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STUDY OF TWO MODELS FOR A CONTINUOUS BEAM REINFORCED CONCRETE FOR THREE DIFFERENT SECTIONS

Arnulfo Luévanos Rojas, Alberto Diosdado Salazar, Facundo Cortés Martinez, Ramón Luévanos Rojas and Armando Cesar Uranga Sifuentes

Facultad de Ingeniería, Ciencias y Arquitectura
Universidad Juárez del Estado de Durango
Av. Universidad S/N, Fracc. Filadelfia, CP 35010
Gómez Palacio, Durango, México
e-mail: arnulfol_2007@hotmail.com

Abstract

In this paper, we develop a comparative analysis using the method of slope-deflection of statically indeterminate beams. The first model is the neglect shear deformations (considering only the flexure deformation), this is the traditional method analysis for continuous beams and the second model is to consider shear deformations (taking into account flexure deformations and shear). For both the models, an analysis is made of three different sections of concrete. The first is a rectangular section, the second is type "I" with the larger web thickness and the third is an also "I" with the thinner web thickness. Each section is analyzed for three different lights, 10.00m, 5.00m and 3.00m. This paper shows the differences between the two models of the problems considered. According to the results obtained, it is shown

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that there is a greater difference in section “I” with the thinner web thickness and in the light of 3.00m. Therefore, the usual practice of not considering the shear deformations is not a recommended solution especially in short beams and sections “I” with the thinner web thickness. Then it proposes the use of taking into account shear deformations and also more attached to reality.

Introduction

Structural analysis is the study of structures such as discrete systems. The theory of the structures is essentially based on the fundamentals of mechanics with which the different structural elements are formulated. The laws or rules that define the balance and continuity of a structure can be expressed in different ways, including partial differential equations of continuous medium three-dimensional, ordinary differential equations that define a member or the various theories of beams, or simply algebraic equations for a discretized structure. The more one delves into the physics of the problem; they are developing theories that are most appropriate for solving certain types of structures that prove more useful for practical calculations. However, in each new theory, hypotheses are made about how the system behaves on member. Therefore, we must always be aware of these hypotheses when evaluating results, the result of applying or developing theories [1].

The analysis of structural systems has been studied by diverse researchers in the past. In 1826, L. M. Navier (1785-1836) published a treatise on the elastic behavior of structures, which is considered as the first textbook on the modern theory of strength of materials. The development of structural mechanics continued at a tremendous step for the rest of the nineteenth century to the mid-twentieth century when it developed most of the classical methods for the analysis of the structures described in this text. The important collaborators of this period included Benoit Paul Emile Clapeyron (1799-1864) who formulated the equation of three moments for the analysis of continuous beams; James Clerk Maxwell (1831-1879) who introduced the method of consistent deformations and the law of the deflections and Mohr circles of stress and unitary deformation; Alberto Castigliano (1847-1884) who formulated the theorem of minimum work; C.

E. Grene (1842-1903) who developed the method of moment-area; Heinrich Müller-Breslau (1851-1925) who presented a principle for the construction of influence lines; G. A. Maney (1888-1947) who developed the method of slope-deflection, which is considered as the precursor of the material for the stiffness method, and Hardy Cross (1885-1959) who developed the method of moment distribution in 1924. The distribution method of moments gives engineers a simple iterative procedure for the analysis of statically indeterminate structures.

The advent of computers in the 1970s revolutionized structural analysis. Because the computer could solve large systems of simultaneous equations, the analysis that lasted and sometimes weeks in the era before the computer, could now be done in seconds. The development of the present methods, aimed at the computer can be attributed, among others, Argyris, Clough, Kelsey, Livesley, Martin, Turner, Wilson and Zienkiewiez [2-4].

Luévanos Rojas developed the theory of slope-deflection method, including flexure deformations and shear.

This paper develops a comparison of the slope-deflection method of neglecting and considering the shear deformations for reinforced concrete continuous beams by analyzing three different sections, with three different lights to see their differences.

Algorithm of the Slope-deflection Method

This method can be used for the analysis of all types of statically indeterminate beams. It is considered that all joints are rigid, i.e., the angles between members on the boards do not change in value when applied load. Then the joints in interior supports of statically indeterminate beams can be considered rigid joints of 180°. When the beams are deformed, the joints rigid are considered rotations, i.e., the tangent remains straight before and after application of the load.

Another consideration is equilibrium in the joints; the sum of the moments must be zero.

In the slope-deflection equations, the moments acting at the ends of the members are expressed in terms of rotations and the loads on members.

Counter-clockwise the end moments are considered positive, and clockwise the end moments are considered negative. Now, with loads applied to the member, the end moments are considered as fixed-end moments [6-12].

The slope-deflection equations, neglecting shear deformations (Model 1) are:

$$M_{AB} = M_{FAB} + \frac{EI}{L}[-4\theta_A - 2\theta_B], \tag{1}$$

$$M_{BA} = M_{FBA} + \frac{EI}{L}[-4\theta_B - 2\theta_A], \tag{2}$$

where

$M_{AB} =$ end moments in the joint A,

$M_{FAB} =$ fixed-end moments at the ends of the beams in the joint A,

E = elasticity modulus,

I = moment of inertia of the section,

$\theta_A =$ rotation in the joint A,

L = length of beam.

The slope-deflection equations, considering the shear deformations (Model 2) are [5] and [13]:

$$M_{AB} = M_{FAB} + \frac{EI}{L}\left[-\left(\frac{4+\phi}{1+\phi}\right)\theta_A - \left(\frac{2-\phi}{1+\phi}\right)\theta_B\right], \tag{3}$$

$$M_{BA} = M_{FBA} + \frac{EI}{L}\left[-\left(\frac{4+\phi}{1+\phi}\right)\theta_B - \left(\frac{2-\phi}{1+\phi}\right)\theta_A\right], \tag{4}$$

where

$$\phi = \frac{12EI}{GA_c L^2}, \tag{5}$$

where

$\phi =$ form factor,

$G =$ shear modulus,

$A_c =$ shear area of the cross-section.

G is obtained as follows:

$$G = \frac{E}{2(1+\nu)}, \tag{6}$$

where $\nu =$ Poisson's ratio.

The procedure of analysis for statically indeterminate beams by the slope-deflection method is as follows (see Figure 1):

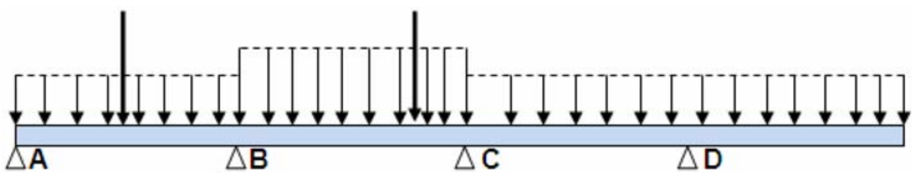
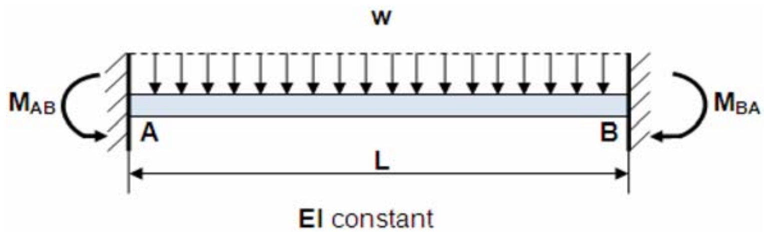


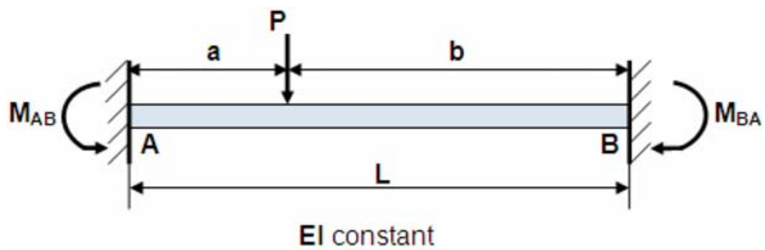
Figure 1. Typical continuous beam.

1. Determine the fixed-end moments at the ends of each span, using the formulas as shown in Figure 2.
2. Express all end moments in terms of the fixed-end moments and the joints rotations by using the slope-deflection equations.
3. Establish simultaneous equations with the rotations at the supports as unknowns by applying the conditions that the sum of the end moments acting on the ends of the two members meeting at the support should be zero.
4. Solve for the rotations at all supports.
5. Substitute the rotations back into the slope-deflection equations, and compute the end moments.
6. Determine all reactions, draw shear and moment diagrams, and sketch the elastic curve.



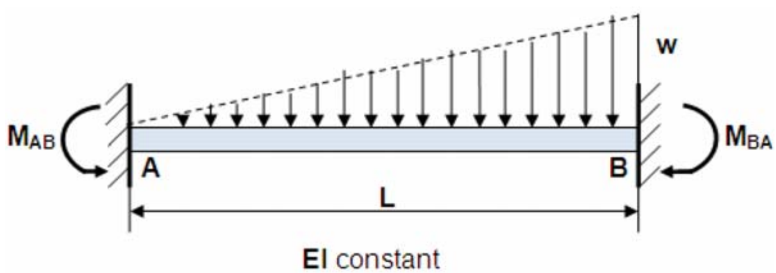
$$M_{FAB} = +\frac{wL^2}{12} \qquad M_{FBA} = -\frac{wL^2}{12}$$

(a)



$$M_{FAB} = +\frac{Pab^2}{L^2} \qquad M_{FBA} = -\frac{Pa^2b}{L^2}$$

(b)



$$M_{FAB} = +\frac{wL^2}{30} \qquad M_{FBA} = -\frac{wL^2}{20}$$

(c)

Figure 2. Fixed-end moments: (a) Uniformly distributed load, (b) Concentrated load and (c) Triangular distributed load.

Application

Developing the following structural analysis of concrete beam for three different sections each for three different lights, as shown in Figure 3, neglecting and considering the shear deformations, based on the following data:

General information:

$w = 3500\,\text{kg/m}$

$F_c' = 210\,\text{kg/cm}^2$

$F_y = 4200\text{kg/cm}^2$

$E = 15000(F_c')^{1/2}$

$\nu = 0.17$

where

$F_c' =$ concrete strength in 28 days

$F_y =$ yield strength of steel.

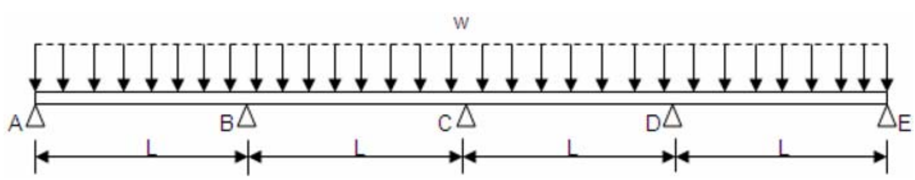


Figure 3. Continuous beam on four equal lengths with uniformly distributed load.

Type 1. Rectangular cross-section

Specific data:

$L = 10.00\text{m};\ 5.00\text{m};\ 3.00\text{m}.$

Beam properties 25×60

$A = 1500\text{cm}^2$

$A_c = 0.85A = 1275\,\text{cm}^2$

$I = 450000\text{cm}^4.$

Unknowns: θ_A, θ_B, θ_C, θ_D and θ_E.

The shear modulus is obtained by equation (6) as follows:

$$G = \frac{217370.65}{2(1+0.17)} = 92893.44\,\text{kg/cm}^2.$$

Once the shear modulus is obtained, the form factor is found by equation (5).

For $L = 10.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(450000)}{(92893.44)(1275)(1000)^2} = 0.009910588254.$$

For $L = 5.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(450000)}{(92893.44)(1275)(500)^2} = 0.03964235301.$$

For $L = 3.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(450000)}{(92893.44)(1275)(300)^2} = 0.1101176473.$$

The fixed-end moments for beams with uniformly distributed load, are found by the equation that appears in Figure 2(a):

For $L = 10.00\text{m}$:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12}$$

$$= +\frac{(3500)(10.00)^2}{12} = +29166.67\,\text{kg-m},$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12}$$

$$= -\frac{(3500)(10.00)^2}{12} = -29166.67\,\text{kg-m.}$$

For $L = 5.00\text{m}$:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12}$$

$$= +\frac{(3500)(5.00)^2}{12} = +7291.67\,\text{kg-m,}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12}$$

$$= -\frac{(3500)(5.00)^2}{12} = -7291.67\,\text{kg-m.}$$

For $L = 3.00\text{m}$:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12}$$

$$= +\frac{(3500)(3.00)^2}{12} = +2625\,\text{kg-m,}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12}$$

$$= -\frac{(3500)(3.00)^2}{12} = -2625\,\text{kg-m.}$$

Calculating “*EI*” for all beams as:

$$EI = (217370.65)(450000) = 97816792500\,\text{kg-cm}^2$$

$$= 9781679.25\,\text{kg-m}^2.$$

Substituting these values for each beam: Model 1 in equations (1) and (2), while for Model 2 in equations (3) and (4).

Once the moments are obtained in each beam as a function of rotations, the condition equilibrium is applied for moments at the joints, which are:

Joint A:

$$M_{AB} = 0. \tag{I}$$

Joint B:

$$M_{BA} + M_{BC} = 0. \tag{II}$$

Joint C:

$$M_{CB} + M_{CD} = 0. \tag{III}$$

Joint D:

$$M_{DC} + M_{DE} = 0. \tag{IV}$$

Joint E:

$$M_{ED} = 0. \tag{V}$$

These equations are presented in terms of rotations, and in this case, there are 5 equations and 5 rotations (unknowns), these are developed to find their values. Subsequently, they are substituted into the slope-deflection equations to localize the final moments at the ends of the beams. Now, by static equilibrium, the shear forces are obtained for each beam. Then the diagrams of shear forces and moments are obtained.

Below there are Tables 1, 2 and 3 with the results.

Table 1. The rotations in each one of the joints in radians

Rotations	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD
θ_A x10^4	+85.19	+85.44	0.9971	+10.65	+10.87	0.9798	+2.30	+2.43	0.9465
θ_B x10^4	−21.30	−21.25	1.0024	−2.66	−2.64	1.0076	−0.58	−0.56	1.0357
θ_C x10^4	0	0	0	0	0	0	0	0	0
θ_D x10^4	+21.30	+21.25	1.0024	+2.66	+2.64	1.0076	+0.58	+0.56	1.0357
θ_E x10^4	−85.19	−85.44	0.9971	−10.65	−10.87	0.9798	−2.30	−2.43	0.9465

θ_i = the angle that forms the tangent due to the deformation in the joint *i*

NSD = neglecting the shear deformations

CSD = considering the shear deformations

Table 2. The shear forces in kg

Shear forces	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD
V_{AB}	+13750	+13758	0.9994	+6875	+6891	0.9977	+4125	+4150	0.9940
V_{BA}	−21250	−21242	1.0004	−10625	−10609	1.0015	−6375	−6350	1.0039
V_{BC}	+18750	+18735	1.0008	+9375	+9346	1.0031	+5625	+5578	1.0084
V_{CB}	−16250	−16265	0.9991	−8125	−8154	0.9964	−4875	−4922	0.9904
V_{CD}	+16250	+16265	0.9991	+8125	+8154	0.9964	+4875	+4922	0.9904
V_{DC}	−18750	−18735	1.0008	−9375	−9346	1.0031	−5625	−5578	1.0084
V_{DE}	+21250	+21242	1.0004	+10625	+10609	1.0015	+6375	+6350	1.0039
V_{ED}	−13750	−13758	0.9994	−6875	−6891	0.9977	−4125	−4150	0.9940

V_{ij} = shear forces of the beam *ij* in end *i*

V_{ji} = shear forces of the beam *ji* in end *j*

Table 3. The moments in kg-m

Moments	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD	NSD	CSD	NSD/CSD
M_{AB}	0	0	0	0	0	0	0	0	0
$M¢_{AB}$	+27009	+27040	0.9989	+6752	+6783	0.9954	+2431	+2460	0.9882
M_{BA}	−37500	−37421	1.0021	−9375	−9297	1.0084	−3375	−3300	1.0227
M_{BC}	−37500	−37421	1.0021	−9375	−9297	1.0084	−3375	−3300	1.0227
$M¢_{BC}$	+12723	+12722	1.0001	+3181	+3180	0.0000	+1145	+1146	0.9991
M_{CB}	−25000	−25070	0.9972	−6250	−6318	0.9892	−2250	−2315	0.9719
M_{CD}	−25000	−25070	0.9972	−6250	−6318	0.9892	−2250	−2315	0.9719
$M¢_{CD}$	+12723	+12722	1.0001	+3181	+3180	0.0000	+1145	+1146	0.9991
M_{DC}	−37500	−37421	1.0021	−9375	−9297	1.0084	−3375	−3300	1.0227
M_{DE}	−37500	−37421	1.0021	−9375	−9297	1.0084	−3375	−3300	1.0227
$M¢_{DE}$	+27009	+27040	0.9989	+6752	+6783	0.9954	+2431	+2460	0.9882
M_{ED}	0	0	0	0	0	0	0	0	0

M_{ij} = negative moment of the beam *ij* in end *i*

$M₵_{AB}$ = positive moment of the beam *ij*

M_{BA} = Negative moment of the beam *ji* in end *j*

Type 2. Cross-section “I” with the larger web thickness (see Figure 4)

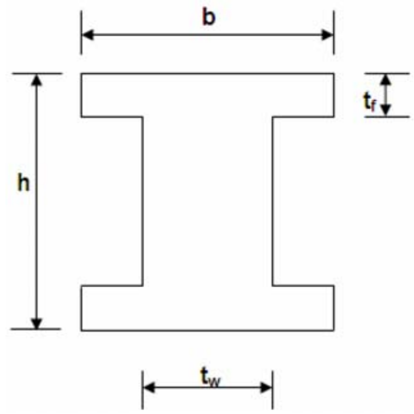


Figure 4. Cross-section type “I”.

Specific data:

$L = 10.00\text{m};\ 5.00\text{m};\ 3.00\text{m}$

$b = 30\text{cm}$

$h = 60\text{cm}$

$t_f = 15\text{cm}$

$t_w = 20\text{cm}$

$A = 1500\text{cm}^2$

$A_c = t_w(h - 2t_f) = 20[60 - 2(15)] = 600\text{cm}^2$

$$I = \frac{bh^3}{12} - \frac{(b - t_w)(h - 2t_f)^3}{12}$$

$$= \frac{(30)(60)^3}{12} - \frac{(30 - 20)[60 - 2(15)]^3}{12} = 517500\text{cm}^4.$$

Unknowns: θ_A, θ_B, θ_C, θ_D and θ_E.

The shear modulus is equal to the type 1:

$$G = 92893.44\,\text{Kg/cm}^2.$$

Once the shear modulus is obtained, the form factor is found by equation (5):

For $L = 10.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(517500)}{(92893.44)(600)(1000)^2} = 0.02421900004.$$

For $L = 5.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(517500)}{(92893.44)(600)(500)^2} = 0.09687600018.$$

For $L = 3.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(517500)}{(92893.44)(600)(300)^2} = 0.2691000005.$$

The fixed-end moments for beams with uniformly distributed load are equal to the type 1.

Calculating "*EI*" for all beams as:

$$EI = (217370.65)(517500) = 112489311400\,\text{kg-cm}^2$$

$$= 11248931.14\,\text{kg-m}^2.$$

Substituting these values for each beam: Model 1 in equations (1) and (2), while for Model 2 in equations (3) and (4).

Once the moments are obtained in each beam as a function of rotations, the condition equilibrium is applied for moments at the joints, the condition equilibrium of moments at the joints is equal to the type 1.

These equations are presented in terms of rotations, and in this case, there are 5 equations and 5 rotations (unknowns), these are developed to find their values. Subsequently, they are substituted into the slope-deflection equations to localize the final moments at the ends of the beams. Now, by static equilibrium, the shear forces are obtained for each beam. Then the diagrams of shear forces and moments are obtained.

Below are Tables 4, 5 and 6 with the results.

Table 4. The rotations in each one of the joints in radians

Rotations	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
θ_A x10^4	+74.08	+75.04	0.9872	+9.26	+9.73	0.9517	+2.00	+2.27	0.8811
θ_B x10^4	−18.52	−18.42	1.0054	−2.32	−2.26	1.0265	−0.50	−0.46	1.0870
θ_C x10^4	0	0	0	0	0	0	0	0	0
θ_D x10^4	+18.52	+18.42	1.0054	+2.32	+2.26	1.0265	+0.50	+0.46	1.0870
θ_E x10^4	−74.08	−75.04	0.9872	−9.26	−9.73	0.9517	−2.00	−2.27	0.8811

Table 5. The shear forces in kg

Shear forces	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
V_{AB}	+13750	+13769	0.9986	+6875	+6912	0.9946	+4125	+4182	0.9864
V_{BA}	−21250	−21231	1.0009	−10625	−10588	1.0035	−6375	−6318	1.0090
V_{BC}	+18750	+18714	1.0019	+9375	+9306	1.0074	+5625	+5522	1.0187
V_{CB}	−16250	−16286	0.9978	−8125	−8194	0.9916	−4875	−4978	0.9793
V_{CD}	+16250	+16286	0.9978	+8125	+8194	0.9916	+4875	+4978	0.9793
V_{DC}	−18750	−18714	1.0019	−9375	−9306	1.0074	−5625	−5522	1.0187
V_{DE}	+21250	+21231	1.0009	+10625	+10588	1.0035	+6375	+6318	1.0090
V_{ED}	−13750	−13769	0.9986	−6875	−6912	0.9946	−4125	−4182	0.9864

Table 6. The moments in kg-m

Moments	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
M_{AB}	0	0	0	0	0	0	0	0	0
$M¢_{AB}$	+27009	+27084	0.9972	+6752	+6825	0.9893	+2431	+2498	0.9732
M_{BA}	−37500	−37308	1.0051	−9375	−9190	1.0201	−3375	−3205	1.0530
M_{BC}	−37500	−37308	1.0051	−9375	−9190	1.0201	−3375	−3205	1.0530
$M¢_{BC}$	+12723	+12722	1.0001	+3181	+3182	0.9997	+1145	+1151	0.9948
M_{CB}	−25000	−25170	0.9932	−6250	−6410	0.9750	−2250	−2390	0.9414
M_{CD}	−25000	−25170	0.9932	−6250	−6410	0.9750	−2250	−2390	0.9414
$M¢_{CD}$	+12723	+12722	1.0001	+3181	+3182	0.9997	+1145	+1151	0.9948
M_{DC}	−37500	−37308	1.0051	−9375	−9190	1.0201	−3375	−3205	1.0530
M_{DE}	−37500	−37308	1.0051	−9375	−9190	1.0201	−3375	−3205	1.0530
$M¢_{DE}$	+27009	+27084	0.9972	+6752	+6825	0.9893	+2431	+2498	0.9732
M_{ED}	0	0	0	0	0	0	0	0	0

Type 3. Cross-section “I” with the thinner web thickness (see Figure 4)

Specific data:

L = 10.00m; 5.00m; 3.00m

$b = 30\text{cm}$

$h = 60\text{cm}$

$t_f = 15\text{cm}$

$t_w = 15\text{cm}$

$A = 1350\text{cm}^2$

$A_c = t_w(h - 2t_f) = 15[60 - 2(15)] = 450\text{cm}^2$

$$I = \frac{bh^3}{12} - \frac{(b - t_w)(h - 2t_f)^3}{12} = \frac{(30)(60)^3}{12} - \frac{(30 - 15)[60 - 2(15)]^3}{12}$$

$$= 506250\text{cm}^4.$$

Unknowns: θ_A, θ_B, θ_C, θ_D and θ_E.

The shear modulus is equal to the type 1:

$$G = 92893.44\,\text{kg/cm}^2.$$

Once the shear modulus is obtained, the form factor is found by equation (5):

For $L = 10.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(506250)}{(92893.44)(450)(1000)^2} = 0.03159000006.$$

For $L = 5.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(506250)}{(92893.44)(450)(500)^2} = 0.1263600002.$$

For $L = 3.00\text{m}$:

$$\phi_{AB} = \phi_{BC} = \phi_{CD} = \phi_{DE} = \frac{12(217370.65)(506250)}{(92893.44)(450)(300)^2} = 0.3510000006.$$

The fixed-end moments for beams with uniformly distributed load are equal to the type 1.

Calculate "*EI*" for all beams as:

$$EI = (217370.65)(506250) = 110043891600\,\text{kg-cm}^2$$

$$= 11004389.16\,\text{kg-m}^2.$$

Substituting these values for each beam: Model 1 in equations (1) and (2), while for Model 2 in equations (3) and (4).

Once the moments are obtained in each beam as a function of rotations, the condition equilibrium is applied for moments at the joints, the condition equilibrium of moments at the joints is equal to the type 1.

These equations are presented in terms of rotations, and in this case, there are 5 equations and 5 rotations (unknowns), these are developed to find their values. Subsequently, they are substituted into the slope-deflection equations to localize the final moments at the ends of the beams. Now, by static equilibrium, the shear forces are obtained for each beam. Then the diagrams of shear forces and moments are obtained.

Below there are Tables 7, 8 and 9 with the results:

Table 7. The rotations in each one of the joints in radians

Rotations	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$
θ_A x10^4	+75.73	+77.00	0.9835	+9.47	+10.08	0.9395	+2.04	+2.39	0.8536
θ_B x10^4	−18.93	−18.80	1.0069	−2.37	−2.29	1.0349	−0.51	−0.45	1.1333
θ_C x10^4	0	0	0	0	0	0	0	0	0
θ_D x10^4	+18.93	+18.80	1.0069	+2.37	+2.29	1.0349	+0.51	+0.46	1.1333
θ_E x10^4	−75.73	−77.00	0.9835	−9.47	−10.08	0.9395	−2.04	−2.27	0.8536

Table 8. The shear forces in kg

Shear forces	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
V_{AB}	+13750	+13775	0.9982	+6875	+6923	0.9931	+4125	+4196	0.9831
V_{BA}	−21250	−21225	1.0012	−10625	−10577	1.0045	−6375	−6304	1.0113
V_{BC}	+18750	+18703	1.0025	+9375	+9287	1.0095	+5625	+5496	1.0235
V_{CB}	−16250	−16297	0.9971	−8125	−8213	0.9893	−4875	−5004	0.9742
V_{CD}	+16250	+16297	0.9971	+8125	+8213	0.9893	+4875	+5004	0.9742
V_{DC}	−18750	−18703	1.0025	−9375	−9287	1.0095	−5625	−5496	1.0235
V_{DE}	+21250	+21225	1.0012	+10625	+10577	1.0045	+6375	+6304	1.0113
V_{ED}	−13750	−13775	0.9982	−6875	−6923	0.9931	−4125	−4196	0.9831

Table 9. The moments in kg-m

Moments	Case 1: L = 10.00m			Case 2: L = 5.00m			Case 3: L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
M_{AB}	0	0	0	0	0	0	0	0	0
M¢$_{AB}$	+27009	+27107	0.9964	+6752	+6846	0.9863	+2431	+2516	0.9662
M_{BA}	−37500	−37250	1.0067	−9375	−9137	1.0260	−3375	−3161	1.0677
M_{BC}	−37500	−37250	1.0067	−9375	−9137	1.0260	−3375	−3161	1.0677
M¢$_{BC}$	+12723	+12722	1.0001	+3181	+3183	0.9994	+1145	+1155	0.9913
M_{CB}	−25000	−25220	0.9913	−6250	−6454	0.9684	−2250	−2421	0.9294
M_{CD}	−25000	−25220	0.9913	−6250	−6454	0.9684	−2250	−2421	0.9294
M¢$_{CD}$	+12723	+12722	1.0001	+3181	+3183	0.9994	+1145	+1155	0.9913
M_{DC}	−37500	−37250	1.0067	−9375	−9137	1.0260	−3375	−3161	1.0677
M_{DE}	−37500	−37250	1.0067	−9375	−9137	1.0260	−3375	−3161	1.0677
M¢$_{DE}$	+27009	+27107	0.9964	+6752	+6846	0.9863	+2431	+2516	0.9662
M_{ED}	0	0	0	0	0	0	0	0	0

Results and Discussions

Tables 10, 11 and 12 show the differences between the two models of the 3 types of sections, for cases 3, as are the cases where the differences are major.

According to Table 10, which presents the rotations in each of the supports, it is observed that the difference is greater in Model 2, with respect to Model 1, in the type 3 and case 3. For example, differences exist in the support "A" of 17.16% are greater in Model 2 and in the support B are greater in Model 1 of 13.33%.

Table 10. The rotations in each one of the joints in the 3 types of cross-sections and case 3 in radians

Rotations	Type 1: Case 3			Type 2: Case 3			Type 3: Case 3		
	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$
$\theta_A \times 10^4$	+2.30	+2.43	0.9465	+2.00	+2.27	0.8811	+2.04	+2.39	0.8536
$\theta_B \times 10^4$	−0.58	−0.56	1.0357	−0.50	−0.46	1.0870	−0.51	−0.45	1.1333
$\theta_C \times 10^4$	0	0	0	0	0	0	0	0	0
$\theta_D \times 10^4$	+0.58	+0.56	1.0357	+0.50	+0.46	1.0870	+0.51	+0.46	1.1333
$\theta_E \times 10^4$	−2.30	−2.43	0.9465	−2.00	−2.27	0.8811	−2.04	−2.27	0.8536

With regard to Table 11, the shear forces are shown at the ends of the bars, the difference being greater in Model 2, with respect to Model 1, in the type 3 and case 3. For example, major differences are in shear forces, V_{BC} of 2.35% in Model 1 and V_{CB} is greater in Model 2 of 2.65%.

Table 11. The shear forces in each one of the joints in the 3 types of cross-sections and case 3 in kg

Shear forces	Type 1: Case 3			Type 2: Case 3			Type 3: Case 3		
	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$
V_{AB}	+4125	+4150	0.9940	+4125	+4182	0.9864	+4125	+4196	0.9831
V_{BA}	−6375	−6350	1.0039	−6375	−6318	1.0090	−6375	−6304	1.0113
V_{BC}	+5625	+5578	1.0084	+5625	+5522	1.0187	+5625	+5496	1.0235
V_{CB}	−4875	−4922	0.9904	−4875	−4978	0.9793	−4875	−5004	0.9742
V_{CD}	+4875	+4922	0.9904	+4875	+4978	0.9793	+4875	+5004	0.9742
V_{DC}	−5625	−5578	1.0084	−5625	−5522	1.0187	−5625	−5496	1.0235
V_{DE}	+6375	+6350	1.0039	+6375	+6318	1.0090	+6375	+6304	1.0113
V_{ED}	−4125	−4150	0.9940	−4125	−4182	0.9864	−4125	−4196	0.9831

In Table 12, illustrating the moments in bars, both negative and positive, the difference is greater in Model 2, with respect to Model 1 in the type 3 and case 3. For example, greater differences are in the moment, M_{CB} in 7.60% is greater in Model 2 and M_{BA} is greater in Model 1 of 6.77%.

Table 12. The moments in each one of the joints in the 3 types of cross-sections and case 3 in kg-m

Moments	Type 1: Case 3			Type 2: Case 3			Type 3: Case 3		
	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$
M_{AB}	0	0	0	0	0	0	0	0	0
$M¢_{AB}$	+2431	+2460	0.9882	+2431	+2498	0.9732	+2431	+2516	0.9662
M_{BA}	−3375	−3300	1.0227	−3375	−3205	1.0530	−3375	−3161	1.0677
M_{BC}	−3375	−3300	1.0227	−3375	−3205	1.0530	−3375	−3161	1.0677
$M¢_{BC}$	+1145	+1146	0.9991	+1145	+1151	0.9948	+1145	+1155	0.9913
M_{CB}	−2250	−2315	0.9719	−2250	−2390	0.9414	−2250	−2421	0.9294
M_{CD}	−2250	−2315	0.9719	−2250	−2390	0.9414	−2250	−2421	0.9294
$M¢_{CD}$	+1145	+1146	0.9991	+1145	+1151	0.9948	+1145	+1155	0.9913
M_{DC}	−3375	−3300	1.0227	−3375	−3205	1.0530	−3375	−3161	1.0677
M_{DE}	−3375	−3300	1.0227	−3375	−3205	1.0530	−3375	−3161	1.0677
$M¢_{DE}$	+2431	+2460	0.9882	+2431	+2498	0.9732	+2431	+2516	0.9662
M_{ED}	0	0	0	0	0	0	0	0	0

Conclusions

According to Table 10, which presents the rotations in each of the supports, it is observed that the difference in the slope-deflection method, neglecting and considering the shear deformations, is quite considerable when the light is reduced between supports of beams in cross-section “I” with the thinner web thickness, and all are not within the safety in the traditional method (Model 1). This implies that the deformations permitted by the rules of construction should be taken into account, because in some situations, it could be the case, that does not comply.

Table 11 shows the shear forces at the ends of the beams between the two models, the differences being larger, when the length between supports is reduced and considering cross-section “I” with the thinner web thickness.

With regard to Table 12, illustrating the moments, both negative and positive, there are big differences when we reduce the length between the two models with cross-section “I” with the thinner web thickness, and not all are on the side of safety.

As for Tables 11 and 12, where shear forces and moments are presented, acting on the beams, these elements are those governing the design of a structure, studied by Models 1 and 2. The results showed the differences between the two models, when members tend to be shorter and considering cross-section "I" with the thinner web thickness, the differences are increased, as in the conservative side as the unsafe side.

This means that this is poorly designed; on one hand, some members are bigger in their transverse dimension, according to what are needed, and in another situation, does not meet the minimal conditions for that a beam is well designed.

Therefore, the usual practice of using the slope-deflection method (neglecting shear deformations) is not a recommended solution when having short length between supports and considering cross-section "I" with web thickness.

So taking into account the numerical approximation, the slope-deflection method (considering shear deformations) happens to be the more appropriate method for structural analysis of continuous beams and also more attached to the real conditions.

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