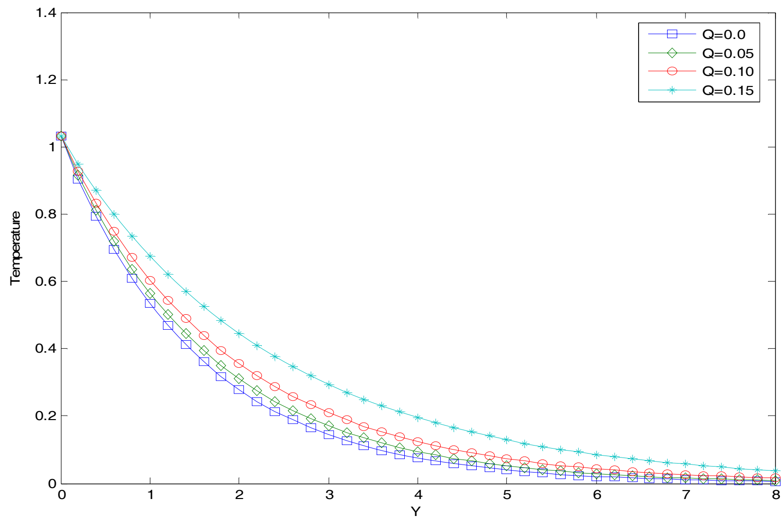

Figure 3. Effect of Q on Temperature field when $P_r = 0.71,$ $A = 0.3,$ $N_r = 0.1.$

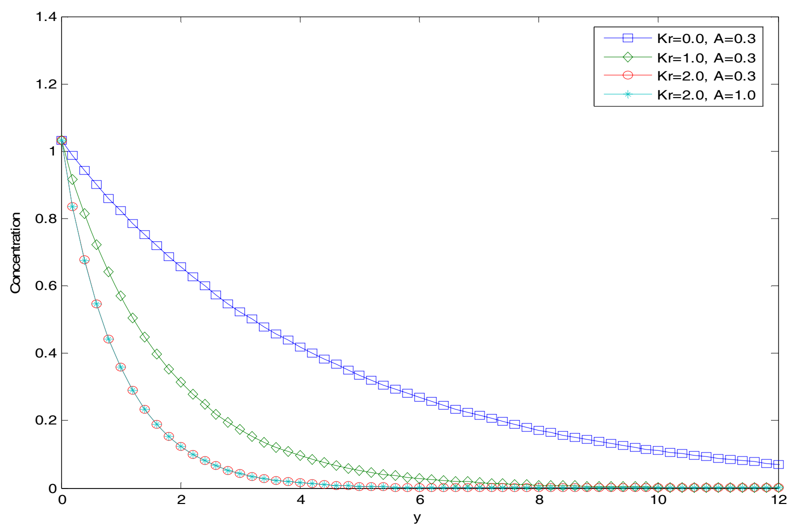

Figure 4. Concentration profiles.

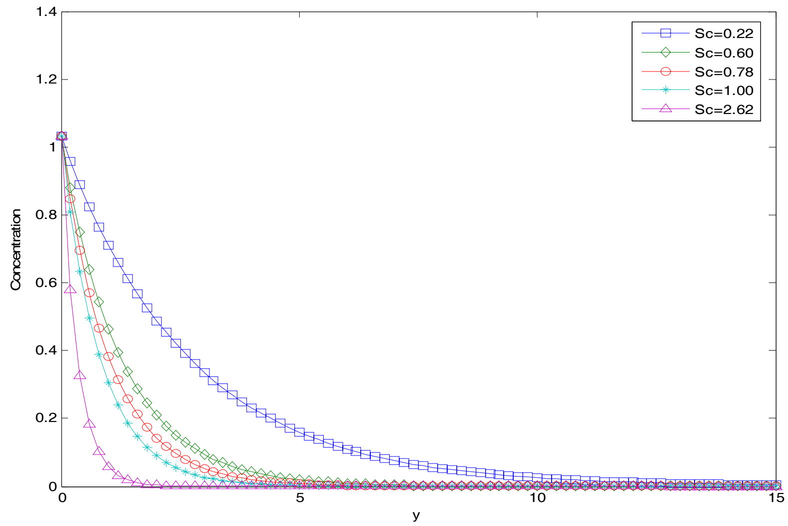

Figure 5. Concentration profiles.

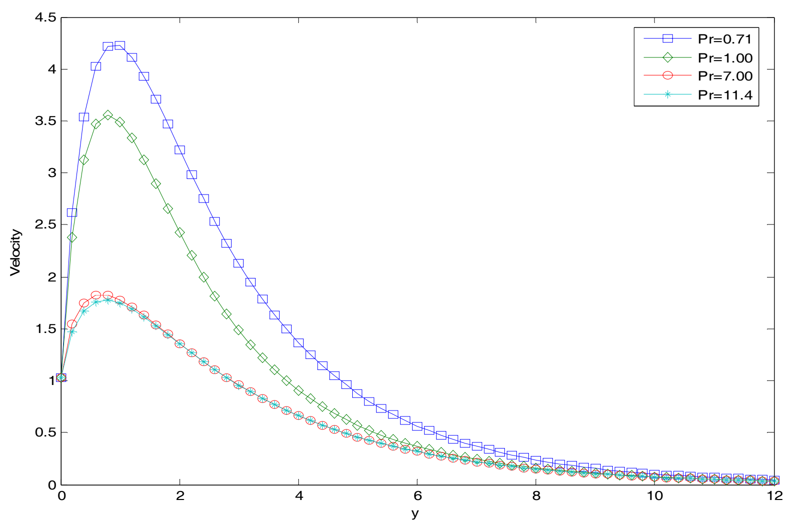

Figure 6. Effect of P_r on velocity field for cooling of the plate when $G_r = 10$, $G_m = 5$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

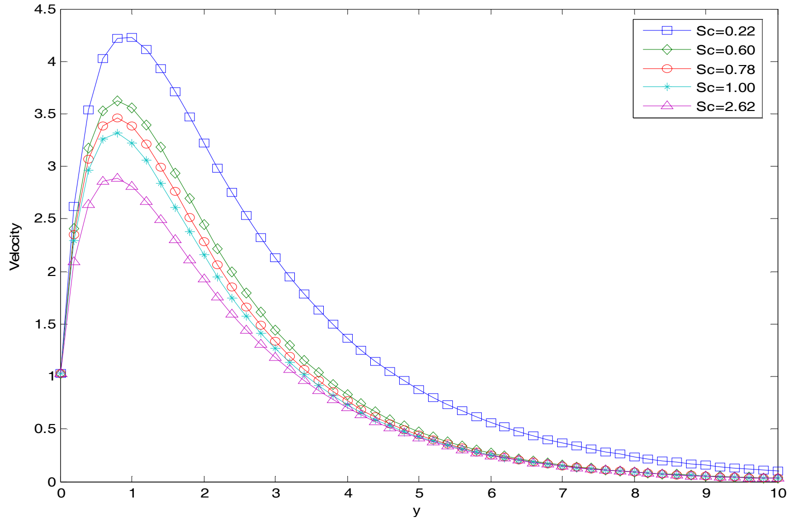

Figure 7. Effect of S_c on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

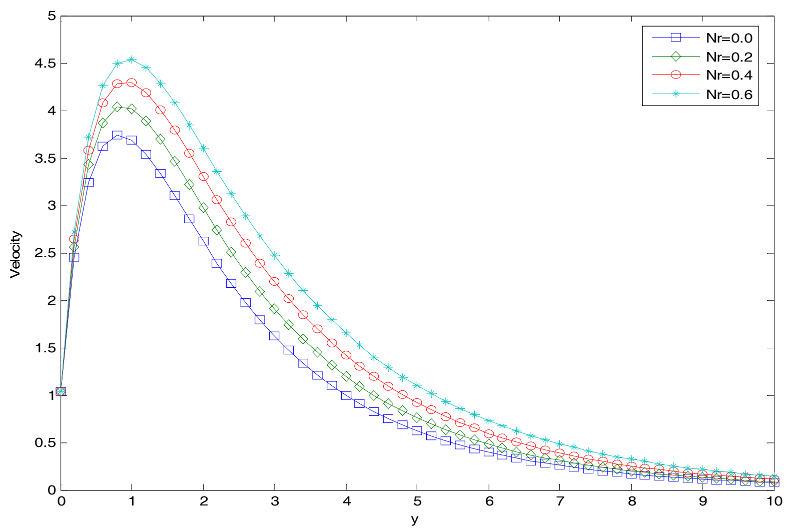

Figure 8. Effect of N_r on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

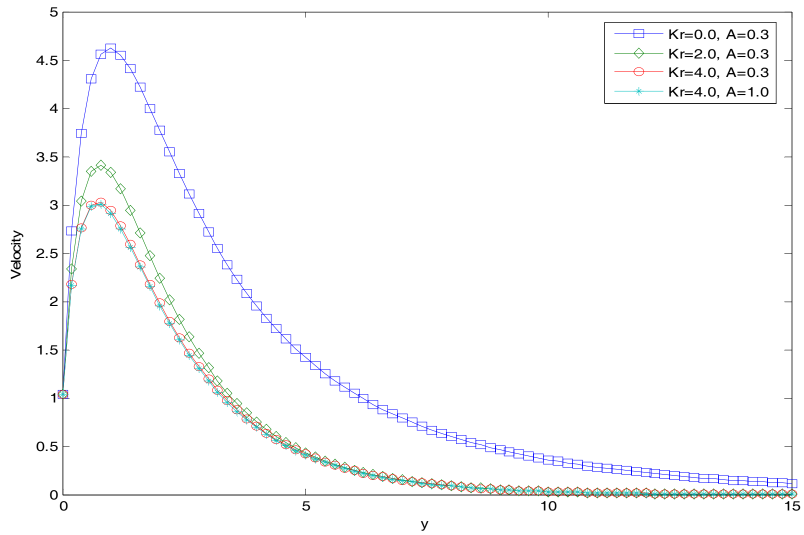

Figure 9. Effect of K_r on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

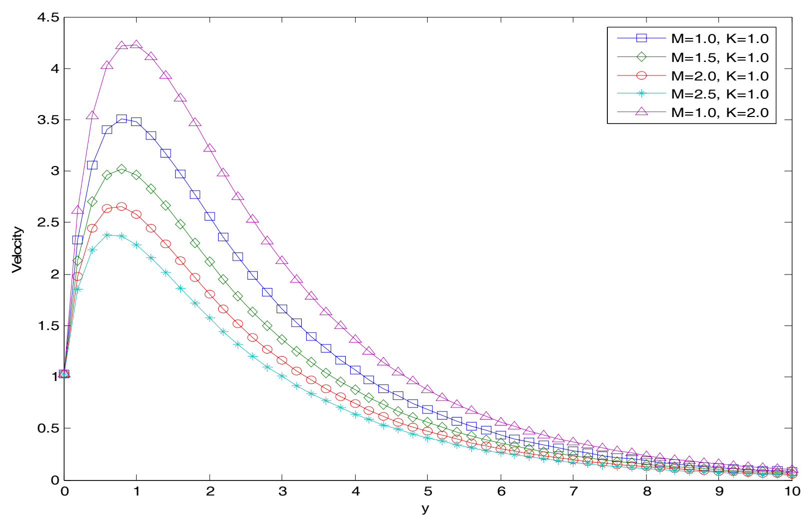

Figure 10. Effect of M and k on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$ and $t = 1.0$.

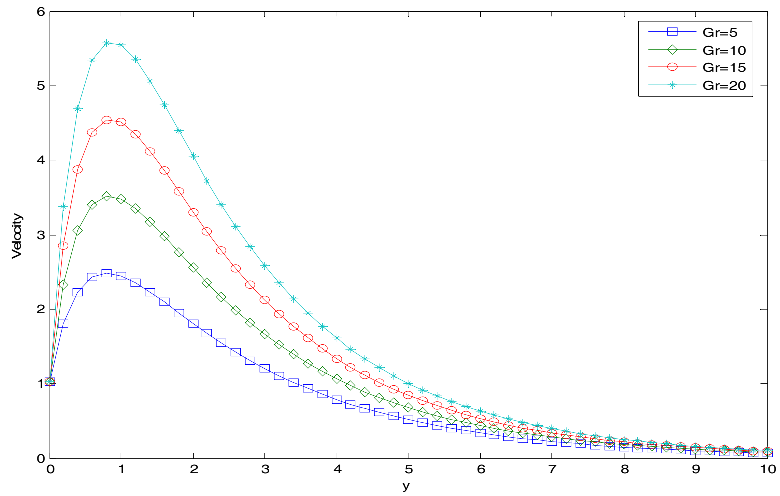

Figure 11. Effect of G_r on velocity field for cooling of the plate $Q = 0.1$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

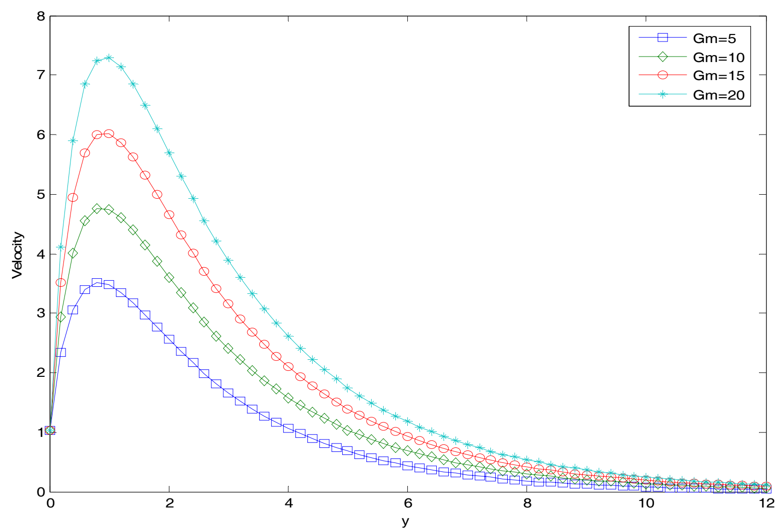

Figure 12. Effect of G_m on velocity field for cooling of the plate $G_r = 10$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

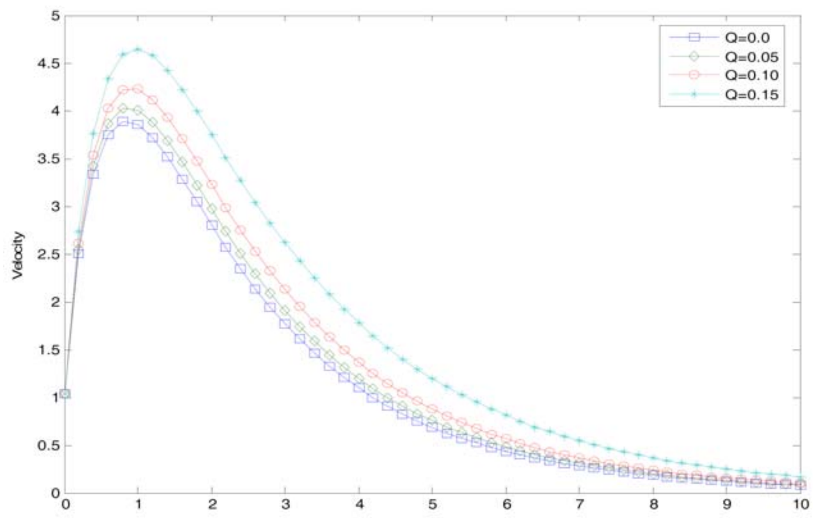

Figure 13. Effect of Q on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

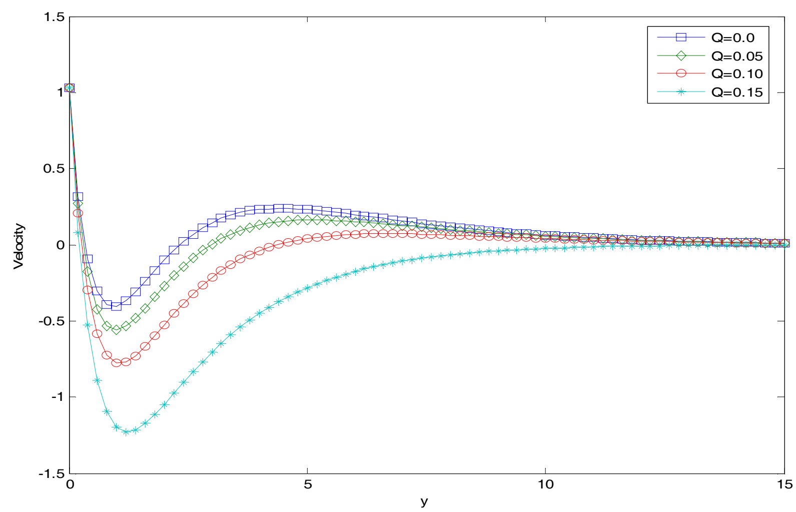

Figure 14. Effect of Q on velocity field for cooling of the plate $G_r = -10$, $G_m = -5$, $P_r = 0.71$, $S_c = 0.22$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

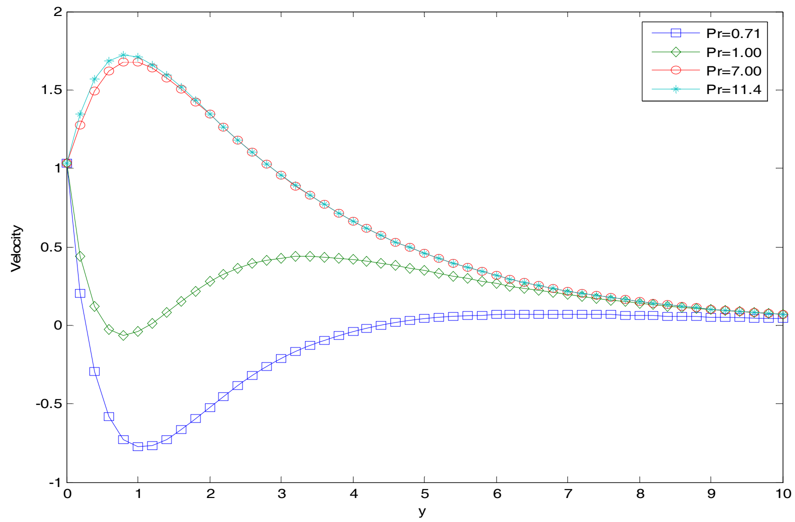

Figure 15. Effect of P_r on velocity field for cooling of the plate when $G_r = 10$, $G_m = 5$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

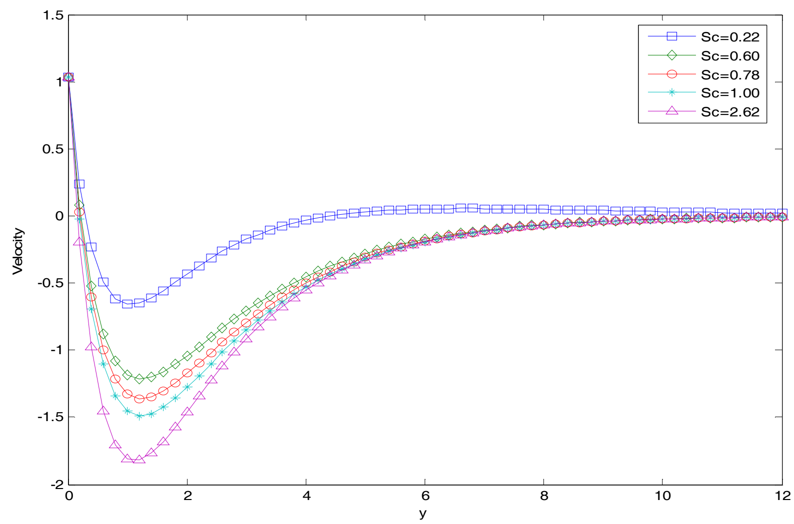

Figure 16. Effect of S_c on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

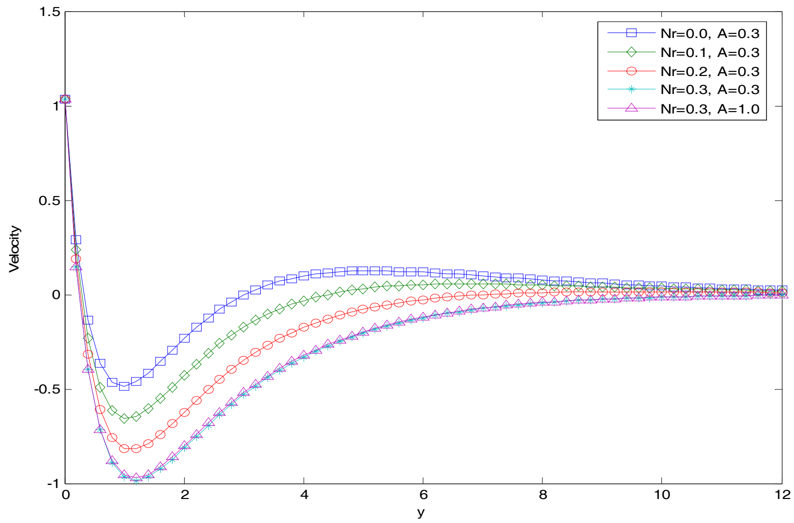

Figure 17. Effect of N_r on velocity field for cooling of the plate $G_r = -10$, $G_m = -5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

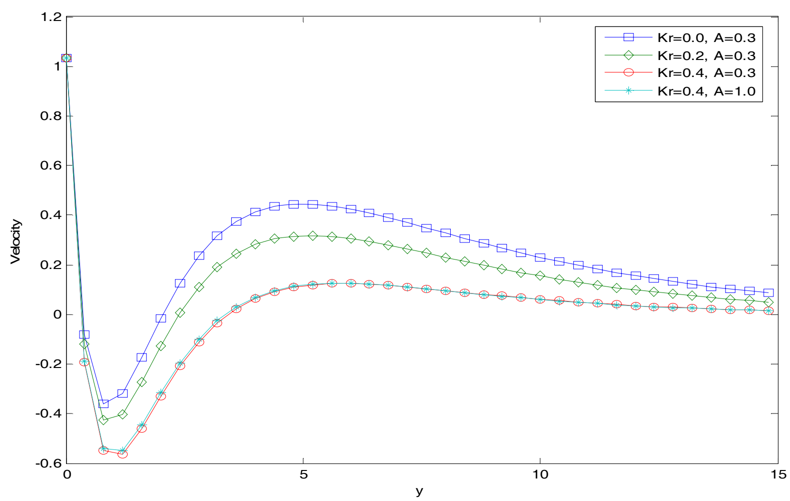

Figure 18. Effect of k_r on velocity field for cooling of the plate $G_r = 10$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $M = 0.5$, $k = 1.0$, $A = 0.3$ and $t = 1.0$.

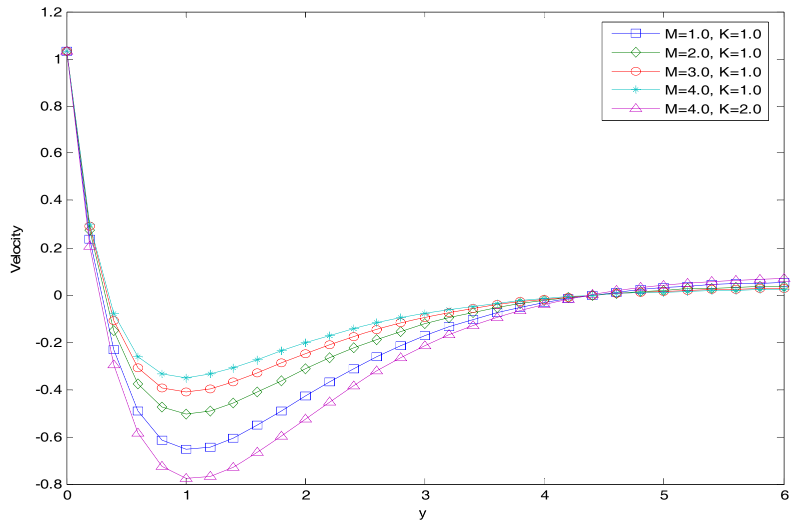

Figure 19. Effect of M and k on velocity field for cooling of the plate $G_r = -10$, $G_m = -5$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$ and $t = 1.0$.

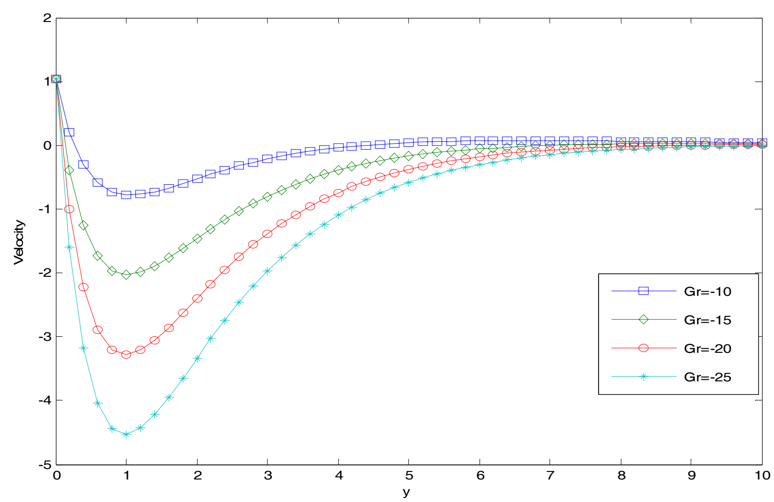

Figure 20. Effect of G_r on velocity field for cooling of the plate $Q = 0.1$, $G_m = 5$, $P_r = 0.71$, $S_c = 0.22$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

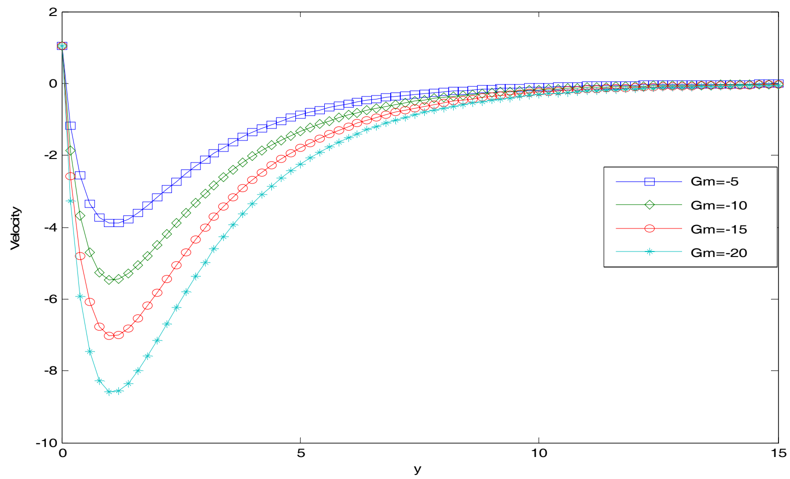

Figure 21. Effect of G_m on velocity field for cooling of the plate $G_r = 10$, $P_r = 0.71$, $S_c = 0.22$, $Q = 0.1$, $N_r = 0.1$, $k_r = 0.5$, $e = 0.02$, $n = 0.5$, $A = 0.3$, $M = 0.5$, $k = 1.0$ and $t = 1.0$.

Figure 6-13 show the effects of material parameters such as P_r, S_c, M, Q, k, N_r, k_r, A, G_r and G_m when the plate is cooled by free convection currents $(G_r > 0)$. It is observed that increase in Prandtl number or Schmidt number or rate of chemical reaction constant or Magnetic field parameter leads to reduction in the velocity field while increase in heat source parameter or thermal radiation or rate of chemical reaction or thermal Grashof number or mass Grashof number results in an increase in velocity. From Figures 6 and 9 it is also observed that with increase of suction parameter (A) leads to fall in velocity for cooling of the plate. Here it is interesting to note that, the velocity increases more rapidly near the plate and after reaches a maximum value it decreases moving away from the plate. Figures 14-21 describe that the effects of material parameters P_r, S_c, M, Q, k, N_r, k_r, A, G_r and G_m when the plate is heated by free-convection currents $(G_r < 0)$. It is observed that with increase of Prandtl number or magnetic field parameter or Thermal Grashof number or mass Grashof number the velocity increases whereas it decreases with increasing values of

Schmidt number or thermal radiation or rate of chemical reaction or permeability parameter. It is also observed, from Figures 17-18 that with increase of suction parameter 'A' leads to increase in the velocity field.

Table 1. Nusselt number

P_r	Q	N_r	**Nusselt Number**
0.71	0.1	0.2	–0.49642244545178
3.00	0.1	0.2	–2.51121892102162
0.71	0.05	0.2	–0.56965445484419
0.71	0.1	0.1	–0.55639311292160

Table 2. Sherwood number

S_c	k_r	**Sherwood number**
0.22	0.5	–0.38772188787622
0.60	0.5	–0.82784945916124
0.22	1.0	–0.61588866696403

Table 3. Skin-friction for cooling of the plate $(G_r > 0)$

P_r	S_c	Q	N_r	k_r	M	k	G_r	G_m	**Skin-friction**
0.71	0.22	0.05	0.1	0.5	0.5	1.0	10.0	5.0	9.65998661811761
7.00	0.22	0.05	0.1	0.5	0.5	1.0	10.0	5.0	3.84933152373352
0.71	0.60	0.05	0.1	0.5	0.5	1.0	10.0	5.0	8.53151490099981
0.71	0.22	0.1	0.1	0.5	0.5	1.0	10.0	5.0	10.02664843725360
0.71	0.22	0.05	0.2	0.5	0.5	1.0	10.0	5.0	9.94976239205124
0.71	0.22	0.05	0.1	1.0	0.5	1.0	10.0	5.0	8.98670622916125
0.71	0.22	0.05	0.1	0.5	1.0	1.0	10.0	5.0	8.12274557119483
0.71	0.22	0.05	0.1	0.5	0.5	2.0	10.0	5.0	11.97394202414897
0.71	0.22	0.05	0.1	0.5	0.5	1.0	20.0	5.0	16.90462585698587
0.71	0.22	0.05	0.1	0.5	0.5	1.0	10.0	10.0	13.97054648726008

Table 4. Skin-friction for heating of the plate $(G_r < 0)$

P_r	S_c	$\boldsymbol{Q}$	N_r	k_r	$\boldsymbol{M}$	$\boldsymbol{k}$	G_r	G_m	**Skin-friction**
0.71	0.22	0.05	0.1	0.5	0.5	1.0	–10.0	5.0	–4.82929185961889
7.00	0.22	0.05	0.1	0.5	0.5	1.0	–10.0	5.0	0.98136323476520
0.71	0.60	0.05	0.1	0.5	0.5	1.0	–10.0	5.0	–5.95776357673670
0.71	0.22	0.10	0.1	0.5	0.5	1.0	–10.0	5.0	–5.19595367875488
0.71	0.22	0.05	0.2	0.5	0.5	1.0	–10.0	5.0	–5.11906763355253
0.71	0.22	0.05	0.1	1.0	0.5	1.0	–10.0	5.0	–5.50257224857526
0.71	0.22	0.05	0.1	0.5	1.0	1.0	–10.0	5.0	–4.76522073537861
0.71	0.22	0.05	0.1	0.5	0.5	2.0	–10.0	5.0	–4.94774424766655
0.71	0.22	0.05	0.1	0.5	0.5	1.0	–20.0	5.0	–12.07393109848715
0.71	0.22	0.05	0.1	0.5	0.5	1.0	–10.0	10.0	–0.51873199047643

Nusselt number (*Nu*), which measures the rate of heat transfer at the plate $y = 0,$ is shown in Table 1 for different values of Prandtl number $P_r,$ Heat source parameter *H* and thermal radiation N_r respectively. It is found that an increase in the Prandtl number (P_r) leads to decrease in the rate of heat transfer while it increases as heat source parameter (Q) or thermal radiation (N_r) increases. Sherwood number (Sh), which measures the rate of mass transfer at the plate $y = 0,$ is shown in Table 2 for different values of Schmidt number and chemical reaction rate constant, respectively. It is observed that Sherwood number decreases with increasing values of Schmidt number *Sc* or chemical reaction rate constant k_r. The numerical values of skin-friction are presented in Tables 3 and 4 for the both cases of the plate is cooled $(G_r > 0)$ and the plate is heated $(G_r < 0)$ respectively. It is observed that, an increase in M, P_r, S_c and k_r leads to decrease in the value of skin-friction while it increases with increase of Q, k, N_r, G_r and G_m for $G_r > 0$

(cooling of the plate). For $G_r < 0$ (heating of the plate), skin-friction is observed to increase with increase of P_r, M, G_r and G_m while it decreases as S_c, Q, N_r, k_r and k increases. These observations show good agreement with the results of Shankar et al. [10] and Prakash and Ogulu [8].

Conclusions

In this paper, we have studied, heat source and thermal radiation effects on unsteady MHD two dimensional, laminar, boundary layer flow of viscous, incompressible, electrically conducting fluid past a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects. A perturbation technique is employed to solve the resulting coupled partial differential equations. The obtained results were compared with previous work Shankar et al. [10] and were found to be good in agreement. It is observed that an increase in Schmidt number or chemical reaction rate constant decrease in concentration and concentration boundary layer. An Increase in the Prandtl number is observed to lead to decrease in temperature boundary layer while increase in the thermal radiation parameter or heat source parameter results in a increase in the thermal boundary layer. And, it is also found that, the suction parameter has negligible effect on temperature and concentration fields. It is observed that increase in Prandtl number or Schmidt number or rate of chemical reaction constant or magnetic field parameter leads to reduction in the velocity field while increase in heat source parameter or thermal radiation or rate of chemical reaction or thermal Grashof number or mass Grashof number results in an increase in velocity for cooling of the plate. And with increase of Prandtl number or magnetic field parameter or thermal Grashof number or mass Grashof number the velocity increases whereas it decreases with increasing values of Schmidt number or thermal radiation or rate of chemical reaction or permeability parameter for heating of the plate.

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