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THE REVIEW OF EUCLIDEAN GEOMETRY AND THE MULTI-PURPOSE GRAPHICS

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Abstract

In this paper, from a basic graphics, a group of extended transformations of the Euclidean geometry of different types of equations and the proofs are given. These play an important role in the study, review and effective remember of Euclidean geometry, to play to win with fewer, a multiplier effect.

As the famous mathematics Professor George Pólya says: “An excellent teacher is one who can use a sample and meaningful subject to help students explore various aspects of the problem. We can introduce the students to the field of a complete theory through this example.” But most of the examples, exercises and the basic graphics are the prototype of the troubling problem which we have to deal with. Therefore, it is not wise for the teacher going by the book or doing as it is.

The wise approach is to value the concept, definition, property and more questions, change transformation and innovate the same kind of topic. The purpose is to broaden the students’ vision, not subject to negative and bounded thoughts with the problem changing and replacing. From the

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observation, association, guessing, thinking activities, the students will broaden their view, act thinking, deepen ideas and expand their ideas. With such a training method, it is helpful for the improvement and cultivation of the students' abilities. Extending and transforming the basic figure, then it will produce a set of exercises. Introducing students to prove them through the knowledge of the Euclidean geometry. This will be more effective and win with fewer.

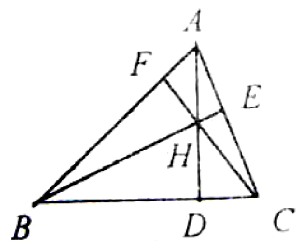

Figure 1

See Figure 1 in the acute angle ΔABC, $AB > AC$, AD, BE, CF are the heights. H is an orthocenter.

(I) The questions are as follows:

(1) How many triangles are there in the diagram?

(2) How many groups are there in the chart which are four points altogether with a circle?

(3) How many groups are there in the chart which are the similar triangles?

(4) How many orthocenters are there in the chart?

After discussing, we will have the enjoyable answers (16 triangles; 6 groups of four-point circles total; 3 groups of similar triangles, each set of 4, 4 vertical heart, that is, A, B, C and D). This will lay the foundation for the following problems.

(II) To verify that

(5) $AH \cdot HD = BH \cdot HE = CH \cdot HF$,

(6) $AE \cdot AC = AH \cdot AD = AF \cdot AB$,

(7) $BF \cdot BA + CE \cdot CA = BH \cdot BE + CH \cdot CF = BC^2$,

(8) $BC = AH \cdot tg\angle BAC$.

To lead the students to find the similar conclusion as

(III) To verify

(9) $AD \cdot BC = BE \cdot CA = CF \cdot AB$,

(10) $\dfrac{HB \cdot HC}{AB \cdot AC} + \dfrac{HA \cdot HC}{BA \cdot BC} + \dfrac{HA \cdot HB}{CA \cdot CB} = 1$,

(11) $\dfrac{HD}{AD} + \dfrac{HE}{BE} + \dfrac{HF}{CE} = 1$,

(12) $\dfrac{AH}{AD} + \dfrac{BH}{BE} + \dfrac{CH}{CF} = 2$,

(13) $\dfrac{AH}{HD} - \dfrac{AF}{FB} - \dfrac{AE}{EC} = 1$.

We can preview the round power theorem, the triangle area formula and related theorems by studying the proof of questions (II) and (III). Connecting DE, EF, FD in Figure 1, we will get a pedal of the triangle DEF , see Figure 2, therefore, we can get the following results.

(IV) To verify

(14) there are sixteen triangles in the figure,

(15) A, B, C are the side centres of ΔDEF,

(16) H is the inner centre of ΔDEF,

(17) $DB \cdot DC = DF \cdot DE = DH \cdot DA$,

(18) $\Delta AEF \backsim \Delta BDF \backsim \Delta CED \backsim \Delta ABC$.

The key idea of the proof: because there are thirteen triangles whose vertex is A (not including ΔABC). Similarly, there are thirteen triangles whose vertices are B and C, respectively. Thus, there are thirty-nine triangles in total. In ΔDEF, there are eight triangles whose vertex is D (not including ΔDEF). Similarly, there are thirteen triangles whose vertices are E and F, respectively. Therefore, there are twenty-four triangles in total. There are three triangles taking AB as one side, and three triangles taking BC and CA as one side, respectively. There are nine triangles in total. In ΔDEF, there are nine triangles which take sides as DE, EF and FD. On one hand, such triangles will be calculated repeatedly, so we have to minus them, on the other hand, we have to plus ΔABC and ΔDEF. So the total number of the triangles is $39 + 24 - 18 + 2 = 47$. Other conclusions are easy to prove. Also, we can draw more conclusions. For example,

(19) $S_{\Delta HBC} : S_{\Delta HAC} : S_{\Delta HAB} = \tan A : \tan B : \tan C$;

(20) $AB + CF > AC + BE$;

(21) $AB - AC < HB - HC$;

∗ (22) Extending three sides of ΔDEF and three sides of ΔABC, the three intersection points will be collinear;

∗ (23) All inscribed triangles of ΔABC, the shortest perimeter triangle is a pedal of ΔDEF.

The key point of the Proof (19) is:

$$\because \angle HBC = \angle HAC \therefore \frac{S_{\Delta HBC}}{S_{\Delta HAC}} = \frac{BH \cdot BC}{AH \cdot AC} = \frac{BC}{AH} : \frac{AC}{BH} = \frac{\tan A}{\tan B}.$$

Similarly, $\frac{S_{\Delta HCA}}{S_{\Delta HAB}} = \frac{\tan B}{\tan C}$.

The key point of the Proof (20) is as $AB > AC$, thus

$$AB \cdot (1 - \sin A) > AC \cdot (1 - \sin A),$$

and we can get the conclusion of the right-angle of trigonometric function. It is easy to prove (21) by using the Pythagorean Theorem. For (25) and (26), we need an auxiliary line.

To make a circumscribed circle $\odot o(R)$ of ΔABC in Figure 2, as in Figure 3, we will have the following conclusion:

(24) $AO \perp EF$;

(25) The symmetrical point *H* of *BC*, that is, H_1, lines on $\odot o(R)$;

(26) The distance between the point *O* and the straight line *BC* is equal to the half length of *AH*;

(27) $AH^2 + BC^2 = BH^2 + CA^2 = CH^2 AB^2 = 4R^2$;

(28) $\frac{AH}{\cos A} = \frac{BH}{\cos B} = \frac{CH}{\cos C} = 2R.$

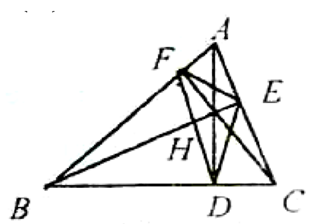

Figure 2

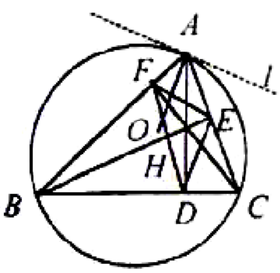

Figure 3

The key point of the proof is:

To (24), make the tangent line *l* through the point *A*, we only need to prove *EF*//*l*. After the proof by contradiction method, we can get (25), and (26) is a common problem. By the conclusion of (26) and the Pythagorean Theorem, (27) is proved.

To (28), as $\sin \angle ACD = \sin \angle AHE = \dfrac{AE}{AH}$ and $AE = AB \cdot \cos \angle BAC$, we have $AH = \dfrac{AB \cdot \cos A}{\sin C}$ and $\dfrac{AB}{\sin C} = 2R$, thus $\dfrac{AH}{\cos A} = 2R$.

Other conclusions can be proved by the same method. The above problems involving the aspect of knowledge are broad, at the same time, we can learn some basic ways to add line, make good use of the fundamental theorem, and learn new one by reviewing old one.

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