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A MATLAB® BASED DEMONSTRATION OF THE EFFECTS OF FADING, SHADOWING AND DIVERSITY IN WIRELESS COMMUNICATIONS FOR ELECTRICAL ENGINEERING STUDENTS

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Abstract

Programs in electrical engineering offer courses in wireless communications, networks and sensors. One of the complex topics in wireless is the phenomenon of fading which degrades the wireless channel and the related issue of fading mitigation to improve the channel conditions through diversity. Often, analytical expressions for the probability density functions of the signal-to-noise ratio are not available making it difficult for the students to understand fading and another coexisting phenomenon of shadowing and the need for diversity. Matlab based approaches can be utilized to supplement the lectures to demonstrate the effects of fading, shadowing and the advantages of diversity and facilitate a better pedagogic experience for the students. This manuscript reports on the use of two Matlab programs to implement diversity combining algorithms to mitigate fading and shadowing. These programs implement diversity techniques, generate the densities, distribution functions, measures of

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the enhancements in channel characteristics in terms of the average signal-to-noise ratio, amount of fading and the shifts of the peaks of the densities. Results illustrate the potential benefits to the EE students in fully grasping the concepts of fading, shadowing and diversity even when the analytical expressions for the density functions do not exist.

1. Introduction

Topics in wireless are now becoming a part and parcel of the electrical engineering curriculum [1-4]. Educators have reported the use of project based approaches along with simulation techniques to complement and supplement the class room teaching to reinforce the concepts. One of the difficult concepts in wireless involves the study of fading and fading mitigation techniques through the use of diversity [9]. The complexity of the concept is compounded by the fact that fading is often accompanied by shadowing. The primary reason for this is the fact that exploration of fading and shadowing involves probability density functions and manipulation of these to quantify the problems in wireless channels and subsequent improvements gained through diversity. Often the density function of the signal-to-noise ratio (SNR) following diversity is not analytically available and students only see the final quantitative measures such as the outage probabilities and error rates, the former defined as the probability that the SNR fails to reach a threshold required to achieve a certain level of performance defined by the latter, namely the error rates. A substantial level of understanding mitigation of fading and shadowing through diversity can be gained through the observations of the density functions and simple quantitative measures such as the peaks of the density functions, average SNR and the amount of fading [9]. Simulation techniques using Matlab® (The MathWorks, Natick, MA, 01760, USA) can make it possible to access the probability density functions and several quantitative measures of wireless channels and such an exploration is pursued in this work.

Following this introductory section, a brief overview of the fading and shadowing is presented. This is followed by a description of the three primary diversity combining algorithms, namely the selection combining

(SC), equal gain combining (EGC), and maximal ratio combining (MRC). Another diversity combining algorithm which utilizes both SC and MRC called the *generalized selection combining* (*GSC*) is also discussed [9]. While these algorithms mitigate the effects of fading, a hybrid scheme used to mitigate both fading and shadowing involving two or more base stations with MRC or SC at each base station followed by SC among the base stations (MRC-SC or SC-SC) is also described [8]. The implementation of the combining algorithms to mitigate fading alone and fading and shadowing jointly are then presented. The results of the simulation are described along with the quantitative measures of the level of fading before and after the implementation of fading. The section on conclusions discusses ways of incorporating such approaches in class rooms and expanding the methodology to include other fading conditions.

2. Fading, Shadowing and Diversity

Wireless channels suffer from the effects of multipath, the phenomenon in which the signal from the transmitter reaches the receiver after traversing through multiple paths. If the number of multipaths is sufficiently high, then the envelope will have Rayleigh distribution [7]. Such a Rayleigh model has been shown to be inadequate to describe multipath fading otherwise known as short term fading in all wireless environments. While several models exist, it has been shown that the Nakagami distribution which also includes Rayleigh distribution as a special case is a more appropriate model [9]. The probability density function (pdf) of the envelope, A, in a Nakagami fading channel is

$$f_A(a) = 2\left(\frac{m}{\Omega}\right)^m \frac{a^{2m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}a^2\right)U(a), \quad m > \frac{1}{2}. \tag{1}$$

In equation (1), m is the Nakagami parameter, often called the *fading parameter* and $\Gamma(\cdot)$ is the gamma function. The unit step function is indicated by $U(\cdot)$. The average power is given by Ω. Higher and higher values of m correspond to lower and lower levels of fading and lower values of m correspond to severe fading. The probability density function of the power P (square of the envelope) is given by

$$f_P(p) = \left(\frac{m}{\Omega}\right)^m \frac{p^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega} p\right) U(p), \quad m > \frac{1}{2}. \tag{2}$$

Since the power is linearly related to the signal-to-noise ratio (SNR), without any loss of generality, we can rewrite equation (2) in terms of the signal-to-noise ratio Z as

$$f_Z(z) = \left(\frac{m}{Z_0}\right)^m \frac{z^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{Z_0} z\right) U(z), \quad m > \frac{1}{2}. \tag{3}$$

In equation (3), Z_0 is the average SNR. We can also notice that equation (3) is also the gamma density function with order $m > 1/2$. If we now consider the effect of multiple scattering in addition to multipath effects, long term fading or shadowing takes place along with short term fading [5, 8]. The consequence of shadowing is that the average SNR itself becomes a random variable and equation (3) needs to be rewritten in terms of a conditioned random variable as

$$f_Z(z|x) = \left(\frac{m}{x}\right)^m \frac{z^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{x} z\right) U(z), \quad m > \frac{1}{2}. \tag{4}$$

In equation (4), the random variable X taking a value of x represents the average SNR in presence of shadowing. The density function of the average SNR, X, has been shown to have a lognormal distribution [9],

$$f_X(x) = \frac{K}{\sqrt{2\pi x^2 \sigma^2}} \exp\left[-\frac{(10\log_{10} x - \mu)^2}{2\sigma^2}\right] U(x). \tag{5}$$

In equation (5), the parameter K is given by

$$K = \frac{10}{\log_e 10}. \tag{6}$$

The quantity μ (dB) is the average SNR in a shadowed fading channel and σ represents the shadowing level (dB). Low values of σ represent weak shadowing while higher values in the range of 6-9 dB correspond to severe levels of shadowing. The pdf of the SNR in the shadowed fading channel now becomes

$$f(z) = \int_0^\infty f(z|x) f(x) dx$$

$$= \int_0^\infty \left(\frac{m}{x}\right)^m \frac{z^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{x} z\right) \frac{K}{\sqrt{2\pi x^2 \sigma^2}} \exp\left[-\frac{(10\log_{10} x - \mu)^2}{2\sigma^2}\right] dx. \quad (7)$$

Note that equation (7) is referred to as the Nakagami-lognormal model for shadowed fading channels. Since equation (7) has no analytical solution available, an approximate model has been shown to be reasonably valid which uses a gamma distribution in place of the lognormal pdf [8]. Using this approach, equation (7) becomes

$$f(z) = \int_0^\infty \left(\frac{m}{x}\right)^m \frac{z^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{x} z\right) \left[\frac{x^{\nu-1}}{\Gamma(\nu) y_0^c} \exp\left(-\frac{x}{y_0}\right)\right] dx. \quad (8)$$

The quantity in the square bracket in equation (8) is the gamma density function of order ν and parameter y_0 and this pdf can more than adequately represent a lognormal pdf. The analytical solution to equation (8) exists and the density function of the SNR in shadowed fading channels can be written as

$$f(z) = \frac{2}{\Gamma(m)\Gamma(\nu)} \left(\sqrt{\frac{m\nu}{Z_0}}\right)^{\nu+m} z^{\left(\frac{\nu+m}{2}\right)-1} K_{m-\nu}\left(2\sqrt{\frac{m\nu}{Z_0} z}\right). \quad (9)$$

In equation (9), $K_{m-\nu}(\cdot)$ is the modified Bessel function and the parameter y_0 has been expressed in terms of the average SNR Z_0. The model that resulted in equation (9) is often referred to as the gamma-gamma or the generalized *K* (GK) model for shadowed fading channels [8]. It is possible to relate the level of shadowing σ to the order of gamma ν as

$$\sigma = K\sqrt{Psi(1, \nu)}. \quad (10)$$

In equation (10), $\mathrm{Psi}(1, \cdot)$ is the trigamma function [8]. Note that equation (9) can also be obtained through the cascaded model where the received signal is the product of several components as [5]:

$$Z = \prod_{j=1}^{N} X_j. \tag{11}$$

In equation (11), N is the number of cascades and X's are gamma distributed. When $N = 2$, one can show that equation. (9) can be obtained by taking two different gamma variables of orders m and ν.

We will now look into ways of quantifying the fading or shadowing in wireless channels. Some of them require additional manipulation of the density functions while some measures can be obtained from the density functions. One of the measures is the amount of fading (AF) in the channel. The amount of fading is defined as [7]:

$$AF = \frac{\langle Z^2 \rangle}{\langle Z \rangle^2} - 1 = \frac{\mathrm{var}(Z)}{\langle Z \rangle^2}. \tag{12}$$

In equation (12), var is the variance and $\langle \cdot \rangle$ is the statistical average. From the moments of the Nakagami density function, the AF in a Nakagami channel becomes

$$AF = \frac{1}{m}. \tag{13}$$

Thus we can see that as the Nakagami parameter increases, the level of fading declines. As $m \to \infty$, fading vanishes. The probability density functions of the signal-to-noise ratio in Nakagami fading channels are shown in Figure 1 for three values of the Nakagami parameter, all having the same average SNR. By observing the densities of the SNR in Nakagami channels, it becomes clear that the peak of the pdf also moves to the right as the Nakagami parameter increases. Thus, if one compares two density functions of equal averages, one can qualitatively argue that the location of the peak would indicate the relative level of fading present. One must be careful to use the peak as an indicator since peaks shift to the higher values when the average SNR also increases. Thus, taken together with the decline in the amount of fading, shift of the peak to higher SNR values will indicate improvement in the channel conditions since the AF is independent of the mean SNR. For the case of the shadowed fading channel as modeled in equation (9), AF can be estimated as [9]:

$$AF = \frac{1}{m} + \frac{1}{\nu} + \left(\frac{1}{m} \cdot \frac{1}{\nu}\right). \tag{14}$$

It is seen that when $\nu \rightarrow \infty$, shadowing vanishes and we have a simple short term faded channel. Equation (10) implies an inverse relationship between ν and σ and thus higher values of ν correspond to low values of σ and vice versa. A value of ν of 5.2 corresponds to a very weak shadowing level of ~2 dB while a value of ν of 0.625 corresponds to a strong shadowing level of ~8 dB.

The other indicator of the channel conditions is the cumulative distribution function (CDF). Note that the outage probability can be defined as the probability that the SNR fails to reach a threshold Z_T set by the operating conditions [9],

$$P_{out} = \int_0^{z_T} f(z)dz = F_Z(z_T). \tag{15}$$

In equation (15), $F(\cdot)$ is the cumulative distribution function of the SNR. The shape and slope of the CDF will provide clues to the levels of outages one can expect in the channel. Since all these measures hold their meaning even after diversity, they also provide measures of improvement following the diversity combining algorithms discussed below. An additional measure is the SNR enhancement gained through diversity obtained by comparing the average SNR before and after the implementation of diversity. All these measures, namely the amount of fading, shift in the peaks of the pdf, SNR enhancement and cumulative distributions can be obtained through random number simulations even in the absence of any analytical expressions for the density functions of the SNR.

It is known that fading leads to declining levels of performance in wireless channels and one way to mitigate the effects of fading is through diversity [9]. Diversity techniques involve the generation of multiple uncorrelated versions of the signal which are then combined using various algorithms. In this work, we will not discuss the diversity techniques such as the spatial, frequency, polarization or others. Instead, we will concentrate on the processing of the multiple signals generated through diversity. Diversity processing techniques fall in two broad categories, micro diversity and macro diversity. The former refers to the case when diversity is implemented at a base station (or a mobile unit some times) while the latter identifies the case when diversity is implemented by taking multiple base stations into consideration [8].

We start with the premise that micro diversity of some form is implemented at the base station and multiple signals, assumed to be independent are available for processing. If the order of diversity is M, we have M branches or M independent signals. The outline of the combining algorithms is shown in Figure 2. Output from the each of the branch is scaled by a gain/weighting factor and three primary algorithms, namely the selection combining (SC), equal gain combining (EGC) and maximal ratio combining (MRC), differ in the manner which the gain factors operate [9].

The selection combining (SC) algorithm is the simplest and easiest of the three algorithms from the view point of practical implementation and it involves picking the output or branch having the highest value of the SNR expressed as

$$Z_{SC} = \max\{Z_k\}_{k=1,2,\ldots,M}. \tag{16}$$

In equation (16), Z_k represent SNR value in the kth branch and the gain factors are irrelevant.

Next, we can seek a set of gains or weights so that the output SNR is maximized. It has been shown that the optimal weights are the signals themselves and the SNR at the MRC output can be expressed as

$$Z_{MRC} = \sum_{k=1}^{M} a_k \cdot a_k^* = \sum_{k=1}^{M} Z_k. \tag{17}$$

In equation (17), *a*'s are the complex signals and $*$ represents the conjugate. Note that MRC does require precise phase matching. The third algorithm for combining the signals is a special case of the MRC algorithm where the gain factors are all equal (EGC). The output SNR of the EGC becomes [9]:

$$Z_{EGC} = \frac{1}{M}\left[\sum_{k=1}^{M} |\, a_k \,|\right]^2 = \frac{1}{M}\left[\sum_{k=1}^{M} \sqrt{Z_k}\right]^2. \tag{18}$$

It has been shown that SC is a simpler means to implement diversity combining while MRC is much more complex as mentioned above because of the need for phase matching. However, the enhancement gained through diversity goes up with M for the case of MRC while the enhancement becomes less and less significant when M goes beyond 2 for the case SC. Thus, use of $M > 2$ can be considered as an inefficient use of resources if SC is implemented.

Another diversity combining algorithm for fading mitigation is the generalized selection combining (GSC) algorithm [9]. The concept is also illustrated in Figure 2. This is a hybrid algorithm since it combines the SC and MRC. Instead of picking the strongest branch, the GSC operates in picking L strongest of the M branches and combining these using the MRC, with $L < M$. This means that if $L = 1$, then we have pure SC and if $L = M$, then we have pure MRC. Thus instead of wasting the resources in the case of a pure SC, we choose a limited number of branches for MRC, thus saving the substantial amount of processing involved with pure MRC. Therefore, we expect the performance of GSC to lie between those of the MRC at the high end and SC at the low end. The output SNR in GSC algorithm can be expressed as

$$Z_{GSC} = \sum_{k=1}^{M>L>1} Z_k, \;\; Z_1 > Z_2 > \cdots > Z_L. \tag{19}$$

As mentioned in the introduction, one of the difficulties with the study of the diversity algorithms is the absence of any analytical expressions for the density functions of the SNR following diversity in several cases. In this manuscript, we will only consider the Nakagami fading channel for short term fading since we have the availability of exact analytical expressions for the SC and MRC and none for the rest. This will allow us to test the efficacy of the approach in our work with cases where analytical expressions are available to demonstrate the strength of the approach even when analytical expressions are unavailable as it is the case with EGC and GSC. The Nakagami fading model is also a reasonably general model since the Nakagami pdf includes Rayleigh, Rician and even lognormal densities as limiting cases.

When shadowing is also present, mitigation requires the use of both micro diversity and macro diversity [8, 9]. In this form of hybrid diversity, MRC or SC is implemented at two or more base stations and the outputs from these base stations are then combined using the SC algorithm. The concept is sketched in Figure 3 for two base stations. At each of the base stations, SC or MRC is implemented (micro diversity). The outputs from the base stations are then combined using the SC algorithm resulting in either MRC-SC or SC-SC as the final output of the hybrid diversity involving both micro diversity and macro diversity [8]. We now examine the simulation methods employed.

3. Simulation Methodology

The various combining algorithms are implemented in Matlab (version 2009b). The statistics toolbox is used to access some of its features. The random number generator provides samples of faded signals. Since the SNR in a Nakagami channel is gamma distributed, the SNR samples can be simulated using gamma random numbers. If the average SNR in each branch is Z_0, M sets of samples can be obtained as $gamrnd(m, Z_0/m, M, numb)$, each set with the number of samples *numb*. The program to accomplish the micro diversity project is given in Appendix 1. The inputs required are the Nakagami parameter, average SNR, the number of diversity branches and the number of samples. If we wish to see the results for different values of m, Z_0 or M, then a short for loop segment can be written in Matlab run the function for different values of m. Using the *ecdf* and *ksdensity* functions, both the CDF and pdf of the SNR before and after the diversity implementation, are obtained. The program also finds the peaks of the density functions, amount of fading as well as the signal-to-noise ratio enhancement before and after the implementation of the diversity techniques. The peaks of the pdf are obtained using *sort* while the moments can be obtained using the function *mean* in Matlab.

For SC, the analytical expression for the pdf of the SNR following diversity is given as [9]:

$$f_{SC}(z) = M\left(\frac{\gamma\left(m, \frac{mz}{Z_0}\right)}{\Gamma(m)}\right)^{M-1}\left(\frac{m}{Z_0}\right)^m \frac{z^{m-1}}{\Gamma(m)}\exp\left(-\frac{mz}{Z_0}\right)U(z). \tag{20}$$

In equation (20), $\gamma(\cdot, \cdot)$ is the incomplete gamma function. For MRC, the pdf of SNR after diversity is [7]:

$$f_{MRC}(z) = \left(\frac{m}{Z_0}\right)^{mM}\frac{z^{mM-1}}{\Gamma(mM)}\exp\left(-\frac{mz}{Z_0}\right)U(z). \tag{21}$$

As mentioned earlier, there are no exact analytical expressions for the pdf of the output SNR of EGC and GSC. Thus, we can only compare the analytical pdfs to the pdfs of the simulated cases for SC and MRC. The program in Appendix 1 will accomplish this task as well. Next, the program sorts the M sets and performs the generalized selection combining. There will be no GSC output if $M < 3$. The program also estimates the amount of fading, SNR enhancements and the peaks of the pdf. The program is written to undertake GSC with 2 or 3 out of the M diversity branches.

Appendix 2 contains the program to implement the hybrid diversity to mitigate fading and shadowing simultaneously. The program first performs SC and MRC alone at one base station (micro diversity). The samples are obtained from the product of gamma random variables of orders m and ν with appropriate scaling parameters so that the sample set has an average SNR of Z_0. We make use of the fact that a shadowed fading channel can be created through a cascaded gamma channel with $N = 2$ as discussed in connection with equation (11). Simulation is repeated creating the data for a second base station as illustrated in Figure 3. MRC and SC are performed at each base station and then macro diversity is implemented to achieve MRC-SC and SC-SC. The program has been written for the case of two base stations (macro diversity of order 2) with the SC algorithm implemented at the outputs of the two base stations. All the density functions, cumulative distribution functions, SNR enhancements, amount of fading and the peaks of the pdf are estimated as discussed previously.

4. Results and Discussion

Simulations were carried out for micro diversity as well as hybrid diversity. The number of samples chosen was 50,000, ensuring a large enough sample size to ensure stability of results. Figure 4 shows the probability density functions of the SNR SC, EGC and MRC for $M = 5$ along with the pdf in the absence of any diversity. The shift of the peaks to increasing values of SNR is clearly seen. This aspect, taken together with the increasing values of the SNR enhancement can confirm the performance enhancement predicted through analytical work. This is discussed below. The cumulative distribution functions plotted in Figure 5 show how the CDF goes up with the SNR. For a given value of outage probability (CDF value) such as 1e-2, the outage probability is reached at a lower value of SNR for SC than for MRC indicating that outage probabilities in SC will be higher than those in MRC. The analytical and simulated densities are compared in Figure 6 for the cases of SC and MRC. The match seems excellent suggesting that the approach proposed here should illustrate the learning aspects of the density functions of the SNR of other diversity combining algorithms even when the analytical expressions are unavailable.

Next, the GSC algorithm is compared to MRC and SC. The probability density functions of the output SNR of SC, MRC and GSC (2 out of 5 and 3 out of 5) are shown in Figure 7. As expected, trends show that the peaks of the pdf's move to increasing values of the SNR as one goes from SC, GSC through MRC. To probe the effect of shift of peaks and the relationship to SNR enhancements and amount of fading, simulations were repeated for $M = 4, 6,$ and 7. The peaks, SNR and amount of fading were normalized to the corresponding values for the case of no diversity. The amount of fading given as the inverse suggesting that a higher value indicates a lower level of fading. Table 1 provides comparison of the performance measures. The shifts in the peaks correspond to increased values of SNR enhancements and reduced levels of fading. One can also see that the performance of EGC is almost a match to that of the MRC and as the order of diversity increases, GSC loses out to EGC. But, at low values of M (4 and 5) GSC(5, 3) is either better than EGC or as good as EGC. The SC is always at the lowest end of the performance level as expected from published research [9].

Figure 8 shows the density functions of the SNR in shadowed fading channel when only micro diversity is implemented as well as when hybrid diversity is implemented $(M = 4)$ in presence of strong shadowing $(\sigma = 8\,\text{dB})$. Figure 9 shows the corresponding CDFs. The effect of implementing micro diversity along with micro diversity is seen in Figure 8 and Figure 9. The peaks of the pdf clearly move farther with the inclusion of macro diversity. The differences between results expected from hybrid diversity will be significantly better than those with the micro diversity alone. The cumulative distributions functions demonstrate the trends in outage probabilities similar to the ones discussed in connection with pure short term fading case earlier. It is also abundantly clear that in presence of shadowing, micro diversity alone is not sufficient to improve the performance and macro diversity must be considered as a necessary option to mitigate the worsening conditions existing in shadowed fading channels.

The results of the simulations are in Table 2. In terms of shifts in the peak, and the reduction in fading, the benefits of macro diversity are evident when shadowing is strong as seen by the higher values for strong shadowing. Also, one observes that SC-SC also performs better than or at least as good as MRC alone. On the other hand, at low levels of shadowing, even though SC-SC outperforms SC alone, MRC alone as a micro diversity fares better than SC and SC-SC. The improvement gained by having MRC-SC over MRC alone is not substantial suggesting that macro diversity might be unnecessary if shadowing is weak.

5. Conclusions

The programs in the appendices rely on a few simple Matlab routines, random number generators, *sort*, *ksdensity*, and *ecdf*. Working with these programs can facilitate a better understanding of the concepts such as the ranking of the combining algorithms, the metrics needed to undertake such a ranking, and the need for additional mitigation through macro diversity because of shadowing. By observing the density functions and distribution functions as M is varied, it is possible to see changes in performance of the combining algorithms and the relative enhancements in the channel characteristics.

The Matlab codes in the two appendices are sufficient to investigate effects of other fading models such as those based on the Weibull or generalized gamma pdf for fading [9] or even the exact lognormal model for shadowing. Such studies only require the change of one or two lines in the program to generate those random variables and the rest of the programs follow smoothly. The only caveat in these programs is the fact that a low input SNR is assumed to reduce the computation time needed to estimate the CDFs and pdfs since a very narrow spacing is used to get fairly continuous plots and to display the peaks of the pdfs. One can certainly increase the average SNR at the cost of higher computational time. It is expected that the studies undertaken here will benefit the EE students pursuing programs in wireless communications.

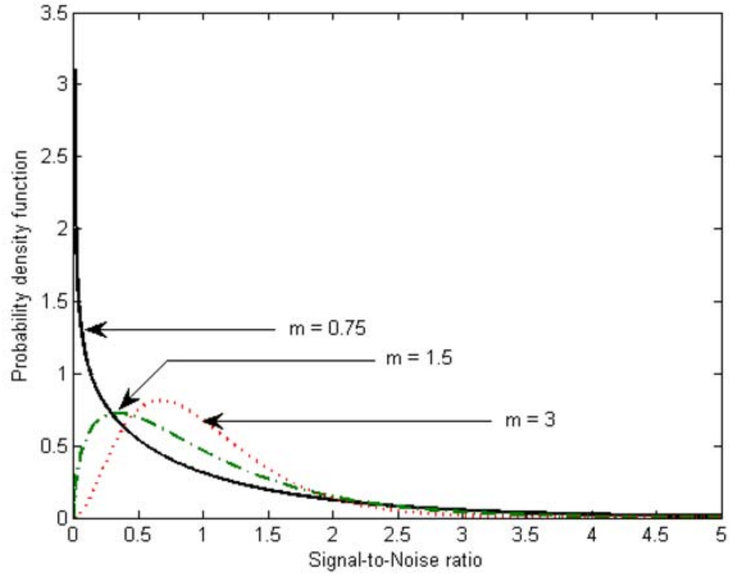

Figure 1. Probability density functions of the signal-to-noise ratio in Nakagami fading channels for $m = 0.75,\ 1.5$ and 3. The average SNR Z_0 is 0 dB.

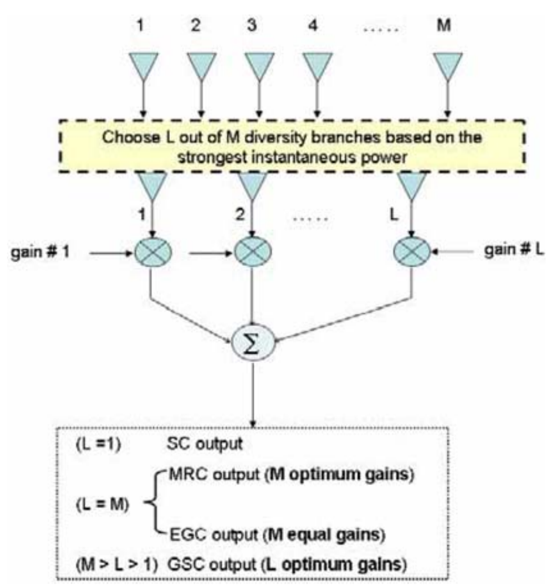

Figure 2. Concept of diversity combining (micro diversity). The various combining algorithms including GSC are indicated.

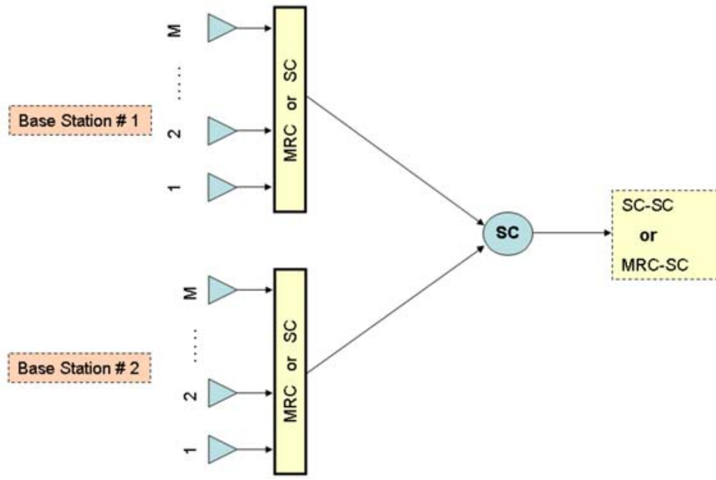

Figure 3. Concept of hybrid diversity (micro diversity and macro diversity). Two base stations are shown. At each Base Station, either MRC or SC is implemented. At the macro diversity level, only SC is implemented.

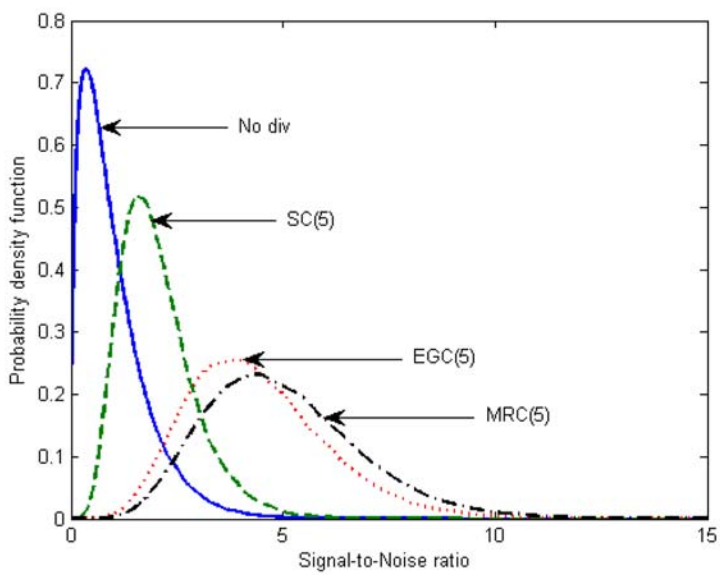

Figure 4. The probability density functions of the SNR for SC, EGC and MRC for the case of $M = 5$ from simulation $(m = 1.5;\ Z_0 = 0 \text{ dB})$. The pdf in the absence of diversity is also shown.

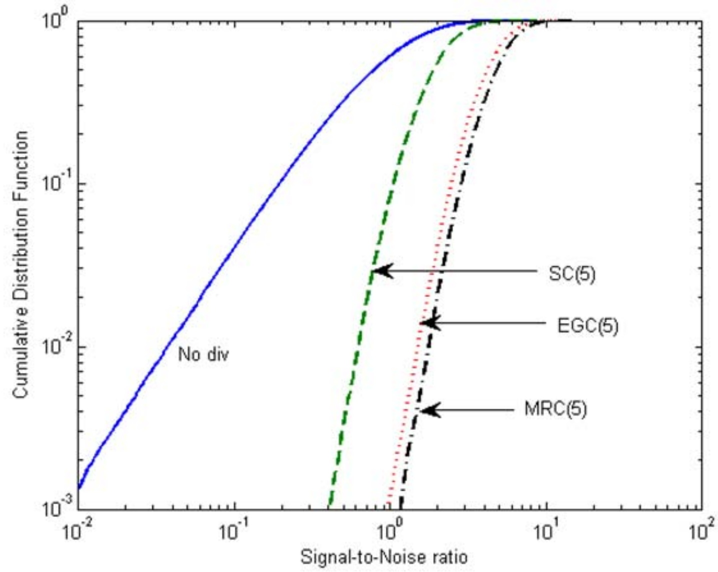

Figure 5. The cumulative distributions of the SNR for SC, EGC and MRC for the case of $M = 5$ from simulation $(m = 1.5;\ Z_0 = 0\text{ dB})$. The CDF in the absence of diversity is also shown.

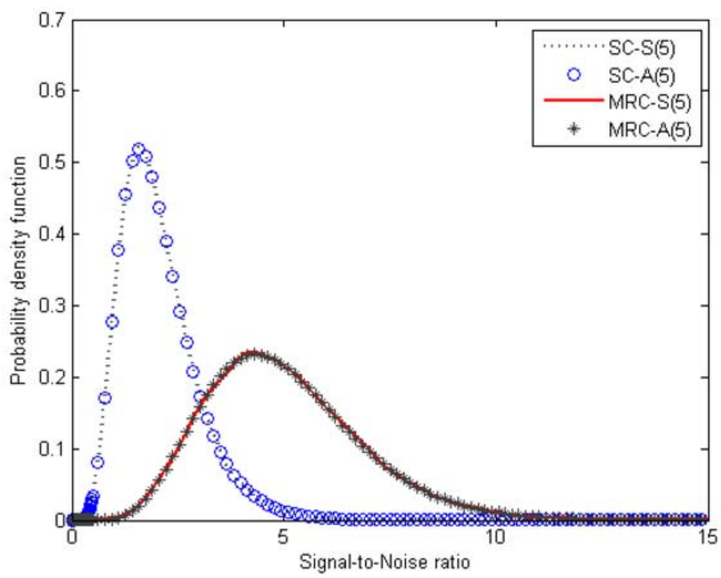

Figure 6. Comparison of the analytical and simulated pdfs for SC and MRC $(M = 5,\ m = 1.5 \text{ and } Z_0 = 0\,\text{dB})$. A(Analytical), S(Simulation).

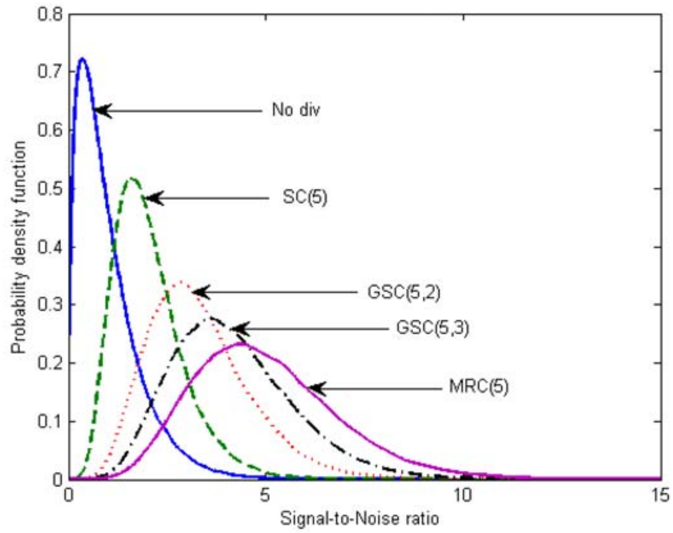

Figure 7. The probability density functions of the SNR for SC, GSC(5, 2), GSC(5, 3) and MRC for $M = 5\,(m = 1.5;\, Z_0 = 0\,\text{dB})$. The pdf in the absence of diversity is also shown.

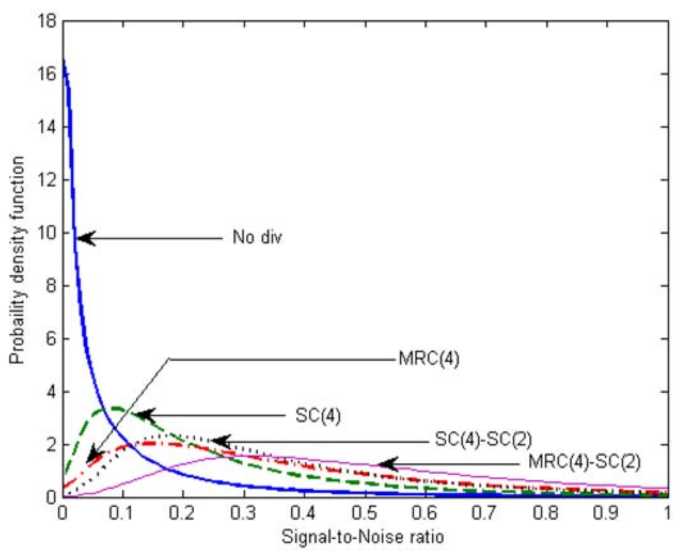

Figure 8. Probability density functions of the SNR under strong shadowing $(\sigma = 8\,\text{dB})$ for a 4-branch micro diversity $(M = 4,\, m = 1.7;\, Z_0 = 0\,\text{dB})$. Only two base stations are considered.

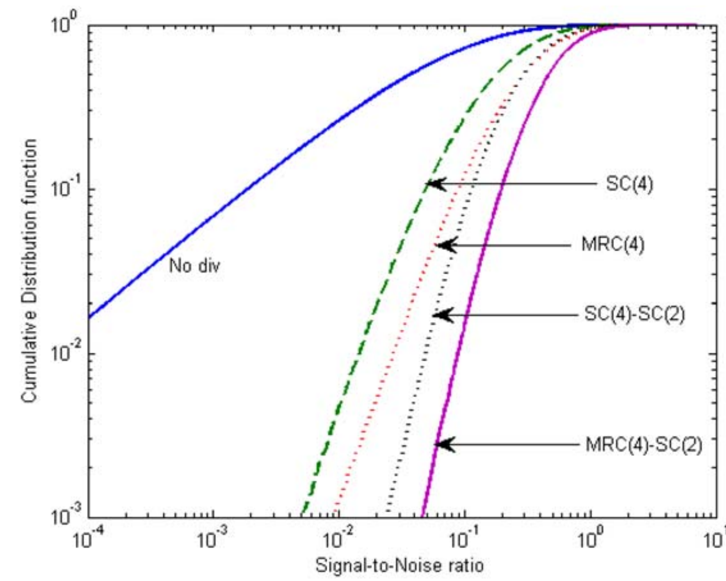

Figure 9. Cumulative distribution functions of the SNR under strong shadowing $(\sigma = 8\,\text{dB})$ for a 4-branch micro diversity $(M = 4,\ m = 1.7;\ Z_0 = 0\,\text{dB})$. Only two base stations are considered.

Table 1. Comparison of the performance measures, Peak, SNR and 1/AF (fading only) for $M = 4,\ 5, 6, 7$. All measures are normalized to the case of no diversity and in place of amount of fading (AF) its inverse (1/AF) is given

	m = 1.7											
	M = 4			M = 5			M = 6			M = 7		
	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$
No Div	1	1	1	1	1	1	1	1	1	1	1	1
SC(M)	2.9	1.9	4.1	3.4	2.0	4.5	3.7	2.2	5.0	4.1	2.3	5.3
GSC(M,2)	3.6	3.0	7.1	4.3	3.3	8.0	4.9	3.6	9.1	5.4	3.8	9.3
GSC(M,3)	3.9	3.7	8.8	4.7	4.2	10.6	5.5	4.6	11.5	6.0	4.9	12.3
EGC(M)	3.7	3.5	8.0	4.7	4.4	10.6	5.7	5.2	13.1	6.7	6.1	15.7
MRC(M)	3.9	4.0	9.5	5.0	5.0	12.9	6.0	6.0	15.1	7.0	7.0	17.7

Table 2. Comparison of the performance measures, Peak, SNR and 1/AF (shadowed fading) for $M = 4$, 5 for two levels of shadowing of 8 dB (strong) and 2 dB (weak). All measures are normalized to the case of no diversity and in place of amount of fading (AF) its inverse (1/AF) is given

	m = 1.7 , σ = 8 dB						m = 1.7 , σ = 2 dB					
	M = 4			M = 5			M = 4			M = 5		
	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$	$1/AF_N$	SNR_N	$Peak_N$
No Div	1	1	1	1	1	1	1	1	1	1	1	1
SC	3.0	2.6	20.0	3.5	3.0	26.0	2.5	2.0	4.5	2.9	2.2	5.4
MRC	4.1	4.0	39.5	5.1	5.0	50.0	4.0	4.0	10.3	4.9	5.0	14.8
SC-SC	4.6	3.9	45.5	5.3	4.3	50.0	3.7	2.6	6.7	4.0	2.8	7.4
MRC-SC	6.4	5.7	75.0	7.9	7.0	100.0	6.1	5.0	13.8	5.6	6.2	18.5

Appendix 1

```
function []=diversitysim(m,ZdB,M);
% m: Nakagami para; ZdB ave snr/div branch; M div order
%SC, EGC, MRC if M=3 or more,  also GSC for two cases
GSC(M,2) GSC(M,3)
%SNR,AF,peak of pdf;uncorrelated cases; analytical match
to simulated SC,MRC.  P. M. Shankar
close all
numb=50000;%numbers used for simulation
Z=10^(ZdB/10); x=gamrnd(m,Z/m,M,numb);
xm=max(x);%SC
xs=sqrt(x);%for EGC
xem=(1/M)*sum(xs).^2;%EGC
xmr=sum(x);%MRC
%estimate the CDF
[ff1,xx1]=ecdf(x(:));
[ff2,xx2]=ecdf(xm(:));[ff3,xx3]=ecdf(xem(:));
[ff4,xx4]=ecdf(xmr(:));
figure
loglog(xx1,ff1,xx2,ff2,xx3,ff3,xx4,ff4),xlabel('Signal-
to-Noise  ratio  (dB)'),ylabel('Cumulative  Distribution
Function')
tt=strcat('(',num2str(M),')');
name1=strcat('SC',tt);name2=strcat('EGC',tt);name3=strcat
('MRC',tt);
legend('No Div',name1,name2,name3,'Location','SouthEast')
```

```
xlim([1e-2 1e2]),ylim([1e-3 1])
x0=0.0:.002:.499;  x01=.5:.02:3*M*Z;%adjust the SNR range
pdf estimation
x1=[x0 x01];%adjust the SNR range for the plots
[f1]=ksdensity(x(:),x1); [f2]=ksdensity(xm(:),x1);
[f3]=ksdensity(xem(:),x1);[f4]=ksdensity(xmr(:),x1);
figure
plot(x1,f1,x1,f2,x1,f3,x1,f4),xlabel('Signal-to-Noise
ratio '),ylabel('Probability density function')
legend('No Div',name1,name2,name3)
%to estimate the shift in the peak of the pdf
w1=[f1;x1]';          w2=[f2;x1]';          w3=[f3;x1]';
w4=[f4;x1]';L=length(x1);
ww1=sortrows(w1);ww2=sortrows(w2);ww3=sortrows(w3);ww4=so
rtrows(w4); %sort rows
peak1=ww1(L,2);peak2=ww2(L,2);peak3=ww3(L,2);peak4=ww4(L,
2);
ftM=gampdf(x1,M*m,Z/m);%MRC
ftS=M*((gamcdf(x1,m,Z/m).^(M-1))).*gampdf(x1,m,Z/m);%SC
figure
plot(x1,f2,x1,ftS,':',x1,f4,'-.',x1,ftM,'--')
legend(strcat('SC-Sim',tt),strcat('SC-Anal',tt),
strcat('MRC-Sim',tt),strcat('MRC-Anal',tt))
xlabel('Signal-to-Noise   ratio   '),ylabel('Probability
density function')
%generalized selection combining
if M>2
 y=sort(x,'descend');%sorts  the  columns  in  descending
order
    ys=max(y);% selection combining
    ym2=sum(y(1:2,:));%GSC of 2 out of M
    ym3=sum(y(1:3,:));%GSC of 3 out of M
[fg2]=ksdensity(ym2(:),x1);[fg3]=ksdensity(ym3(:),x1);
[Fg2,xg2]=ecdf(ym2(:)); [Fg3,xg3]=ecdf(ym3(:));
name2g=strcat('GSC(',num2str(M),',2)');
name3g=strcat('GSC(',num2str(M),',3)');
figure
loglog(xx1,ff1,xx2,ff2,xg2,Fg2,xg3,Fg3,xx4,ff4),
xlabel('Signal-to-Noise  ratio  (dB)'),ylabel('Cumulative
Distribution Function')
legend('No
Div',name1,name2g,name3g,name3,'Location','SouthEast')
xlim([1e-2 1e2]),ylim([1e-3 1])
```

```
figure
plot(x1,f1,x1,f2,x1,fg2,x1,fg3,x1,f4),xlabel('Signal-to-
Noise ratio (dB)'),ylabel('Probability density function')
legend('No Div',name1,name2g,name3g,name3)
wg2=[fg2;x1]';wg3=[fg3;x1]';wwg2=sortrows(wg2);wwg3=sortr
ows(wg3);
peakg2=wwg2(L,2);peakg3=wwg3(L,2);
'peak--  nodiv;   SC;  GSC(M,2)  GSC(M,3);  EGC; MRC'
[peak1 peak2 peakg2 peakg3 peak3 peak4]
'SNR nodiv;   SC;  GSC(M,2)  GSC(M,3);  EGC  MRC'
[mean(x(:))   mean(xm)   mean(ym2)   mean(ym3)   mean(xem)
mean(xmr)]
'AF  nodiv;   SC;  GSC(M,2)  GSC(M,3); EGC   MRC'
[var(x(:))/(mean(x(:)))^2            var(xm)/(mean(xm))^2
var(ym2)/mean(ym2)^2                 var(ym3)/mean(ym3)^2
var(xem)/mean(xem)^2 var(xmr)/(mean(xmr))^2]
else
end;
```

Appendix 2

```
function []=shadowfadingsim(m,nu,Zdb,M)
%m: Naka Para; c: gamma order (shadowing);M(order micro
div)
%two base stations (SC-macro diversity);MRC and SC micro
diversity
%plots the pdf, CDF, estimates the SNR, AF and peak of
pdf
%Shankar, P. M. March 6, 3:00 PM; corrected March 19 4:00
PM
close all
Z=10^(Zdb)/10;%SNR in absolute units
%converts the parameter of the gamma to shadowing
parameter to sigma dB
sigma=(10/log(10))*sqrt(psi(1,nu));%shadowing level dB
for legends only
sig=round(sigma*100)/100;%to keep the decimal place
display of sigma to one
xt=strcat('m       =       ',num2str(m),';','\sigma       =
',num2str(sig),' dB');
numb=50000; b=Z/(m*nu);%to have the same SNR/branch
x=gamrnd(m,1,M,numb).*gamrnd(nu,b,M,numb);%
xm=sum(x);%MRC only
```

```
xmax=max(x);%SC only
%second Base Station
y=gamrnd(m,1,M,numb).*gamrnd(nu,b,M,numb);%
ym=sum(y); ymax=max(y);
 %%%perform micro diversity & macro diversity
xm1=max(xm,ym);%MRC-SC
xmax1=max(xmax,ymax);%SC-SC
 %choose one BS to examine the performance in micro
diversity
 x0=0.:.01:.5;  x01=0.5:.1:5*M*Z;  x1=[x0  x01];%adjust
values pdf estimation
      [f1]=ksdensity(x(:),x1);%input channel
   [fm]=ksdensity(xm(:),x1); %MRC  order M
   [fs]=ksdensity(xmax(:),x1);%SC  order M
   [F1,xx1]=ecdf(x(:));%CDF input
   [Fm,xxm]=ecdf(xm(:));%CDF MRC
   [Fs,xxs]=ecdf(xmax(:));%CDF SC
    mm=num2str(M);            nameS=strcat('SC(',mm,')');
nameM=strcat('MRC(',mm,')');
    figure
plot(x1,f1,x1,fs,'-',x1,fm,'--'),xlabel('Signal-to-Noise
Ratio'),ylabel('Probaility density function')
text(0.75*max(x1),.25*max(f1),xt)%puts the text in the
plot
legend('No div',nameS,nameM)
figure
loglog(xx1,F1,xxs,Fs,'-',xxm,Fm,'--'),xlabel('Signal-to-
Noise Ratio'),ylabel('Cumulative Distribution function')
 xlim([1e-2 1e2]),ylim([1e-3 1])
 text(M*Z,.5,xt)%puts the text in the plot
legend('No div',nameS,nameM,'Location','SouthEast')
x0=0.:.01:.5; x01=0.5:.1:5*M*Z; x1=[x0 x01];% adjust the
values for
   [fmm]=ksdensity(xm1(:),x1); %MRC-SC
   [fss]=ksdensity(xmax1(:),x1);%SC-SC
  [Fmm,xmm]=ecdf(xm1(:));%MRC-SC
  [Fss,xss]=ecdf(xmax1(:));%SC-SC
  figure
  name2=strcat(nameS,'-SC(2)');     name3=strcat(nameM,'-
SC(2)');
  plot(x1,f1,x1,fs,x1,fm,x1,fss,x1,fmm),xlabel('Signal-
to-Noise Ratio'),ylabel('Probaility density function')
  legend('No div',nameS,nameM,name2,name3)
```

```
  text(0.75*max(x1),.25*max(f1),xt)%puts the text in the
plot
  figure
loglog(xx1,F1,xxs,Fs,xxm,Fm,xss,Fss,xmm,Fmm),xlabel('Sign
al-to-Noise    Ratio'),ylabel('Cumulative    distribution
function')
  xlim([1e-2 1e2]),ylim([1e-3 1])
 text(10,.5,xt)%puts the text in the plot
  legend('No
div',nameS,nameM,name2,name3,'Location','SouthEast')
  'SNR: input; SC; MRC; SC-SC; MRC-SC'
  [mean(x(:))   mean(xmax(:))  mean(xm(:))  mean(xmax1(:))
mean(xm1(:))]
  'AF: input; SC; MRC; SC-SC; MRC-SC'
 [var(x(:))/mean(x(:))^2     var(xmax(:))/mean(xmax(:))^2
var(xm(:))/mean(xm(:))^2   var(xmax1(:))/mean(xmax1(:))^2
var(xm1(:))/mean(xm1(:))^2 ]
  w1=[f1;x1]';          w2=[fs;x1]';          w3=[fm;x1]';
w4=[fss;x1]';w5=[fmm;x1]';LL=length(x1);
ww1=sortrows(w1);ww2=sortrows(w2);ww3=sortrows(w3);ww4=so
rtrows(w4);ww5=sortrows(w5);
 p1=ww1(LL,2);p2=ww2(LL,2);p3=ww3(LL,2);p4=ww4(LL,2);p5=
ww5(LL,2);
   'pdfpeak: input; SC; MRC; SC-SC; MRC-SC'
  [p1 p2 p3 p4 p5]
   ' m sigma(dB)'
[m sigma]
```

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