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MODELLING FRAUD DETECTION BY ATTACK TREES AND CHOQUET INTEGRAL

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Abstract

Modelling an attack tree is basically a matter of associating a logical "and" and a logical "or", but in most of real world applications related to fraud management the "and/or" logic is not adequate to effectively represent the relationship between a parent node and its children, most of all when information about attributes is associated to the nodes and the main problem to solve is how to promulgate attribute values up the tree through recursive aggregation operations occurring at the "and/or" nodes. OWA-based aggregations have been introduced to generalize "and" and "or" operators starting from the observation that in between the extremes "for all" (and) and "for any" (or), terms (quantifiers) like "several", "most", "few", "some", etc. can be introduced to represent

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the different weights associated to the nodes in the aggregation. The aggregation process taking place at an OWA node depends on the ordered position of the child nodes but it does not take care of the possible interactions between the nodes. In this paper, we propose to overcome this drawback introducing the Choquet integral whose distinguished feature is to be able to take into account the interaction between nodes. At first, the attack tree is valuated recursively through a bottom-up algorithm whose complexity is linear versus the number of nodes and exponential for every node. Then, the algorithm is extended assuming that the attribute values in the leaves are unimodal *LR* fuzzy numbers and the calculation of Choquet integral is carried out using the alpha-cuts.

1. Introduction

The fraud surveys carried out in the last five years by leading international consulting companies (see, e.g., KPMG Fraud Survey [33]) demonstrate that fraud is an increasing phenomenon depending most of all on behavioural aspects. Therefore, when addressing fraud detection processes the adoption of traditional statistical techniques comes out to be not as adequate as those based on the evaluations of experts using computational intelligence techniques based on network models.

One of the most widely used techniques to serve as a formal representation of the possible sequences of events/actions leading to an attack to some kind of asset are attack trees developed by Schneier [45, 46]. Basically, attacks against a system/person are represented in a tree structure where the goal is the root node and child nodes denote the different ways of achieving that goal. When using attack trees to model a fraud detection process, evaluation procedures involving the information about attributes are to be carried on. The information is usually associated to the child nodes and transferred to the root node via operations occurring at the “and” and/or” operators.

As Yager [55] pointed out, in many real world applications related to security problems the “and/or” logic is not adequate to effectively represent the relationships between a parent node and its children, and therefore he proposed an extension of the "and/or" attack trees to what he called *OWA trees*. The OWA trees are based on a class of aggregating operators (Yager, [51, 53], Yager and Kacprzyk [54]) called *Ordered Weighted Averaging* (OWA) operators and aim at providing a generalization of the "and" and "or" operators.

The aggregation process taking place at an OWA node depends on the ordered position of the child nodes but it does not take care of the possible interactions between the nodes. One way to overcome this drawback of the OWA operator is to introduce the Choquet integral (Choquet [8]) whose distinguished feature is to be able to take into account the interaction between nodes, ranging from redundancy (negative interaction) to synergy (positive interaction).

In this paper, we show how to use the Choquet integral to extend the OWA-based attack trees (Yager [55]) assuming that the attack tree is valuated recursively through a bottom-up algorithm whose complexity is linear versus the number of nodes and exponential for every node. Then, the algorithm is further extended assuming that the attribute values in the leaves are unimodal *LR* fuzzy numbers and the calculation of Choquet integral is based on alpha-cuts.

The paper is organized as follows. In Section 2, we introduce the notion of attack tree and its extension to the OWA-based setting. In Section 3, we summarize some basic results regarding the Choquet integral. In Section 4, we show how to use the Choquet integral to valuate the attack tree via a bottom-up algorithm, starting from the attributes' values associated to the leaves. In Section 5, the algorithm is extended assuming that values on the leaves are unimodal *LR* fuzzy numbers and the computation of Choquet integral is performed through the introduction of alpha-cuts and thus based on interval-valued functions. The last section is devoted to conclusions and perspectives on future work.

2. Attack Trees and Aggregation Operators

The attack tree technique, as proposed by Schneier [45, 46], provides a structured approach to fraud detection showing the possible attack goals, their respective technical difficulty, severity (cost) of impact, and likelihood of detection. Using attack trees the different ways in which a system can be attacked are systematically classified, and the out-coming graphical notation is appealing to practitioners and easy to be automated.

An attack tree is a tree in which the nodes represent the elementary components of an attack. While the root node is the global goal the potential fraudster would like to achieve, children of a node are components representing refinements of this goal, and leaves are components than cannot be refined anymore. Figure 1 shows an example from Schneier [45] where the main goal is to open a safe.

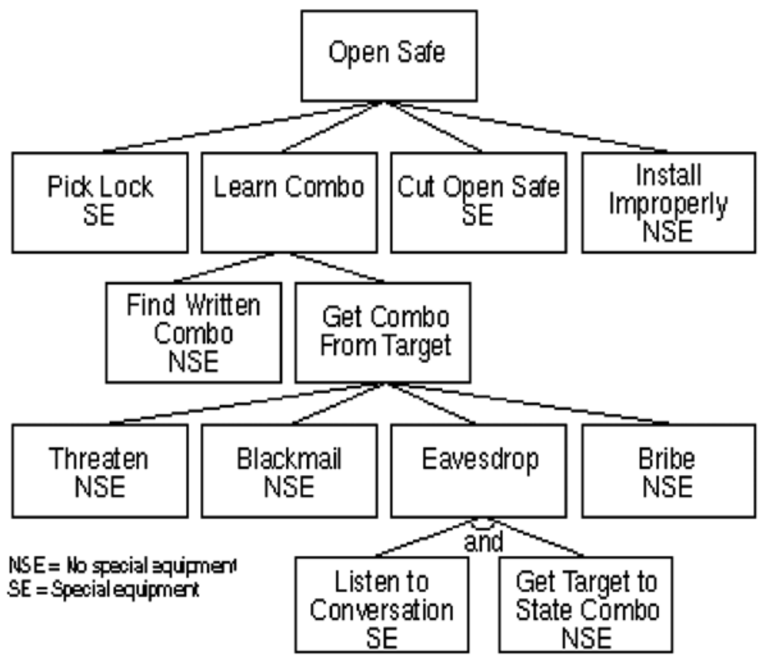

Figure 1. Schneier's attack tree for opening safe.

The basic components of an attack tree are therefore "and" nodes and "or" nodes, where an "and" node describes a situation in which all the children must be satisfied while an "or" node requires at least one of the children to be satisfied.

In many applications of attack trees, information about attributes is commonly associated to the leaves and to the other nodes as well and one of the main problem to be solved becomes how to promulgate the information up the tree until it reaches the root node. The process of transferring attribute information up the tree is carried out by the introduction of aggregation operations occurring at the "and/or" nodes.

In most of the applications addressing the design of fraud detection systems "and/or" nodes are not suitable to represent the complex relationships between the parents nodes and their children and therefore the problem to solve is to introduce a more rich aggregation apparatus able to extend the capabilities of "and/or" tree structure.

The semantics of aggregation are actually so many that is not easy to classify the corresponding aggregation operators. There are cases where it is required that high and low inputs average each other, in some cases aggregation should model conjunction and disjunction logical connectives in order to reinforce the inputs each other, etc. For an extensive treatise on aggregation functions see Beliakov et al. [2] and Grabisch et al. [29].

The most commonly used aggregation operator is the weighted average which computes a convex linear combination of the criteria through a pre-defined set of nonnegative weights. The weighted average is widely used for many real world problems. Even if simple and easy to be understood and explained also to non-expert users, the weighted average is a compensative method. In fact, given its linear nature, an highly scored attribute can compensate a lower scored one, implying the satisfaction of the independent preference axiom. As a consequence, no interactions among the attributes is possible, and this characterization strongly conveys the preference structure defined by it to very particular cases.

In some cases, as for the attack trees, the attributes are organized into a hierarchical form, and the evaluation is top-down computed for each node of the tree.

The aggregation of attributes along the tree can be carried out using the OWA operators (Yager [51]) which provide a generalization of "and" and "or" operators observing that "and" and "or" are specific examples of the quantifiers introduced by Zadeh [60]. Therefore, in between the extreme quantifiers "for all" (and) and "for any" (or), terms like "several", "most", "few", "some", "at least one half", etc. can be used to highlight the different weight associated to the nodes in the aggregation process. According to Yager [55], the basic component of the attack tree becomes the OWA node which is characterized by a vector **w** whose dimension is equal to the number of children emanating from this node.

The aggregation process taking place at an OWA node depends on the ordered position of the child nodes but it does not take care of the possible interactions between the nodes. One way to overcome this drawback of the OWA operator is to introduce the Choquet integral (Choquet [8]) whose distinguished feature is to be able to take into account the interaction between nodes, ranging from redundancy (negative interaction) to synergy (positive interaction).

Moreover, the Choquet integral is mathematically well founded and characterized (Klement et al. [32]). Many applications in multi-criteria and multi-attribute decision aid are reported (Grabisch [22]; Grabisch et al. [28]; Grabisch and Labreuche [30]), but we remark that even if its generalizing properties are surely a gain, an heavier computational load is required, since many more parameters are required than in simpler cases. To wit, a fuzzy measure assigns a weight to every possible coalition of criteria, and a monotonicity condition needs to be satisfied. The weight of a coalition can be greater or lower than the sum of the weights of the elements of every its partition. If the equality holds, then the operator degenerates to the weighted average. The Choquet integral includes the weighted average, the min and the max operator, the OWA, the *k*-statistic and other aggregation operators, showing a wide generality. The most critical issue to address when introducing the Choquet integral is the definition of the measure sets. In many problems their value can be directly obtained from fuzzy measure, even if a nonlinear high dimensional optimization problem needs to be solved (Grabisch et al. [26]). In fact, since a weight is assigned to every possible criteria subsets, the number of the parameters exponentially increases with the cardinality of the criteria set. Anywise, for human decision problems, since the fuzzy measure values reflect the expert's preference structure, they can be indirectly obtained by means of, e.g., a questionnaire (Despic and Simonovic [10]). Moreover, the expert's attitude to pessimism or to optimism can be computed from the fuzzy measure, by means of a so called *andness* (*orness*) *index*. To downsize the numerical complexity, a reduced order model can be applied, admitting interactions only for low cardinality criteria coalitions (Grabisch [23]; Grabisch and Roubens [25]).

3. Preliminary Concepts on Choquet Integral

Consider a finite set of elements $N = \{1, 2, 3, ..., n\}$. A (*discrete*) *fuzzy measure* μ (also called *capacity*) defined on N is a set function $\mu : 2^N \to [0, 1]$ satisfying

(1) $\mu(\emptyset) = 0, \ \ \mu(N) = 1$ (boundary conditions),

(2) $S \subseteq T \Rightarrow \mu(S) \leq \mu(T), \forall S, T \subseteq N$ (monotonicity condition).

Let we remark that removing the monotonicity condition one can model strong conflicting effects (Cardin and Giove [4]; De Waegenaere and Wakker [9]; Murofushi et al. [41]; Rèbillè [43]). Moreover, the values of fuzzy measure can be fuzzified (Giove [19]; Meyer and Roubens [38]; Yang et al. [56]).

Assigning a different weight to every coalition, this fuzzy measure can represent positive and negative interactions among the criteria (Marichal [35, 36]).

Given two coalitions $S, T \subseteq N$, with $S \cap T = \emptyset$, the fuzzy measure is said to be *additive* if $\mu(S \cup T) = \mu(S) + \mu(T)$, *sub-additive* if $\mu(S \cup T) < \mu(S) + \mu(T)$, and *super-additive* if $\mu(S \cup T) > \mu(S) + \mu(T)$, with respect to the two coalitions S, T.

A sub-additive fuzzy measure models a redundant effect, a super-additive models a synergic effect, while in the additive case we obtain a linear fuzzy measure.

Given a fuzzy measure μ an equivalent representation is obtained through the Möbius transform (Rota [44])

$$m_\mu(T) = \sum_{S \subseteq T} (-1)^{t-s} \mu(S), \quad T \subseteq N, \tag{3.1}$$

where s and t denote the cardinality of the coalitions S and T.

Conversely, given the Möbius transform m_μ, the associated fuzzy measure μ is obtained as

$$\mu(T) = \sum_{S \subseteq T} m_\mu(S), \quad T \subseteq N. \tag{3.2}$$

Anywise, not every set of 2^n real numbers can be the Möbius values of a fuzzy measure, they need to satisfy (Chateauneuf and Jaffray [6])

$$m(\varnothing) = 0, \quad \sum_{T \subseteq N} m(T) = 1, \ \sum_{S \subseteq T} m(S \cup i) \geq 0, \quad T \subseteq N \backslash i, \ \ i \in N. \tag{3.3}$$

A fuzzy measure μ is said to be 2-*additive* (Grabisch [23]) if $m_\mu(T) = 0$ for all $T \subseteq N$, with $t > 2$, and there exists at least one coalition $T \subseteq N$ with $t = 2$ such that $m_\mu(T) \neq 0$.

For 2-additive fuzzy measures we have simply

$$\mu(T) = \sum_{i \in T} m_\mu(i) + \sum_{\{i, j\} \subseteq T} m_\mu(ij), \qquad T \subseteq N. \tag{3.4}$$

Definition 3.1. Let μ be a fuzzy measure on N. The *discrete Choquet integral* of a function $f : N \to [0, 1]$ with respect to μ is defined by

$$\int f d\mu = C_\mu(f(1), \ldots, f(n)) = \sum_{i=1}^{n} [f((i)) - f((i-1))] \mu(A_{(i)}), \tag{3.5}$$

where (i) indicates a permutation on N so that $f((1)) \leq f((2)) \leq \cdots \leq f((n))$, and $A_{(i)} = \{(i), \ldots, (n)\}$. Also $f((0)) = 0$.

From now on we call $f(i) = x_i$, so the Choquet integral of the vector $(x_1, x_2, ..., x_n) \in [0, 1]^n$ with respect to the fuzzy measure μ is as follows:

$$C_\mu(x_1, ..., x_n) = \sum_{i=1}^{n} (x_{(i)} - x_{(i-1)})\mu(A_{(i)}), \tag{3.6}$$

where $x_{(i)} \in (x_1, x_2, ..., x_n)$, $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$, $A_{(i)} = \{(i), ..., (n)\}$, and $x_{(0)} = 0$.

The Choquet integral is nothing else than a linear combination of the marginal gains (differences) between the ordered criteria. In this sense, it extends the weighted average, but in it is not a linear operator, because it requires a preliminary ordering between the criteria. Alternatively, the Choquet integral can be written as

$$C_\mu(x_1, ..., x_n) = \sum_{i=1}^{n} x_{(i)}(\mu(A_{(i)}) - \mu(A_{(i+1)})), \tag{3.7}$$

where $A_{(n+1)} = \emptyset$.

For instance, if $x_2 \leq x_3 \leq x_1$, we have

$$C_\mu(x_1, ..., x_n) = x_2[\mu(2, 3, 1) - \mu(3, 1)] + x_3[\mu(3, 1) - \mu(1)] + x_1\mu(1). \tag{3.8}$$

The Choquet integral satisfies several properties (see, e.g., Grabisch [22]; Marichal [34]; Ghirardato and Le Breton [17]) and for the purposes of our paper we recall the following ones (for all $\mathbf{x}, \mathbf{x}' \in [0, 1]^n$)

$$x_i \leq x_i' \quad \forall i \in N \Rightarrow C_\mu(x_1, x_2, ..., x_n) \leq C_\mu(x_1', x_2', ..., x_n') \text{ (Monotonicity)} \tag{3.9}$$

$$x_i < x_i' \ \forall i \in N \Rightarrow C_\mu(x_1, x_2, ..., x_n) < C_\mu(x_1', x_2', ..., x_n') \text{ (Strict Monotonicity)} \tag{3.10}$$

$$\min(x_1, x_2, ..., x_n) \leq C_\mu(x_1, x_2, ..., x_n) \leq \max(x_1, x_2, ..., x_n). \tag{3.11}$$

Moreover, Choquet integral is continuous.

Let we remark that if the fuzzy measure is additive, it coincides with the weighted average, and if every subset with the same cardinality has the same measure, it collapses into the OWA operator (Fodor et al. [14]; Grabisch, [20]).

Using the Möbius transform, the Choquet integral can be alternatively written as (Marichal [34])

$$C_\mu(x_1, x_2, ..., x_n) = \sum_{T \subseteq N} m_\mu(T) \min_{i \in T}\{x_i\}. \tag{3.12}$$

In the case of 2-additive fuzzy measures, we have simply

$$C_\mu(x_1, x_2, ..., x_n) = \sum_{i \in N} m_\mu(i) x_i + \sum_{\{i,\, j\} \subseteq N} m_\mu(ij) \min\{x_i, x_j\}. \tag{3.13}$$

The more or less tendency to pessimism or to optimism of an expert can be featured by his own fuzzy measure. In particular, we consider the andness index together with its complementary orness index (Dujmovic [12]). If the orness index is close to 0 (andness $= 1$), the fuzzy measure reflects a pessimistic behaviour of the decision maker, and its Choquet integral is nothing else that the min operator, i.e., the logical conjunction of the criteria values (conservative behaviour of the decision maker). On the opposite, when orness $= 1$, the fuzzy measure reflects an optimistic behaviour of the decision maker and the Choquet integral becomes the max operator, i.e., the logical disjunction of the criteria values (fully compensative). For any Choquet integral we have

$$orness(C_\mu) = \frac{1}{n-1} \sum_{T \subseteq N} \frac{n-t}{t+1} m_\mu(T) \tag{3.14}$$

being $m_\mu(T)$ the Möbius values of the fuzzy measure. Moreover, $andness(C_\mu) = 1 - orness(C_\mu)$ for any fuzzy measure μ on N. Both indices can be easily computed given the values of the fuzzy measure. Shapley power and interaction indices as well can be computed (Shapley [47]; Murofushi [40]; Grabisch and Roubens [24]) together with other ones like veto and favour indices (Marichal [35]).

4. The CAT Algorithm

The main scope we would like to achieve through the use of Choquet integral is to combine the inputs in such a way that not only the importance of individual inputs as in weighted means, or their magnitude as in OWA, matter, but the importance of their coalitions as well. It means that an input might be not relevant by itself becoming very important when merged with some other inputs.

We start considering the Choquet-based aggregation in a single node of the attack tree. According to the scheme proposed by Yager [55] for the OWA trees, we associate to each node a fuzzy measure $\mu : 2^N \to [0, 1]$, defined on the set N of his child nodes.

As special cases, if we consider the fuzzy measure

$$\mu_{and}(T) = \begin{cases} 1 & T = N, \\ 0 & \text{otherwise}, \end{cases} \tag{4.1}$$

then the node corresponds to an "and" node, while if we consider the fuzzy measure

$$\mu_{or}(T) = \begin{cases} 1 & T \neq \emptyset, \\ 0 & \text{otherwise}, \end{cases} \tag{4.2}$$

then the node corresponds to an "or" node.

Now we want to evaluate the attributes associated with the tree. Suppose that a node has n children, and denote x_i is the attribute value associated with the child i, see Figure 2. We want to assign to this node a single attribute value, aggregating the values $\{x_1, x_2, ..., x_n\}$. We do this by means of the Choquet integral with respect to the fuzzy measure μ associated to this node. Therefore, we obtain for the node the attribute value $C_\mu(x_1, x_2, ..., x_n)$, see Equation (3.7).

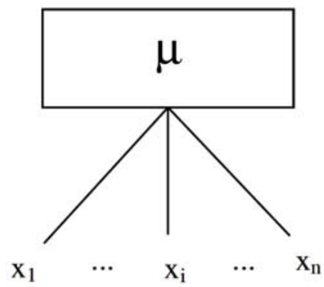

Figure 2. A node with attribute valued children.

Again, as special cases, if we consider the fuzzy measure μ_{and}, we obtain

$$C_{\mu_{and}}(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} x_{(i)}(\mu_{and}(A_{(i)}) - \mu_{and}(A_{(i+1)}))$$

$$= x_{(1)} = \min(x_1, ..., x_n), \quad (4.3)$$

and if we consider the fuzzy measure μ_{or}, we obtain

$$C_{\mu_{or}}(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} x_{(i)}(\mu_{or}(A_{(i)}) - \mu_{or}(A_{(i+1)}))$$

$$= x_{(n)} = \max(x_1, ..., x_n), \quad (4.4)$$

thus μ_{and} performs the same as an “and”, and μ_{or} performs the same as an “or”.

For any other choice of the fuzzy measure μ, according to Equation (3.11), we obtain

$$\min(x_1, x_2, ..., x_n) \le C_{\mu}(x_1, x_2, ..., x_n) \le \max(x_1, x_2, ..., x_n). \quad (4.5)$$

From now on, we simply say nodes values instead of attribute values on the nodes.

Now we show how Choquet integral can be used to valuate the attack tree via a bottom-up algorithm, starting from the leaves. At first, each leaf contains a numerical value, while the remaining nodes are empty. Moreover, each empty node will be characterized by its own fuzzy measure referring to every coalition of child nodes converging to it. To each node i of the tree, except leaves nodes, is assigned a fuzzy measure $\mu_i : 2^{N_i} \to [0, 1]$, where N_i is the set of the child nodes of the ith node. We will obtain the value of the ith node aggregating, through Choquet integral, the values of the child nodes converging to it.

The attack tree is then described as a triplet $\mathbf{Tree} = \{\mathbf{Node}, \mathbf{Val}, \boldsymbol{\mu}\}$, where

- **Node** is a vector of dynamical arrays whose ith row contains the parents of the ith node. For the root node, it is the null set

- **Val** is a vector containing for the leaves the original value and for the remaining the aggregated value

- $\boldsymbol{\mu}$ is a vector of dynamical arrays whose ith row contains the values of fuzzy measure associated to the ith node.

Note that we require $2^{|N_i|} - 2$ coefficients in $[0, 1]$ in order to define the fuzzy measure μ_i being $N_i = \{j : Node(j) = i\}$ the set of the child nodes of the ith node.

For instance, consider the following tree containing 8 nodes and assume that the values attached to the 5 leaves are (0.5, 0.9, 0.1, 0.6, 0.4).

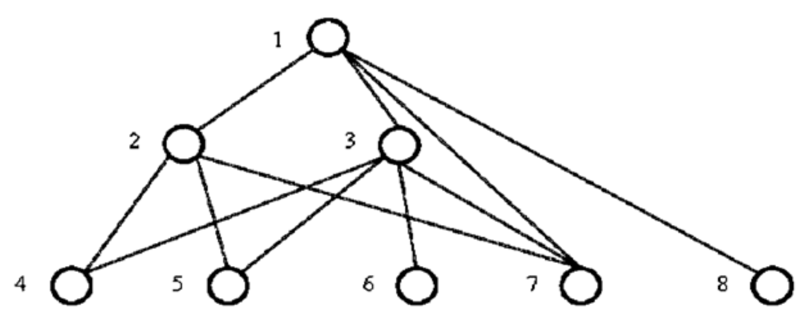

Figure 3. A tree containing 8 nodes.

Accordingly, the tree can be represented as $\mathbf{Tree} = \{\mathbf{Node}, \mathbf{Val}, \boldsymbol{\mu}\}$, where

$$\mathbf{Node} = \begin{bmatrix} - & & \\ 1 & & \\ 1 & & \\ 2 & 3 & \\ 2 & 3 & \\ 3 & & \\ 2 & 3 & 1 \\ 1 & & \end{bmatrix}, \qquad \mathbf{Val} = \begin{bmatrix} * \\ * \\ * \\ 0.5 \\ 0.9 \\ 0.1 \\ 0.6 \\ 0.4 \end{bmatrix}, \qquad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \boldsymbol{\mu}_3 \\ - \\ - \\ - \\ - \\ - \end{bmatrix}.$$

Clearly, the fuzzy measures associated to the nodes 1, 2, 3, need to be specified trough vectors with cardinality $2^4 - 2 = 14$, 6, and 6 respectively; in particular:

$$\boldsymbol{\mu}_1 = (\mu_1(2), \mu_1(3), \mu_1(7), \mu_1(8), \mu_1(2, 3), \mu_1(2, 7), \mu_1(2, 8), \mu_1(3, 7),$$
$$\mu_1(3, 8), \mu_1(7, 8), \mu_1(2, 3, 7), \mu_1(2, 3, 8), \mu_1(2, 7, 8), \mu_1(3, 7, 8)).$$

The following pseudo-Pascal code describes the CAT algorithm, a top-down recursive algorithm which computes the aggregate value.

// input: the vector Tree

// output: the value of the aggregated Choquet integral

READ $\mathbf{Tree} = \{\mathbf{Node}, \mathbf{Val}, \boldsymbol{\mu}\}$

PROCEDURE CV := Choquet($\boldsymbol{\mu}$, N, $\mathbf{x}$); // input: set N of child nodes,

// vector $\mathbf{x}$ of values and values

// of fuzzy measure μ; output:

// the aggregated value CV

$CV = \sum_{i=1}^{n} (x_{(i)} - x_{(i-1)}) \cdot \mu(A_{(i)})$ // Choquet computation, see formula (3.6)

```
END CV;

PROCEDURE CAT(μ, B);

    j := 1

    REPEAT

    IF VAL(B(j)) ≠ * THEN

        x(j) := VAL(B(j))

    ELSE

        A := {k ∈ N : Node(k) = B(j)}

        x(j) := CAT(μ_B(j), A);

    j := j + 1;

    UNTIL j > card(B)

CAT := Choquet(μ, B, x);

END CAT;

B := {j ∈ N : Node(j) = 1}; // determine the children set of the root node

WRITE Output := CAT(μ_1, B) // call the procedure with the root values and

                            // print the result
```

The algorithm runs top-down in a recursive way, and furnishes the aggregated Choquet values. The complexity is linear versus the number of nodes, and exponential for every node, but usually the cardinality of the set of the child nodes of each node is small enough to make the computation efficient.

As an illustrative example, consider the attack tree in Figure 1 and label the 13 nodes as OpenSafe = 1, PickLock = 2, LearnCombo = 3, …, GetTargetToStateCombo = 13.

Moreover, suppose that the (normalized) leaves values are:

PickLock (node 2) = 0.5

CutOpenSafe (node 4) = 0

Install-Improperly (node 5) = 0.2

FindWritenCombo (node 6) = 0.9

Threaten (node 8) = 0

BlackMail (node 9) = 0.4

Bribe (node 11) = 0.5

ListenToConversation (node 12) = 0.6

GetTargetToStateCombo (node 13) = 0.2.

Thus Tree is defined as $\mathbf{Tree} = \{\mathbf{Node}, \mathbf{Val}, \boldsymbol{\mu}\}$, where

$$\mathbf{Node} = \begin{bmatrix} - \\ 1 \\ 1 \\ 1 \\ 1 \\ 3 \\ 3 \\ 7 \\ 7 \\ 7 \\ 7 \\ 10 \\ 10 \end{bmatrix}, \qquad \mathbf{Val} = \begin{bmatrix} * \\ 0.5 \\ * \\ 0 \\ 0.2 \\ 0.9 \\ * \\ 0 \\ 0.4 \\ * \\ 0.5 \\ 0.6 \\ 0.2 \end{bmatrix}, \qquad \boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ - \\ \boldsymbol{\mu}_3 \\ - \\ - \\ - \\ \boldsymbol{\mu}_7 \\ - \\ - \\ \boldsymbol{\mu}_{10} \\ - \\ - \\ - \end{bmatrix}$$

and the capacities associated to the nodes 1, 3, 7, 10 need to be specified trough vectors with cardinality 14, 2, 14 and 2 respectively. Introduce now the following measures:

$$\boldsymbol{\mu}_1 = (\mu_1(2), \mu_1(3), \mu_1(4), \mu_1(5), \mu_1(2,3), \mu_1(2,4), \mu_1(2,5), \mu_1(3,4),$$
$$\mu_1(3,5), \mu_1(4,5), \mu_1(2,3,4), \mu_1(2,3,5), \mu_1(2,4,5), \mu_1(3,4,5))$$
$$= (0.2, 0.1, 0.3, 0.1, 0.3, 0.3, 0.3, 0.7, 0.2, 0.6, 0.7, 0.4, 0.6, 1)$$

$$\boldsymbol{\mu}_3 = \{\mu_3(6), \mu_3(7)\} = \{0.6, 0.9\}$$

$$\boldsymbol{\mu}_7 = \{\mu_7(8), \mu_7(9), \mu_7(10), \mu_7(11), \mu_7(8,9), \mu_7(8,10), \mu_7(8,11),$$
$$\mu_7(9,10), \mu_7(9,11), \mu_7(10,11), \mu_7(8,9,10), \mu_7(8,9,11),$$
$$\mu_7(8,10,11), \mu_7(9,10,11)\} = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$$

$$\boldsymbol{\mu}_{10} = \{\mu_{10}(12), \mu_{10}(13)\} = \{0, 0\}.$$

Note that the nodes 7 and 10 correspond to "and" and "or" nodes respectively. Therefore, C_{μ_7} corresponds to the max operator and $C_{\mu_{10}}$ corresponds to the min operator.

Moreover, μ_1 is a 2-additive fuzzy measure, and if we compute the orness indices, see Equation (3.10), we obtain

$$orness(C_{\mu_1}) = \frac{5}{12},\ orness(C_{\mu_3}) = \frac{3}{4},\ orness(C_{\mu_7}) = 1,\ orness(C_{\mu_{10}}) = 0.$$

Applying the CAT algorithm we obtain

$$x_{10} = C_{\mu_{10}}(x_{12}, x_{13}) = \min(x_{12}, x_{13}) = \min(0.6, 0.2) = 0.2$$

$$x_7 = C_{\mu_7}(x_8, x_9, x_{10}, x_{11}) = \max(x_8, x_9, x_{10}, x_{11})$$
$$= \max(0, 0.4, 0.2, 0.5) = 0.5$$

$$x_3 = C_{\mu_3}(x_6, x_7) = C_{\mu_3}(0.9, 0.5) = 0.5 + (0.9 - 0.5) \cdot 0.6 = 0.74$$

$$x_1 = C_{\mu_1}(x_2, x_3, x_4, x_5) = C_{\mu_1}(0.5, 0.74, 0, 0.2)$$
$$= 0 + (0.2 - 0) \cdot 0.4 + (0.5 - 0.2) \cdot 0.3 + (0.74 - 0.5) \cdot 0.1 = 0.194.$$

So far, the risk of an attack is low even though the first two child nodes of the root, PickLock and LearnCombo are medium-high valued (0.5 and 0.74 respectively), because the relative coalition weights are relatively small.

Finally, let us remark that among the existing methods used to assign the values of the capacities, we decided to adopt the one proposed by Despic and Simonovic [10], based on a questionnaire to be submitted to the experts in order to achieve their subjective evaluations of the leaves.

5. Extension to Fuzzy-valued Attributes

The estimation of the attributes' values along the attack tree is usually based on data type depending on subjective judgments, most commonly represented by natural language expressions. Following Zadeh [58, 59], here we assume to translate these expressions into the mathematical formalism of possibility measures and to represent the numeric imprecision of attributes' values using unimodal *LR* fuzzy numbers, as fuzzy subsets of the set of real numbers (Dubois and Prade [11]).

Definition 5.1. A unimodal *LR* fuzzy number *A* is defined by

$$A(x) = \begin{cases} L\left(\dfrac{a-x}{a_1}\right) & \text{for } a - a_1 \leq x \leq a, \\ R\left(\dfrac{x-a}{a_2}\right) & \text{for } a \leq x \leq a + a_2, \\ 0 & \text{else}, \end{cases} \tag{5.1}$$

where $a \in R$ is the peak of *A*, $\alpha > 0$ and $\beta > 0$ are the left and the right spread, respectively, and $L, R : [0, 1] \to [0, 1]$ are two strictly decreasing continuous shape function such that $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$.

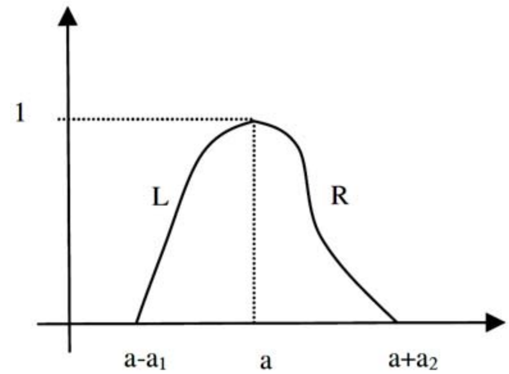

Figure 4. Unimodal *LR* fuzzy number.

In this section, the CAT algorithm is extended assuming that values on the leaves are unimodal *LR* fuzzy numbers. Extending the Choquet integral to a fuzzy domain several forms of information can be handled at the same time, i.e., crisp data, interval values, fuzzy numbers and linguistic variables (Yang et al. [56]).

At first, the Choquet integral is defined for a measurable interval-valued function (Aumann [1]), and then it is extended to fuzzy integrand using the alpha-cuts (Grabisch et al. [21]).

From now on, we introduce the following notations:

- $\mathcal{I}$ the set of interval numbers (rectangular fuzzy numbers).
- $N = \{1, 2, ..., n\}$ a set of elements.
- $F : N \to \mathcal{I}$ an interval-valued function.
- $F_L(i)$ and $F_R(i)$ respectively the left end point and the right end point of the interval $F(x)$.
- $\mathcal{F}$ the set of all unimodal *LR*-type fuzzy numbers.
- $[{}^{L}A^{\alpha}, {}^{R}A^{\alpha}]$ the alpha-cut of fuzzy number *A*.
- $\Phi : N \to \mathcal{F}$ a unimodal *LR* fuzzy-valued function.

- $\mathcal{F}$-tree, an attack tree whose leaves' values are unimodal *LR* fuzzy numbers.

The following definitions are due to Yang et al. [56]:

Definition 5.2. $F(i)$ is *measurable* if both $F_L(i)$ and $F_R(i)$ are measurable functions.

Definition 5.3. The *Choquet integral* of $F(i)$ with respect to a fuzzy measure μ is defined as

$$\int F d\mu = \left\{ \int G d\mu \,|\, G(i) \in F(i)\ \forall i \in N, \text{ and } G(i) \text{ measurable} \right\}.$$

Definition 5.4. $\Phi(i)$ is *measurable* if its alpha-cuts $\Phi^{\alpha}(i)$ are measurable interval-valued functions for every $\alpha \in (0, 1]$.

Definition 5.5. Given a measurable fuzzy-valued function $\Phi(i)$ on N and a fuzzy measure μ on 2^N, the *Choquet integral* of $\Phi(i)$ with respect to μ is defined as

$$\int \Phi d\mu = \bigcup_{0 \le \alpha \le 1} \alpha \int \Phi^{\alpha} d\mu. \tag{5.2}$$

Accordingly, the calculation of the Choquet integral with a fuzzy-valued function depends on the calculation of the Choquet integral with interval-valued functions, and the following proposition can be proved (Grabisch et al. [21]).

Proposition 5.1. *Given the measurable interval-valued function* Φ^{α} *and the fuzzy measure* μ *on* 2^N, *the Choquet integral of* Φ^{α} *with respect to* μ *is*

$$\int \Phi^{\alpha} d\mu = \left[\int \Phi_L^{\alpha} d\mu, \int \Phi_R^{\alpha} d\mu \right]. \tag{5.3}$$

Therefore (5.2) becomes

$$\int \Phi d\mu = \bigcup_{0 \le \alpha \le 1} \alpha \left[\int \Phi_L^{\alpha} d\mu, \int \Phi_R^{\alpha} d\mu \right]. \tag{5.4}$$

Consider now an $\mathcal{F}$-tree whose leaves' values are unimodal *LR* fuzzy numbers.

To prove that the root value is still a unimodal *LR* fuzzy number, we introduce the following.

Proposition 5.2. *The Choquet integral of unimodal LR fuzzy numbers is still a unimodal LR fuzzy number*.

Proof. A generic unimodal *LR* fuzzy number *A* is characterized by an alpha-cut $[{}^{L}A^{\alpha},\ {}^{R}A^{\alpha}]$, where L^{α} and R^{α} are strictly monotonic continuous functions (with respect to α).

Consider now a set of unimodal *LR* fuzzy numbers $\{A_1, ..., A_k\}$. If we aggregate these fuzzy numbers through Choquet integral with respect to a fuzzy measure μ, we obtain a fuzzy number *A* characterized by the alpha-cut $[{}^{L}A^{\alpha},\ {}^{R}A^{\alpha}]$, where, see Equation (5.3),

$$ {}^{L}A^{\alpha} = C_{\mu}[{}^{L}A_1^{\alpha}, ..., {}^{L}A_k^{\alpha}], \quad {}^{R}A^{\alpha} = C_{\mu}[{}^{R}A_1^{\alpha}, ..., {}^{R}A_k^{\alpha}]. \tag{5.5}$$

In fact, from the strict monotonicity of the Choquet integral, and given that the lower bound of each alpha-cut is less than the relative upper bound, we have ${}^{L}A^{\alpha} < {}^{R}A^{\alpha}$.

Moreover, if we consider $0 \le \alpha_1 < \alpha_2 \le 1$ since ${}^{L}A_i^{\alpha_1} < {}^{L}A_i^{\alpha_2}$ and ${}^{R}A_i^{\alpha_1} > {}^{R}A_i^{\alpha_2}$ $\forall i = 1, ..., k,$ from the strict monotonicity of the Choquet integral we have

$$ {}^{L}A^{\alpha_1} < {}^{L}A^{\alpha_2} \qquad {}^{R}A^{\alpha_1} > {}^{R}A^{\alpha_2}. \tag{5.6}$$

Then L^{α} and R^{α} are strictly monotonic functions (with respect to α). Moreover, since Choquet integral is a continuous aggregation function, all L_i^{α} and R_i^{α} are continuous functions $\forall i = 1, ..., k,$ and the composition of continuous functions is continuous, then it follows that L^{α} and R^{α} are continuous functions (with respect to α).

Therefore, the Choquet integral of unimodal *LR* fuzzy numbers is still a unimodal *LR* fuzzy number.

Then, as an immediate consequence of Prop. 5.2, starting from the leaves and carrying on a bottom-up Choquet aggregation, the obtained tree root's value is again a unimodal (continuous) *LR* fuzzy number.

This result holds even when the same fuzzy sub-node's value is referred to two or more different higher-level nodes, like the nodes 4, 5 and 7 in the Figure 3, all of them connected to the nodes 2, 3. The two interactive fuzzy nodes 2 and 3 can be separately computed and again aggregated to the node 1 (with the leave node 8). Let us remark that the nodes 2 and 3 are interactive because they are functions of the same two variables 4, 5 and 7.

The fuzzy attribute value in the root can be computed applying the algorithm described in the Section 4 to every alpha-cut of the fuzzy attribute values on the leaves.

The algorithm proceeds as described below. First of all, the alpha-cuts of each unimodal *LR* fuzzy number in the leaves will be considered, using a suitable grid. The procedure receives the extremes of the alpha-cut, and computes the aggregated value for both the lower and the upper bounds. Increasing the values of alpha in between $[0, 1]$, the two computed values form and interval included in the previous ones (for lower value of alpha). Thus the obtained intervals form the alpha-cuts of the fuzzy root, the required solution.

The pseudo-Pascal code describes the generalized fuzzy-CAT algorithm. For simplicity, we consider only the alpha-cut computation; an external tool will be devoted to the computation of the alpha-cut extremes of the leaves fuzzy nodes, using a loop over a pre-fixed number of iterations.

Fuzzy-CAT algorithm (for the alpha-cut)

// input: the vector fuzzy Tree

// output: the value of the aggregated Choquet integral

// here $Val(i) = [Val_L(i), Val_R(i)]$, the two extremes of the alpha-cut; the same for $x(i) = [x_L(i),\ x_R(i)]$

READ **Tree** = {**Node**, **Val**, **μ**}

PROCEDURE CV := Choquet($\boldsymbol{\mu}$, N, $\mathbf{x}$); // input: set N of child nodes,

// vector **x** of fuzzy values and

// values of fuzzy measure μ,

// output: the aggregated fuzzy

// value CV

$$CV_L = \sum_{i=1}^{n} (x_{(i)}^{L} - x_{(i-1)}^{L}) \cdot m(A_{(i)})$$ // Choquet computation for the lower

// bound

$$CV_R = \sum_{i=1}^{n} (x_{(i)}^{R} - x_{(i-1)}^{R}) \cdot m(A_{(i)})$$ // Choquet computation for the upper

// bound

END CV;

PROCEDURE CAT($\boldsymbol{\mu}$, B);

j := 1

REPEAT

IF VAL(B(j)) ≠ ∗ THEN

$x_L(j) := VAL_L(B(j))$

$x_R(j) := VAL_R(B(j))$;

ELSE

$A = \{k \in N : Node(k) = B(j)\}$

$x_L(j) := CAT(B_L(j), A)$;

$x_R(j) := \mathrm{CAT}(B_R(j), A)$;

$j := j + 1$;

UNTIL $j > \mathrm{card}(B)$

CAT := Choquet$(\boldsymbol{\mu}, B, \mathbf{x})$;

END CAT;

$B := \{j \in N : \mathrm{Node}(j) = 1\}$; // determine the children set of the root node

WRITE Output $:= \mathrm{CAT}(\boldsymbol{\mu}_1, B)$ // call the procedure with the root values and

// print the result

The algorithm computes the lower and the upper bounds for the aggregated fuzzy Choquet values. The complexity is obviously the same of the basic CAT algorithm.

6. Conclusions

In this paper, our main concern has centered on the development on a framework for improving the effectiveness of the aggregation of the information along the so called attack trees (Schneier [45]). When addressing fraud detection problems, a suitable approach is represented by the introduction of a tree structure where the goal is represented by the root node and child nodes represent the different ways of achieving that goal. Accordingly, evaluation procedures involving the information about attributes associated to leave nodes can be carried on via "and" and/or" operators. As Yager [54] pointed out, in many real world applications related to security problems the "and/or" logic is not adequate to effectively represent the relationships between a parent node and its children, and therefore he proposed an extension of the "and/or" attack trees to what he called OWA-based attack trees.

In this paper, observing that OWA operators do not consider possible interactions between the nodes, we have shown how to use the Choquet integral to overcome this weakness of OWA-based attack trees. Then, the algorithm was extended assuming that the attribute values in the leaves are unimodal *LR* fuzzy numbers and the calculation of Choquet integral was implemented using the alpha-cuts.

Future work will be devoted to the extension of our approach to a multi experts framework aiming at representing the negotiation process involved in the representation and valuation of the attack tree, exploiting the results achieved in Giove [18] and Bortot et al. [3].

References

[1] R. J. Aumann, Integrals of set-valued functions, J. Math. Anal. Appl. 12 (1965), 1-12.

[2] G. Beliakov, A. Pradera and T. Calvo, Aggregation Functions: A Guide to Practitioners, Springer-Verlag, Heidelberg, 2007.

[3] S. Bortot, M. Fedrizzi and S. Giove, Choquet-based consensus modelling in multiple criteria group decision making, 21st International Conference on Multiple Criteria Decision Making, Jyväskylä, Finland, 2011.

[4] M. Cardin and S. Giove, Aggregation functions with non-monotonic measures, Fuzzy Economic Review 13(2) (2008), 3-15.

[5] M. Ceberio and F. Modave, An interval valued 2-additive Choquet integral for multicriteria decision making, 10th Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems, (IPMU), Perugia. Italy, 2004.

[6] A. Chateauneuf and J. Y. Jaffray, Some characterizations of lower probabilities and other monotone capacities through the use of Möbius inversion, Math. Social Sciences 17 (1989), 263-283.

[7] S. J. Chen and C. L. Hwang, Fuzzy Multiple Attribute Decision Making: Methods and Applications, Springer, Berlin, 1992.

[8] G. Choquet, Theory of capacities, Ann. Inst. Fourier 5 (1953), 131-295.

[9] A. De Waegenaere and P. P. Wakker, Nonmonotonic Choquet integrals, J. Math. Econom. 36 (2001), 45-60.

[10] O. Despic and S. P. Simonovic, Aggregation operators for soft decision making in water resources, Fuzzy Sets and Systems 115 (2000), 11-33.

[11] D. Dubois and H. Prade, Fuzzy numbers: An overview, Analysis of Fuzzy

Information - Vol. I: Mathematics and Logic, J. Bezdek, ed., CRC Press, Boca Raton, 1987, pp. 3-39.

[12] J. J. Dujmovic, Weighted conjunctive and disjunctive means and their application in system evaluation, Univ. Beograd. Publ. Elektrotehn. Fak. (1974), 147-158.

[13] J. Fodor and M. Roubens, Fuzzy Preference Modelling and Multicriteria Decision Support, Kluwer, Dordrecht, 1994.

[14] J. Fodor, J. L. Marichal and M. Roubens, Characterization of the ordered weighted averaging operators, IEEE Trans. Fuzzy Systems 3(2) (1995), 236-240.

[15] K. Fujimoto and T. Murofushi, Some characterizations of the systems represented by Choquet and multi-linear functional through the use of Möbius inversion, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 5(5) (1997), 547-561.

[16] K. Fujimoto, T. Murofushi and M. Sugeno, Canonical hierarchical decomposition of Choquet integral over a finite set with respect to null additive fuzzy measure, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 6(4) (1998), 345-363.

[17] P. Ghirardato and M. Le Breton, Choquet rationality, J. Economic Theory 90(2) (2000), 277-285.

[18] S. Giove, Fuzzy Measures and the Choquet Integral for Group Multicriteria Decision Making, Neural Nets, Springer, 2001, pp. 77-84.

[19] S. Giove, Non additive measures and Choquet integral with uncertain coefficients, Ratio Sociologica, J. Social Sciences: Theory Appl. 1(2) (2008), 3-20.

[20] M. Grabisch, On equivalence classes of fuzzy connectives - the case of fuzzy integrals, IEEE Trans. Fuzzy Systems 3(1) (1995a), 96-109.

[21] M. Grabisch, H. T. Nguyen and E. A. Walker, Fundamentals of Uncertainty Calculi, with Applications to Fuzzy Inference, Kluwer, Boston, MA, 1995b.

[22] M. Grabisch, The applications of fuzzy integrals in multicriteria decision making, European J. Oper. Res. 89(3) (1996), 445-456.

[23] M. Grabisch, *K*-order additive discrete fuzzy measures and their representation, Fuzzy Sets and Systems 92(2) (1997), 167-189.

[24] M. Grabisch and M. Roubens, An axiomatic approach to the concept of interaction among players in cooperative games, Int. J. Game Theory 28(4) (1999), 547-565.

[25] M. Grabisch and M. Roubens, Application of the Choquet integral in multicriteria decision making, Fuzzy Measures and Integrals - Theory and Applications, M. Grabisch, T. Murofushi and M. Sugeno, eds., Physica Verlag, Heidelberg, 2000, pp. 415-434.

[26] M. Grabisch, C. Labreuche and J. C. Vansnick, Construction of a decision model in the presence of interacting criteria, Proceedings of AGOP 2001, Asturias (Spain) (2001), 28-33.

[27] M. Grabisch, C. Labreuche and J. C. Vansnick, On the extension of pseudo-boolean functions for the aggregation of interacting criteria, European J. Oper. Res. 148(1) (2003), 28-47.

[28] M. Grabisch, I. Kojadinovic and P. Meyer, A review of methods for capacity identification in Choquet integral based multi-attribute utility theory, Applications of the Kappalab R package, European J. Oper. Res. 186 (2008), 766-785.

[29] M. Grabisch, J. L. Marichal, R. Mesiar and E. Pap, Aggregation Functions, Cambridge University Press, 2009.

[30] M. Grabisch and C. Labreuche, A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid, Ann. Oper. Res. 175 (2010), 247-286.

[31] E. P. Klement, R. Mesiar and E. Pap, Triangular Norms, Kluwer Academic Publishers, Dordrecht, Boston, London, 2000.

[32] E. P. Klement, R. Mesiar and E. Pap, A universal integral as common frame for Choquet and Sugeno integral, IEEE Trans. Fuzzy Systems 18(1) (2010), 178-187.

[33] KPMG Fraud Survey, htpp://kpmginstitutes.com/aci/insights/2009/pdf/kpmg-fraud-survey-2009.pdf, 2009.

[34] J. L. Marichal, Aggregation operators for multicriteria decision aid, Ph.D. Thesis, University of Liège, Belgium, 1998.

[35] J. L. Marichal, Aggregation of interacting criteria by means of the discrete Choquet integral, Preprint 9910, GEMME, Faculty of Economics, University of Liège, Liège, Belgium, 1999.

[36] J. L. Marichal, On Choquet and Sugeno integrals as aggregation functions, Fuzzy Measures and Integrals - Theory and Applications, M. Grabisch, T. Murofushi and M. Sugeno, eds., Physica Verlag, Heidelberg, 2000, pp. 247-272.

[37] Y. Matsushita and H. Kambara, Partition type fuzzy integral model for evaluation process, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 5(5) (1997), 531-545.

[38] P. Meyer and M. Roubens, On the use of the Choquet integral with fuzzy numbers in multiple criteria decision support, Fuzzy Sets and Systems 157(7) (2006), 927-938.

[39] T. Murofushi and M. Sugeno, An interpretation of fuzzy measures and the Choquet integral as an integral with respect to a fuzzy measures, Fuzzy Sets and Systems 29 (1989), 201-227.

[40] T. Murofushi, A technique for reading fuzzy measures (I): the Shapley value with respect to a fuzzy measure, Proc. 2nd Fuzzy Workshop, Nagaoka, Japan, 1992, pp. 39-48 (in Japanese).

[41] T. Murofushi, M. Sugeno and D. Machida, Non-monotonic fuzzy measures and the Choquet integral, Fuzzy Sets and Systems 64 (1994), 73-86.

[42] T. Murofushi, K. Fujimoto and M. Sugeno, Canonical separated hierarchical decomposition of the Choquet integral over a finite set, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 6(3) (1998), 257-272.

[43] Y. Rébillé, Sequentially continuous non-monotonic Choquet integrals, Fuzzy Sets and Systems 153 (2005), 79-94.

[44] G. C. Rota, On the foundations of combinatorial theory I. Theory of Möbius functions, Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebeite 2 (1964), 340-368.

[45] B. Schneier, Attack trees, Dr Dobb's J. 24(12) (1999).

[46] B. Schneier, Secrets and Lies: Digital Security in a Networked World, Wiley and Sons, NY, 2000.

[47] L. S. Shapley, A value for *n*-person games, Contributions to the Theory of Games, Vol. II, H. W. Kuhn and A. W. Tucker, eds., Annals of Mathematics Studies, Princeton University Press, 1953, pp. 307-317.

[48] M. Sugeno and K. Fujimoto, A hierarchical decomposition of Choquet integral model, Internat. J. Uncertain. Fuzziness Knowledge-Based Systems 3(1) (1995), 1-15.

[49] P. Vincke, Multicriteria Decision Aid, John Wiley and Sons, NY, 1992.

[50] C. Von Altrock, Fuzzy Logic and Neurofuzzy Applications Explained, Prentice Hall, Inc., Upper Saddle River, NJ, 1995.

[51] R. R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Trans. Systems, Man and Cybernetics 18 (1988), 183-190.

[52] R. R. Yager, Applications and extensions of OWA aggregation, Internat. J. Man Machine Studies 37 (1992), 103-132.

[53] R. R. Yager, Families of OWA operators, Fuzzy Sets and Systems 59(2) (1993), 125-148.

[54] R. R. Yager and J. Kacprzyk, The Ordered Weighted Averaging Operators: Theory and Application, Kluwer Academic Publisher, Norwell, MA, 1997.

[55] R. R. Yager, OWA trees and their role in security modelling using attack trees, Inform. Sci. 176 (2006), 2933-2959.

[56] R. Yang, Z. Wang, P. A. Heng and K. S. Leung, Fuzzy numbers and fuzzification of the Choquet integral, Fuzzy Sets and Systems 153 (2005), 95-113.

[57] Z. Wang, R. Yang, P. A. Heng and K. S. Leung, Real-valued Choquet integrals with fuzzy valued integrand, Fuzzy Sets and Systems 157 (2006), 256-259.

[58] L. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems (1978), 3-28.

[59] L. Zadeh, A theory of approximate reasoning, Machine Intelligence 9, J. Hayes, D. Michie and L. Mikulich, eds., Halsted Press, New York, 1979, pp. 149-194.

[60] L. Zadeh, A computational approach to fuzzy quantifiers in natural language, Comp. Math. Appl. 9(1) (1983), 149-184.