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SOME REMARKS ON FUZZY BIPARTITE GRAPHS

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Abstract

In this paper, the relation between fuzzy bipartite graph and bipartite fuzzy graph is discussed. It is shown that fuzzy trees are fuzzy bipartite. It is proved that a fuzzy graph is fuzzy bipartite graph if and only if it has no odd strong cycles. The concepts of k-partite fuzzy graph, complete fuzzy bipartite graph and full fuzzy bipartite graph are introduced. The number of strong arcs and δ-arcs of a full fuzzy bipartite graph is obtained.

1. Introduction

Graphs and hyper graphs have been applied in a large number of problems including cancer detection, robotics, human cardiac functions, networking and designing [7, 13]. It was Zadeh [21] who introduced fuzzy sets and fuzzy logic into mathematics to deal with problems of uncertainty. As most of the phenomena around us involve much of ambiguity and

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vagueness, fuzzy logic and fuzzy mathematics have to play a crucial role in modelling real time systems with some level of uncertainty.

Fuzzy graphs are very new in mathematics. In 1975, Rosenfeld [10] introduced fuzzy graphs to model relations involving uncertainty. After that several authors including Mordeson and Nair [7], Bhattacharya and Suraweera [1], Bhutani and Rosenfeld [2-5], Sameena et al. [11-14, 16-20], etc. have took initiatives in growing fuzzy graph theory as an applicable area of mathematics.

The concept of strong arcs by Bhutani and Rosenfeld [5] helped in formulating several fuzzy analogues of concepts in graph theory. Also, Mathew and Sunitha [12] studied strong arcs in detail and obtained an arc analysis algorithm. Ramakrishnan and Lakshmi [9] introduced the concept of fuzzy bipartite graphs. In this paper, the authors reframe the definition using strong arcs and characterize them. Also *k*-partite fuzzy graphs are introduced.

A fuzzy graph (*f*-graph) [10] is a pair $G : (\sigma, \mu)$, where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric [10]. In all the examples, σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in S : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in S \times S : \mu(u, v) > 0\}$. A fuzzy graph $H : (\tau, \nu)$ is called a *partial fuzzy subgraph* of $G : (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for every $u \in \tau^*$ and $\nu(u, v) \leq \mu(u, v)$ for every $(u, v) \in \nu^*$ [7]. A path P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0,\ i = 1, 2, ..., n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n$ and $n \geq 3$, then P is called a *cycle* and a cycle P is called a *fuzzy cycle* (*f-cycle*) if it contains more than one weakest arc [7]. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$ and an $x - y$ path P is called a *strongest* $x - y$ *path* if its strength equals $CONN_G(x, y)$ [7]. An *f*-graph $G : (\sigma, \mu)$ is connected if for every x, y in σ^*, $CONN_G(x, y) > 0$. Throughout, we assume that G is connected. An arc of an *f*-graph is called *strong* if its weight is at least as great as the connectedness of its end nodes when it is deleted and an $x - y$ path P is called a *strong path* if P contains only strong arcs [5]. A fuzzy cut node w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w [7]. An arc is called a *fuzzy bridge* of G if its removal reduces the strength of connectedness between some pair of nodes in G [7]. A connected fuzzy graph G is called a *block* (or *non-separable*) if it has no fuzzy cut nodes [7]. If (u, v) is a strong arc in G, then u and v are called *strong neighbors* and a node is called a *fuzzy end node* of G if it has exactly one strong neighbor in G [3]. An arc (x, y) in G is called α-*strong* if $\mu(x, y) > CONN_{G-(x, y)}(x, y)$. It is called β-*strong* if $\mu(x, y) = CONN_{G-(x, y)}(x, y)$ and a δ-arc if $\mu(x, y) < CONN_{G-(x, y)}(x, y)$ [12].

2. Fuzzy Bipartite Graphs

The concept of strong arcs introduced by Bhutani and Rosenfeld [5] made revolutionary changes in fuzzy graph theory and its applications. Researchers tried to generalize or reformulate several concepts in graph theory and fuzzy graph theory using the concepts of strong arcs. Even now there are many concepts in graph theory without a fuzzy graph generalization. In this section, we generalize the concept of a bipartite graph. First similar to the classical definition, we have the definition of a bipartite fuzzy graph (BFG).

Definition 1 [9]**.** A fuzzy graph $G : (\sigma, \mu)$ is said to be *bipartite* if its underlying graph $G^* : (\sigma^*, \mu^*)$ is bipartite. Similarly a fuzzy graph $G : (\sigma, \mu)$ is n-partite if $G^* : (\sigma^*, \mu^*)$ is n-partite.

Example 1. Let $G : (\sigma, \mu)$ be with $\sigma^* = \{u, v, w, x\}$ and $\mu(u, v) = \mu(x, u) = \mu(v, w) = \mu(w, x) = 0.5.$ Here G^* is an even cycle and hence is bipartite.

Now we define fuzzy bipartite graph as follows.

Definition 2. Let $G : (\sigma, \mu)$ be a fuzzy graph. Then G is said to be a *fuzzy bipartite graph* (*FBG*) with fuzzy bipartition (σ_1, σ_2) if there exist two fuzzy subsets σ_1 and σ_2 on σ^* with $\sigma = \sigma_1 \cup \sigma_2$ such that for each strong arc (u, v) in G, $\sigma_1(u) > 0$ and $\sigma_2(v) > 0$ and such that for every arc of G with $\sigma_1(u) > 0$ and $\sigma_1(v) > 0$ or $\sigma_2(u) > 0$ and $\sigma_2(v) > 0$ there exists a $u - v$ path with strength more than $\mu(u, v)$.

From the definition, it follows that if there is an arc in one particular fuzzy bipartition, then it must be a δ-arc [12]. Also, note that there may be δ-arcs with one end in σ_1^* and other in σ_2^*.

Example 2. Let $G : (\sigma, \mu)$ be with $\sigma^* = \{A, B, C\}$ and $\mu(A, B) = 0.2$, $\mu(A, C) = 0.3$, $\mu(B, C) = 0.1$. Then if we let $\sigma_1 = B/0.3 + C/0.3$ and $\sigma_2 = A/0.3$. Then $\sigma = \sigma_1 \cup \sigma_2$ and for each strong arc, namely, (A, B) and (A, C), $\sigma_1(B) > 0, \sigma_2(A) > 0$ and $\sigma_2(A) > 0, \sigma_1(C) > 0$. Also for the arc BC with $\sigma_1(B) > 0$ and $\sigma_1(C) > 0$, there exists a strong path BAC from B to C whose strength is 0.2 which is greater than $\mu(B, C) = 0.1$. Thus, (σ_1, σ_2) is a fuzzy bipartition of G. Hence, G is an *FBG*.

Example 3. Let $G : (\sigma, \mu)$ be with $\sigma^* = \{a, b, c, d\}$ and $\mu(a, b) = 1$, $\mu(b, c) = 0.9$, $\mu(c, d) = 0.8$, $\mu(a, d) = 0.7$, $\mu(a, c) = 0.2$, $\mu(b, d) = 0.2$. Let $\sigma_1^* = \{a, c\}$ and $\sigma_2^* = \{b, d\}$. Then $\sigma = \sigma_1 \cup \sigma_2$. For all strong arcs in G, that is for (a, b), (b, c) and (c, d), we have one end in σ_1^* and other end in σ_2^*. There are three δ-arcs in G, namely, (a, c), (b, d) and (a, d). For the arc (a, d), we have $\sigma_1(a) > 0$ and $\sigma_2(d) > 0$. For (a, c) and (b, d), there exist paths $P_1 = abc$ and $P_2 = bcd$ such that strength of $P_1 = 0.9 > 0.2 = \mu(a, c)$ and strength of $P_2 = 0.8 > 0.1 = \mu(b, d)$. Thus, (σ_1, σ_2) is a fuzzy bipartition of G. Hence, G is an *FBG*.

Any bipartite graph with bipartition (V_1, V_2) can be viewed as a fuzzy bipartite graph with fuzzy bipartition (σ_1, σ_2), where $\sigma_1^* = V_1$ and $\sigma_2^* = V_2$. Thus, we have the following proposition.

Proposition 1. *Any bipartite fuzzy graph* (*BFG*) *is a fuzzy bipartite graph* (*FBG*).

Proof. Let $G : (\sigma, \mu)$ be a BFG. Then G^* is bipartite. Let $V(G^*) = (V_1, V_2)$. Each arc of G^* has one end in V_1 and other end in V_2. Let $V_1 = \{u_1, u_2, ..., u_m\}$ and $V_2 = \{v_1, v_2, ..., vn\}$. Then define $\sigma_1 = \sum_{i=1}^{m} u_i / \sigma(u_i)$ and $\sigma_2 = \sum_{j=1}^{n} v_j / \sigma(v_j)$.

Now for each arc *uv*, $u = u_i$ for $i = 1, 2, ..., m$ and $v = v_j$ for $j = 1, 2, ..., n$. Therefore, $\sigma_1(u_i) > 0$ and $\sigma_2(v_j) > 0$. Also since there exist no arcs between vertices of V_1 or V_2, it follows that there exist no arcs *uv* in *G* such that $\sigma_1(u) > 0$ and $\sigma_1(v) > 0$ or $\sigma_2(u) > 0$ and $\sigma_2(v) > 0$. Thus, it follows that *G* is an FBG.

The converse of Proposition 1 is not true as seen from Example 2.

The result that any tree is bipartite is true in the case of fuzzy trees also even if fuzzy trees are cyclic. This is because of the fact that a fuzzy tree has no strong cycles [5].

Proposition 2. *Fuzzy trees are fuzzy bipartite graphs.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy tree with *F* its unique maximum spanning tree. Since any tree is bipartite, *F* is bipartite. Let $V(G) = V(F) = (V_1, V_2)$. Let $V_1 = \{u_1, u_2, ..., u_m\}$ and $V_2 = \{v_1, v_2, ..., vn\}$. Then define $\sigma_1 = \sum_{i=1}^{m} u_i / \sigma(u_i)$ and $\sigma_2 = \sum_{j=1}^{m} v_j / \sigma(v_j)$.

Now for each arc uv in F, $u = u_i$ for $i = 1, 2, ..., m$ and $v = v_j$ for $j = 1, 2, ..., n$. Therefore $\sigma_1(u) > 0$ and $\sigma_2(v) > 0$. If (x, y) is not an arc in F, then by definition, there exists a strong path P from x to y such that strength of $P > \mu(x, y)$. Hence, for all arcs uv in G with $\sigma_1(u) > 0$ and $\sigma_1(v) > 0$ or $\sigma_2(u) > 0$ and $\sigma_2(v) > 0$ (if exists) such strong paths exist and hence it follows that G is fuzzy bipartite.

Proposition 3. *A strong cycle* $G : (\sigma, \mu)$ *with* $|\sigma^*| = n$ *is fuzzy bipartite if and only if* $n = 2t$, $t \in Z$.

Proof. Let $G : (\sigma, \mu)$ be a strong cycle. First suppose that G is fuzzy bipartite with fuzzy bipartition (σ_1, σ_2). Now since each arc (x, y) of G is strong, $\sigma_1(x) > 0$ and $\sigma_2(y) > 0$. Then G^* is bipartite with bipartition (σ_1^*, σ_2^*). Hence, G^* has no odd cycles [6]. Since G and G^* are cycles, it follows that G^* is an even cycle and hence $|\sigma^*| = n = 2t$, $t \in Z$.

Conversely, suppose that $n = 2t, t \in Z$. Then G^* is an even cycle, which is bipartite. Then G is a bipartite fuzzy graph. But any BFG is an FBG. Hence, G is fuzzy bipartite.

Now we characterize fuzzy bipartite graphs using odd strong cycles.

Theorem 1. *Let* $G : (\sigma, \mu)$ *be a connected fuzzy graph. Then G is a fuzzy bipartite graph if and only if G has no odd strong cycles*.

Proof. Let $G : (\sigma, \mu)$ be a connected fuzzy graph. First suppose that G is fuzzy bipartite with fuzzy bipartition (σ_1, σ_2). Let $C = u_1u_2u_3 \dots u_mu_1$ be a strong cycle in G. Without loss of generality, let $\sigma_1(u_1) > 0$. Then $\sigma_2(u_2) > 0$, since (u_1, u_2) is a strong arc. Then $\sigma_1(u_3) > 0, \sigma_2(u_4) > 0$, etc. Finally, since (u_m, u_1) is a strong arc and $\sigma_1(u_1) > 0$, $\sigma_2(u_m) > 0$. Since all indices of nodes in σ_2^* are even, it follows that $n = 2t$, even. Thus, C is a strong cycle. Since C is arbitrary, it follows that G has no odd strong cycles.

Conversely, suppose that G has no odd strong cycles. To prove G is fuzzy bipartite, we have to find a fuzzy bipartition (σ_1, σ_2) of σ such that for each strong arc (x, y) of G, $\sigma_1(x) > 0$ and $\sigma_2(y) > 0$ and if there exists an arc (x, y) such that $\sigma_1(x) > 0$ and $\sigma_1(y) > 0$ or if $\sigma_2(x) > 0$ and $\sigma_2(y) > 0$, then there exists a $u - v$ path P such that strength of $P > \mu(u, v)$.

Let G' be the fuzzy graph obtained by removing all δ-arcs in G. Since G has no odd strong cycles, G' has no odd strong cycles. Since G' has only strong arc, it follows that G' has only even cycles. Hence, G'^* contains only even cycles. Hence, G'^* is bipartite. Let (V_1, V_2) be a bipartition of G'^*. Define $\sigma_1 = \sum_{u \in V_1} u/\sigma(u)$ and $\sigma_2 = \sum_{v \in V_2} v/\sigma(v)$. Clearly, $\sigma = \sigma_1 \cup \sigma_2$. Now for each strong arc (x, y) in G, (x, y) is an arc in G'^* and hence $x \in V_1$ and $y \in V_2$. Thus, it follows that $\sigma_1(x) > 0$ and $\sigma_2(y) > 0$.

If (x, y) is an arc of G, which is not strong such that $\sigma_1(x) > 0$ and $\sigma_1(y) > 0$ or if $\sigma_2(x) > 0$ and $\sigma_2(y) > 0$, then (x, y) is a δ-arc. By definition, $\mu(x, y) < CONN_G(x, y)$. That is there exists a strong path P in G from x to y such that strength of $P > \mu(x, y)$.

Also, note that if (x, y) is not strong, but $\sigma_1(x) > 0$ and $\sigma_2(y) > 0$, then there is nothing to prove as the arc is between the fuzzy bipartition. Thus, it follows that G is a fuzzy bipartite graph.

Definition 3 (Fuzzy k-partite graph)**.** Let $G : (\sigma, \mu)$ be a fuzzy graph. Then G is fuzzy k-partite if there exist k fuzzy subsets $\sigma_1, \sigma_2, \ldots, \sigma_k$ with $\sigma = \bigcup_{i=1}^{k} \sigma_i$ such that for any strong arc (x, y) in G, $\sigma_i(x) > 0$ and $\sigma_j(y) > 0$, $i \neq j$, where $i, j = 1, 2, \ldots, k$ and if for some arc (x, y), $\sigma_i(x) > 0$ and $\sigma_i(y) > 0$, then there exists a strong path between x and y with more strength than $\mu(x, y)$.

Example 4. Let $G : (\sigma, \mu)$ be with $\sigma^* = \{u, v, w, x, y, z, t\}$ and $\mu(u, v) = 0.1$, $\mu(u, w) = 0.5$, $\mu(v, x) = 0.4$, $\mu(w, x) = 0.6$, $\mu(w, y) = 0.8$, $\mu(x, z) = 0.7$, $\mu(y, z) = 0.9$, $\mu(y, t) = \mu(z, t) = 1$. Then G is a fuzzy 4-partite graph with partition $\sigma^* = \bigcup_{i=1}^{4} \sigma_i^*$, where $\sigma_1^* = \{u, v\}$, $\sigma_2^* = \{w, x\}$, $\sigma_3^* = \{y, z\}$, $\sigma_4^* = \{t\}$.

Definition 4 (Complete fuzzy bipartite graph)**.** A complete fuzzy bipartite graph $C_{m,n}(\sigma, \mu)$ is a fuzzy bipartite graph with fuzzy bipartition (σ_1, σ_2), where $|\sigma_1| = m$ and $|\sigma_2| = n$ with the property that $\mu(x, y) = \sigma_1(x) \wedge \sigma_2(y)$ for every $x \in \sigma_1^*$ and $y \in \sigma_2^*$.

Example 5. Let $G : (\sigma, \mu)$ be with $\sigma(u) = 0.5$, $\sigma(v) = 0.7$, $\sigma(w) = 0.4$, $\sigma(x) = 1$. Let $\mu(u, v) = 0.1$, $\mu(u, w) = 0.4$, $\mu(u, x) = 0.5$, $\mu(v, w) = 0.4$, $\mu(v, x) = 0.7$. Then G is a complete fuzzy bipartite graph with $\sigma = \sigma_1 \cup \sigma_2$, where $\sigma_1^* = \{u, v\}$ and $\sigma_2^* = \{w, x\}$. There are four strong arcs between the fuzzy bipartition and a δ-arc, namely, arc (u, v) in σ_1^*.

It is clear that the maximum possible number of arcs in $C_{m,n}(\sigma, \mu)$ is $mn + {}^nC_2 + {}^mC_2$ and it has at least mn arcs.

So we have a trivial proposition.

Proposition 4. *If e denotes the number of arcs of* $C_{m,n}(\sigma, \mu)$, *then* $mn \leq e \leq mn + {}^nC_2 + {}^mC_2$.

From the above proposition, it follows that the number of arcs in $C_{m,n}(\sigma, \mu)$ can vary. Motivated by this, we have the definition.

Definition 5. A complete fuzzy bipartite graph with mn strong arcs and ${}^nC_2 + {}^mC_2$ δ-arcs is called a *full fuzzy bipartite graph* denoted by $FC_{m,n}$ (σ, μ).

Note that in full fuzzy bipartite graph, any pair of nodes in σ_1^* and σ_2^* is joined by a δ-arc.

Example 6. Let $G : (\sigma, \mu)$ be with $\sigma(u) = 0.5$, $\sigma(v) = 0.7$, $\sigma(w) = 1$. Let $\mu(u, v) = 0.5$, $\mu(u, w) = 0.5$, $\mu(v, w) = 0.1$. Then G is a full fuzzy bipartite graph $FC_{1, 2}$ with $\sigma = \sigma_1 \cup \sigma_2$, where $\sigma_1^* = \{u\}$ and $\sigma_2^* = \{v, w\}$. There are two strong arcs between the fuzzy bipartition and a δ-arc, namely, arc (v, w) in σ_2^*.

Example 7. Let $G : (\sigma, \mu)$ be with $\sigma(u) = 1$, $\sigma(v) = 0.8$, $\sigma(w) = 0.8$, $\sigma(x) = 0.9$. Let $\mu(u, v) = 0.1$, $\mu(u, w) = 0.8$, $\mu(u, x) = 0.9$, $\mu(v, w) = 0.8$, $\mu(v, x) = 0.8$, $\mu(w, x) = 0.1$. Then G is a full fuzzy bipartite graph $FC_{2, 2}$ with $\sigma = \sigma_1 \cup \sigma_2$, where $\sigma_1^* = \{u, v\}$ and $\sigma_2^* = \{w, x\}$. There are four strong arcs between the fuzzy bipartition and two δ-arcs, namely, arc (u, v) and (w, x) in σ_1^*.

We can generalize the concepts of complete fuzzy bipartite graphs and full fuzzy bipartite graphs to n-partite fuzzy graphs as seen from the following definitions.

Definition 6. A complete fuzzy n-partite graph $C_{m_1, m_2, \ldots, m_n}(\sigma, \mu)$ is a fuzzy n-partite graph with $\sigma = \bigcup_{i=1}^{n} \sigma_i$ with $|\sigma_i| = m_i$ for $i = 1, 2, \ldots, n$ with the property that for any arc (x, y) with $\sigma_i(x) > 0$ and $\sigma_j(y) > 0$, $i \neq j$, $\mu(x, y) = \sigma_i(x) \wedge \sigma_j(y)$.

Example 8. Let $G : (\sigma, \mu)$ be with $\mu(u, w) = \mu(u, x) = \mu(v, w) = \mu(v, x) = \mu(w, y) = \mu(x, y) = 1$, $\mu(w, x) = 0.3$. Then G is a $C_{2, 2, 1}$ since $\sigma = \sigma_1 \cup \sigma_2 \cup \sigma_3$ with $\sigma_1^* = \{u, v\}$, $\sigma_2^* = \{x, w\}$ and $\sigma_3^* = \{y\}$. Note that (x, w) is a δ-arc in σ_2^*. All other arcs are strong and are between the fuzzy partitions.

Definition 7. A complete fuzzy n-partite graph is said to be a *full fuzzy n-partite graph* if it has $\sum_{i=1}^{n-1} m_i m_{i+1}$ strong arcs and ${}^{(m_1+m_2+\cdots+m_n)}C_2$ δ-arcs. It is denoted by $FC_{m_1, m_2, \ldots, m_n}(\sigma, \mu)$.

Example 9. Let $G : (\sigma, \mu)$ be with $\sigma^* = \{u, v, w, x\}$ and $\mu(u, w) = \mu(u, x) = \mu(v, w) = \mu(v, x) = \mu(w, y) = \mu(x, y) = 1$, $\mu(w, x) = \mu(u, v) = 0.3$. Then G is an $FC_{2, 2, 1}$ since $\sigma = \sigma_1 \cup \sigma_2 \cup \sigma_3$ with $\sigma_1^* = \{u, v\}$, $\sigma_2^* = \{x, w\}$ and $\sigma_3^* = \{y\}$. Note that (u, v) is a δ-arc in σ_1^* and (x, w) is a δ-arc in σ_2^*. All other arcs are strong and are between the fuzzy partitions.

3. Concluding Remarks

In this paper, we introduced the concepts of fuzzy bipartite graphs and fuzzy n-partite graphs. Complete fuzzy bipartite graphs and full bipartite graphs are also discussed. Similar to the graph analogue, we characterized the fuzzy bipartite graphs using strong cycles.

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