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FUZZY PAIRWISE STRONGLY (r, s)-SEMICONTINUOUS MAPPINGS

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Abstract

We define $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosures and $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiinteriors. By these concepts, we characterize fuzzy pairwise strongly (r, s)-semicontinuous mappings, fuzzy pairwise strongly (r, s)-semiopen mappings and fuzzy pairwise strongly (r, s)-semiclosed mappings in smooth bitopological spaces.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [13] in his classical paper. Using the concept of fuzzy sets, Chang [2] introduced fuzzy topological spaces and several other authors continued the investigation of such spaces. Chattopadhyay et al. [4] and Ramadan [10] introduced new definition of smooth topological spaces as a generalization of fuzzy topological spaces. Kandil [6] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee et al. [8] introduced and studied the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

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In this paper, we introduce $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosures and $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiinteriors. By these notions, we characterize fuzzy pairwise strongly (r, s)-semicontinuous mappings, fuzzy pairwise strongly (r, s)-semiopen mappings and fuzzy pairwise strongly (r, s)-semiclosed mappings in smooth bitopological spaces.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line and $I_0 = (0, 1]$. For a set X, I^X denotes the collection of all mappings from X to I. A member μ of I^X is called a *fuzzy set* of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with values 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are the standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X [2] is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$, then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_k \in T$ for each k, then $\bigvee \mu_k \in T$.

The pair (X, T) is called a *Chang's fuzzy topological space*.

A system (X, T_1, T_2) consisting of a set X with two Chang's fuzzy topologies T_1 and T_2 on X is called a *Kandil's fuzzy bitopological space* [6].

A *smooth topology* on X [4, 10] is a mapping $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.

(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$.

(3) $\mathcal{T}(\bigvee \mu_k) \geq \bigwedge \mathcal{T}(\mu_k)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*.

A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *smooth bitopological space*. Throughout this paper the indices i, j take values in $\{1, 2\}$ and $i \neq j$.

Let $(X, \mathcal{T})$ be a smooth topological space. Then it is easy to see that for each $r \in I_0$, an r-cut

$$\mathcal{T}_r = \{\mu \in I^X \mid \mathcal{T}(\mu) \geq r\}$$

is a Chang's fuzzy topology on X.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Then the map $T^r : I^X \to I$ defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise} \end{cases}$$

becomes a smooth topology.

Hence, we obtain that if $(X, \mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopological space and $r, s \in I_0$, then $(X, (\mathcal{T}_1)_r, (\mathcal{T}_2)_s)$ is a Kandil's fuzzy bitopological space. Also, if (X, T_1, T_2) is a Kandil's fuzzy bitopological space and $r, s \in I_0$, then $(X, (T_1)^r, (T_2)^s)$ is a smooth bitopological space.

Definition 2.1 [8]**.** Let $(X, \mathcal{T})$ be a smooth topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *fuzzy r-closure* is defined by

$$\mathcal{T}\text{-Cl}(\mu, r) = \bigwedge\{\rho \mid \mu \leq \rho,\ \mathcal{T}(\rho^c) \geq r\}$$

and the *fuzzy r-interior*

$$\mathcal{T}\text{-Int}(\mu, r) = \bigvee\{\rho \mid \mu \geq \rho,\ \mathcal{T}(\rho) \geq r\}.$$

Lemma 2.2 [8]**.** *For a fuzzy set* μ *of a smooth topological space* $(X, \mathcal{T})$

and $r \in I_0$, *we have*

(1) $\mathcal{T}\text{-Int}(\mu, r)^c = \mathcal{T}\text{-Cl}(\mu^c, r)$.

(2) $\mathcal{T}\text{-Cl}(\mu, r)^c = \mathcal{T}\text{-Int}(\mu^c, r)$.

Definition 2.3 [7]**.** Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy strongly* (r, s)*-semiopen set* if there is a $\mathcal{T}_i$-fuzzy (r, s)-open set ρ in X such that $\rho \leq \mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r)$,

(2) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy strongly* (r, s)*-semiclosed set* if there is a $\mathcal{T}_i$-fuzzy (r, s)-closed set ρ in X such that $\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\rho, s), r) \leq \mu \leq \rho$.

3. Fuzzy Pairwise Strongly (r, s)-semicontinuous Mappings

Definition 3.1. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a smooth bitopological space and $r, s \in I_0$. For each $\mu \in I^X$, the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy strongly* (r, s)*-semiclosure* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu, r, s)$$
$$= \bigwedge\{\rho \in I^X \mid \mu \leq \rho,\ \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy strongly } (r, s)\text{-semiclosed}\}$$

and the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy strongly* (r, s)*-semiinterior* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s)$$
$$= \bigvee\{\rho \in I^X \mid \mu \geq \rho,\ \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy strongly } (r, s)\text{-semiopen}\}.$$

Obviously, we have that $(\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu, r, s)$ is the smallest $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosed set which contains μ and $(\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu, r, s) = \mu$ for any $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosed set μ. Also, we have that $(\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s)$ is the greatest $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiopen set which is contained in μ and $(\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s) = \mu$ for any $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiopen set μ.

Also, we have the following results:

(1) $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\tilde{0}, r, s) = \tilde{0}$, $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\tilde{1}, r, s) = \tilde{1}$.

(2) $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu, r, s) \geq \mu$.

(3) $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu \vee \rho, r, s) \geq (\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu, r, s) \vee (\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\rho, r, s)$.

(4) $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$((\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu, r, s), r, s) = (\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu, r, s)$.

(5) $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\tilde{0}, r, s) = \tilde{0}$, $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\tilde{1}, r, s) = \tilde{1}$.

(6) $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s) \leq \mu$.

(7) $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu \wedge \rho, r, s) \leq (\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s) \wedge (\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\rho, r, s)$.

(8) $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$((\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s), r, s) = (\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s)$.

Theorem 3.2. *For a fuzzy set* μ *of a smooth bitopological space* $(X, \mathcal{T}_1, \mathcal{T}_2)$ *and* $r, s \in I_0$, *we have*

(1) $((\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s))^c = (\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu^c, r, s)$.

(2) $((\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu, r, s))^c = (\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu^c, r, s)$.

Proof. (1) Note that $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s) \leq \mu$ and $(\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiopen set. So $\mu^c \leq ((\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s))^c$ and $((\mathcal{T}_i, \mathcal{T}_j)$-ssInt$(\mu, r, s))^c$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosed set of X. Thus

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu^c, r, s) \leq (\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(((\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s))^c, r, s)$$

$$= ((\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s))^c.$$

Conversely, note that $\mu^c \leq (\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu^c, r, s)$ and $(\mathcal{T}_i, \mathcal{T}_j)$-ssCl$(\mu^c, r, s)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiclosed set. So $((\mathcal{T}_i, \mathcal{T}_j)$-

$\text{ssCl}(\mu^c, r, s))^c \leq \mu$ and $((\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu^c, r, s))^c$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy strongly (r, s)-semiopen set of X. Thus

$$((\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu^c, r, s))^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(((\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu^c, r, s))^c, r, s)$$

$$\leq (\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s)$$

and hence $((\mathcal{T}_i, \mathcal{T}_j)\text{-ssInt}(\mu, r, s))^c \leq (\mathcal{T}_i, \mathcal{T}_j)\text{-ssCl}(\mu^c, r, s)$.

(2) Similar to (1).

Definition 3.3 [7]**.** Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is called a *fuzzy pairwise* (r, s)*-continuous* ((r, s)*-open* and (r, s)*-closed*, respectively) *mapping* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is a fuzzy r-continuous (r-open and r-closed, respectively) mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is a fuzzy s-continuous (s-open and s-closed, respectively) mapping.

Definition 3.4 [7]**.** Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is called

(1) *fuzzy pairwise strongly* (r, s)*-semicontinuous* if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy strongly (r, s)-semiopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy strongly (s, r)-semiopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y,

(2) *fuzzy pairwise strongly* (r, s)*-semiopen* if $f(\rho)$ is a $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiopen set of Y for each $\mathcal{T}_1$-fuzzy r-open set ρ of X and $f(\lambda)$ is a $(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiopen set of Y for each $\mathcal{T}_2$-fuzzy s-open set λ of X,

(3) *fuzzy pairwise strongly* (r, s)*-semiclosed* if $f(\rho)$ is a $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiclosed set of Y for each $\mathcal{T}_1$-fuzzy r-closed set ρ

of X and $f(\lambda)$ is a $(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiclosed set of Y for each $\mathcal{T}_2$-fuzzy s-closed set λ of X.

Theorem 3.5. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping and* $r, s \in I_0$. *Then the following statements are equivalent*:

(1) *f is a fuzzy pairwise strongly (r, s)-semicontinuous mapping.*

(2) *For each fuzzy set ρ of X,* $f((\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(\rho, r, s)) \leq \mathcal{U}_1\text{-Cl}(f(\rho), r)$ *and* $f((\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(\rho, s, r)) \leq \mathcal{U}_2\text{-Cl}(f(\rho), s)$.

(3) *For each fuzzy set μ of Y,* $(\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu), r, s) \leq f^{-1}(\mathcal{U}_1\text{-Cl}(\mu, r))$ *and* $(\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu), s, r) \leq f^{-1}(\mathcal{U}_2\text{-Cl}(\mu, s))$.

(4) *For each fuzzy set μ of Y,* $f^{-1}(\mathcal{U}_1\text{-Int}(\mu, r)) \leq (\mathcal{T}_1, \mathcal{T}_2)\text{-ssInt}(f^{-1}(\mu), r, s)$ *and* $f^{-1}(\mathcal{U}_2\text{-Int}(\mu, s)) \leq (\mathcal{T}_2, \mathcal{T}_1)\text{-ssInt}(f^{-1}(\mu), s, r)$.

Proof. (1) $\Rightarrow$ (2) Let ρ be any fuzzy set of X. Then $\mathcal{U}_1\text{-Cl}(f(\rho), r)$ is $\mathcal{U}_1$-fuzzy r-closed and $\mathcal{U}_2\text{-Cl}(f(\rho), s)$ is $\mathcal{U}_2$-fuzzy s-closed in Y. Since f is a fuzzy pairwise strongly (r, s)-semicontinuous mapping, $f^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho), r))$ is $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy strongly (r, s)-semiclosed and $f^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho), s))$ is $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy strongly (s, r)-semiclosed in X. Thus

$$\begin{aligned}(\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(\rho, r, s) &\leq (\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho), r)), r, s)\\ &= f^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho), r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(\rho, s, r) &\leq (\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho), s)), s, r)\\ &= f^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho), s)).\end{aligned}$$

Hence

$$f((\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(\rho, r, s)) \leq ff^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho), r)) \leq \mathcal{U}_1\text{-Cl}(f(\rho), r)$$

and

$$f((\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(\rho, s, r)) \leq ff^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho), s)) \leq \mathcal{U}_2\text{-Cl}(f(\rho), s).$$

(2) $\Rightarrow$ (3) Let μ be any fuzzy set of Y. Then

$$f((\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu), r, s)) \leq \mathcal{U}_1\text{-Cl}(ff^{-1}(\mu), r) \leq \mathcal{U}_1\text{-Cl}(\mu, r)$$

and

$$f((\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu), s, r)) \leq \mathcal{U}_2\text{-Cl}(ff^{-1}(\mu), s) \leq \mathcal{U}_2\text{-Cl}(\mu, s).$$

Thus

$$\begin{aligned}(\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu), r, s) &\leq f^{-1}f((\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu), r, s))\\ &\leq f^{-1}(\mathcal{U}_1\text{-Cl}(\mu, r))\end{aligned}$$

and

$$\begin{aligned}(\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu), s, r) &\leq f^{-1}f((\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu), s, r))\\ &\leq f^{-1}(\mathcal{U}_2\text{-Cl}(\mu, s)).\end{aligned}$$

(3) $\Rightarrow$ (4) Let μ be any fuzzy set of Y. Then

$$(\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu)^c, r, s) \leq f^{-1}(\mathcal{U}_1\text{-Cl}(\mu^c, r))$$

and

$$(\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu)^c, s, r) \leq f^{-1}(\mathcal{U}_2\text{-Cl}(\mu^c, s)).$$

By Theorem 3.2,

$$\begin{aligned}f^{-1}(\mathcal{U}_1\text{-Int}(\mu, r)) = f^{-1}(\mathcal{U}_1\text{-Cl}(\mu^c, r))^c &\leq (\mathcal{T}_1, \mathcal{T}_2)\text{-ssCl}(f^{-1}(\mu)^c, r, s)^c\\ &= (\mathcal{T}_1, \mathcal{T}_2)\text{-ssInt}(f^{-1}(\mu), r, s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\mathcal{U}_2\text{-Int}(\mu, s)) = f^{-1}(\mathcal{U}_2\text{-Cl}(\mu^c, s))^c &\leq (\mathcal{T}_2, \mathcal{T}_1)\text{-ssCl}(f^{-1}(\mu)^c, s, r)^c\\ &= (\mathcal{T}_2, \mathcal{T}_1)\text{-ssInt}(f^{-1}(\mu), s, r).\end{aligned}$$

(4) $\Rightarrow$ (1) Let μ be any $\mathcal{U}_1$-fuzzy r-open set and ν be any $\mathcal{U}_2$-fuzzy s-open set of Y. Then $\mathcal{U}_1\text{-Int}(\mu, r) = \mu$ and $\mathcal{U}_2\text{-Int}(\nu, s) = \nu$. Thus

$$f^{-1}(\mu) = f^{-1}(\mathcal{U}_1\text{-Int}(\mu, r)) \leq (\mathcal{T}_1, \mathcal{T}_2)\text{-ssInt}(f^{-1}(\mu), r, s) \leq f^{-1}(\mu)$$

and

$$f^{-1}(\nu) = f^{-1}(\mathcal{U}_2\text{-Int}(\nu, s)) \leq (\mathcal{T}_2, \mathcal{T}_1)\text{-ssInt}(f^{-1}(\nu), s, r) \leq f^{-1}(\nu).$$

So $f^{-1}(\mu) = (\mathcal{T}_1, \mathcal{T}_2)\text{-ssInt}(f^{-1}(\mu), r, s)$ and $f^{-1}(\nu) = (\mathcal{T}_2, \mathcal{T}_1)\text{-ssInt}(f^{-1}(\nu), s, r)$. Hence $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy strongly (r, s)-semiopen set and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy strongly (s, r)-semiopen set of X. Therefore f is a fuzzy pairwise strongly (r, s)-semicontinuous mapping.

Theorem 3.6. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping and* $r, s \in I_0$. *Then the following statements are equivalent*:

(1) *f is a fuzzy pairwise strongly* (r, s)*-semiopen mapping.*

(2) *For each fuzzy set* ρ *of* X, $f(\mathcal{T}_1\text{-Int}(\rho, r)) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s)$ *and* $f(\mathcal{T}_2\text{-Int}(\rho, s)) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\rho), s, r)$.

(3) *For each fuzzy set* μ *of* Y, $\mathcal{T}_1\text{-Int}(f^{-1}(\mu), r) \leq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(\mu, r, s))$ *and* $\mathcal{T}_2\text{-Int}(f^{-1}(\mu), s) \leq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(\mu, s, r))$.

Proof. (1) $\Rightarrow$ (2) Let ρ be any fuzzy set of X. Clearly, $\mathcal{T}_1\text{-Int}(\rho, r)$ is $\mathcal{T}_1$-fuzzy r-open and $\mathcal{T}_2\text{-Int}(\rho, s)$ is $\mathcal{T}_2$-fuzzy s-open in X. Since f is a fuzzy pairwise strongly (r, s)-semiopen mapping, $f(\mathcal{T}_1\text{-Int}(\rho, r))$ is $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiopen and $f(\mathcal{T}_2\text{-Int}(\rho, s))$ is $(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiopen in Y. Thus

$$f(\mathcal{T}_1\text{-Int}(\rho, r)) = (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\mathcal{T}_1\text{-Int}(\rho, r)), r, s) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s)$$

and

$$f(\mathcal{T}_2\text{-Int}(\rho, s)) = (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\mathcal{T}_2\text{-Int}(\rho, s)), s, r) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\rho), s, r).$$

(2) $\Rightarrow$ (3) Let μ be any fuzzy set of Y. Then

$$f(\mathcal{T}_1\text{-Int}(f^{-1}(\mu), r)) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(ff^{-1}(\mu), r, s) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(\mu, r, s)$$

and

$$f(\mathcal{T}_2\text{-Int}(f^{-1}(\mu), s)) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(ff^{-1}(\mu), s, r) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(\mu, s, r).$$

Thus we have

$$\mathcal{T}_1\text{-Int}(f^{-1}(\mu), r) \leq f^{-1}f(\mathcal{T}_1\text{-Int}(f^{-1}(\mu), r)) \leq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(\mu, r, s))$$

and

$$\mathcal{T}_2\text{-Int}(f^{-1}(\mu), s) \leq f^{-1}f(\mathcal{T}_2\text{-Int}(f^{-1}(\mu), s)) \leq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(\mu, s, r)).$$

(3) $\Rightarrow$ (1) Let ρ be any $\mathcal{T}_1$-fuzzy r-open set and λ be any $\mathcal{T}_2$-fuzzy s-open set of X. Then $\mathcal{T}_1\text{-Int}(\rho, r) = \rho$ and $\mathcal{T}_2\text{-Int}(\lambda, s) = \lambda$. Thus

$$\rho = \mathcal{T}_1\text{-Int}(\rho, r) \leq \mathcal{T}_1\text{-Int}(f^{-1}f(\rho), r) \leq f^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s))$$

and

$$\lambda = \mathcal{T}_2\text{-Int}(\lambda, s) \leq \mathcal{T}_2\text{-Int}(f^{-1}f(\lambda), s) \leq f^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\lambda), s, r)).$$

Hence we have

$$f(\rho) \leq ff^{-1}((\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s)) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s) \leq f(\rho)$$

and

$$f(\lambda) \leq ff^{-1}((\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\lambda), s, r)) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\lambda), s, r) \leq f(\lambda).$$

Thus $f(\rho) = (\mathcal{U}_1, \mathcal{U}_2)\text{-ssInt}(f(\rho), r, s)$ and $f(\lambda) = (\mathcal{U}_2, \mathcal{U}_1)\text{-ssInt}(f(\lambda), s, r)$. Hence $f(\rho)$ is a $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiopen set and $f(\lambda)$ is a $(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiopen set of Y. Therefore f is a fuzzy pairwise strongly (r, s)-semiopen mapping.

Theorem 3.7. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping and* $r, s \in I_0$. *Then f is fuzzy pairwise strongly* (r, s)*-semiclosed if and only if for each fuzzy set* ρ *of* X, $(\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\rho), r, s) \leq f(\mathcal{T}_1\text{-Cl}(\rho, r))$ *and* $(\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\rho), s, r) \leq f(\mathcal{T}_2\text{-Cl}(\rho, s))$.

Proof. Let ρ be any fuzzy set of X. Clearly, $\mathcal{T}_1\text{-Cl}(\rho, r)$ is a $\mathcal{T}_1$-fuzzy r-closed set and $\mathcal{T}_2\text{-Cl}(\rho, s)$ is a $\mathcal{T}_2$-fuzzy s-closed set of X. Since f is a fuzzy pairwise strongly (r, s)-semiclosed mapping, $f(\mathcal{T}_1\text{-Cl}(\rho, r))$ is a $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiclosed set and $f(\mathcal{T}_2\text{-Cl}(\rho, s))$ is a

$(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiclosed set of Y. Thus we have

$$(\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\rho), r, s) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\mathcal{T}_1\text{-Cl}(\rho, r)), r, s) = f(\mathcal{T}_1\text{-Cl}(\rho, r))$$

and

$$(\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\rho), s, r) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\mathcal{T}_2\text{-Cl}(\rho, s)), s, r) = f(\mathcal{T}_2\text{-Cl}(\rho, s)).$$

Conversely, let ρ be any $\mathcal{T}_1$-fuzzy r-closed set and λ be any $\mathcal{T}_2$-fuzzy s-closed set of X. Then $\mathcal{T}_1\text{-Cl}(\rho, r) = \rho$ and $\mathcal{T}_2\text{-Cl}(\lambda, s) = \lambda$. Thus

$$(\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\rho), r, s) \leq f(\mathcal{T}_1\text{-Cl}(\rho, r)) = f(\rho) \leq (\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\rho), r, s)$$

and

$$(\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\lambda), s, r) \leq f(\mathcal{T}_2\text{-Cl}(\lambda, s)) = f(\lambda) \leq (\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\lambda), s, r).$$

So $f(\rho) = (\mathcal{U}_1, \mathcal{U}_2)\text{-ssCl}(f(\rho), r, s)$ and $f(\lambda) = (\mathcal{U}_2, \mathcal{U}_1)\text{-ssCl}(f(\lambda), s, r)$. Hence $f(\rho)$ is a $(\mathcal{U}_1, \mathcal{U}_2)$-fuzzy strongly (r, s)-semiclosed set and $f(\lambda)$ is a $(\mathcal{U}_2, \mathcal{U}_1)$-fuzzy strongly (s, r)-semiclosed set of Y. Therefore f is a fuzzy pairwise strongly (r, s)-semiclosed mapping.

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