



DESIGN METHOD OF OBSERVER-BASED OUTPUT FEEDBACK CONTROLLER FOR UNCERTAIN T-S FUZZY DISCRETE SYSTEMS WITH TIME-DELAY

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Abstract

In this paper, we consider the stability analysis of uncertain discrete systems with time-delay when the states are unmeasurable. Takagi-Sugeno (T-S) fuzzy model is used to describe this kind of systems. Based on Lyapunov functional approach, a new design method of observer-based output feedback controller is proposed. A numerical example is given to illustrate the effectiveness of our method.

1. Introduction

In the past few decades, a number of research activities have been concentrated on the topic of stability analysis of nonlinear time-delay systems with parameter uncertainties. This kind of systems can be found in many real life systems such as electric systems, rolling mill systems, different types of societal systems and so on. Takagi-Sugeno (T-S) fuzzy model [10] can be used to describe this kind of systems.

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For continuous-time systems, Cao et al. [1, 5, 8, 9, 17, 19] considered the robust H_∞ control; and Huang et al. [6, 13, 14, 16, 20] proposed some design methods of fuzzy controller for the discrete-time systems. When the states are measurable, Antai et al. [3] designed a stabilizing state-feedback controller for uncertain time-delay systems, Li and Xia [7] studied the H_2 controller design for uncertain discrete-time delay piecewise systems, and stabilizability of linear quadratic state-feedback control problem has been researched in [12]. Under the unmeasurable states environment, stability and stabilization conditions for uncertain fuzzy systems with time-delay are also considered via output feedback controller in [11, 15, 16]. In a previous paper [4], we proposed an observer-based output feedback controller design method for a class of uncertain T-S fuzzy systems with time-delay, this method fixed an error in [11], and now we pay attention to stability analysis of the discrete-time systems when the states are unmeasurable in this paper.

Notation. For a symmetric matrix X , the notation $X > 0$ means that the matrix X is positive definite. I is an identity matrix of appropriate dimension. X^T denotes the transpose of matrix X . For any nonsingular matrix X , X^{-1} denotes the inverse of matrix X . R^n denotes the n -dimensional Euclidean space. $R^{m \times n}$ is the set of all $m \times n$ matrices. $*$ denotes the transposed element in the symmetric position of a matrix.

2. System Description

Consider the following parameter uncertain discrete system with time-delay described by Takagi-Sugeno fuzzy model:

Plant rule i . If $z_1(t)$ is λ_{i1} and $z_2(t)$ is λ_{i2} and \dots and $z_g(t)$ is λ_{ig} , then

$$\begin{cases} x(t+1) = \tilde{A}_{i1}x(t) + \tilde{A}_{i2}x(t-d) + \tilde{B}_i u(t), \\ y(t) = C_{i1}x(t) + C_{i2}x(t-d), \\ x(t) = \varphi(t), \quad t \in [-d, 0], \end{cases} \quad (1)$$

where $i = 1, 2, \dots, n$, n is the number of rules; $z_1(t), z_2(t), \dots, z_g(t)$ are the premise variables; λ_{ij} ($i = 1, 2, \dots, n, j = 1, 2, \dots, g$) is the fuzzy set; $x(t) \in R^q$ is the state vector; $u(t) \in R^m$ is the input vector; $y(t) \in R^l$ is the output vector; $d > 0$ is the

upper bound of time-delay; $\varphi(t)$ is the initial condition of system (1); $\tilde{A}_{i1} \triangleq A_{i1} + \Delta A_{i1}(t)$, $\tilde{A}_{i2} \triangleq A_{i2} + \Delta A_{i2}(t)$ and $\tilde{B}_i \triangleq B_i + \Delta B_i(t)$; A_{i1} , A_{i2} , B_i , C_{i1} and C_{i2} ($i = 1, 2, \dots, n$) are constant matrices of appropriate dimensions; $\Delta A_{i1}(t)$, $\Delta A_{i2}(t)$, $\Delta B_i(t)$ ($i = 1, 2, \dots, n$) are real valued unknown matrices representing time-varying parameter uncertainties of (1) and satisfy the following assumption:

Assumption 1.

$$[\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)] = U_i F_i(t) [E_{i1}, E_{i2}, E_i], \quad (2)$$

where U_i , E_{i1} , E_{i2} and E_i ($i = 1, 2, \dots, n$) are known real constant matrices of appropriate dimensions. $F_i(t)$ ($i = 1, 2, \dots, n$) is an unknown real time-varying matrix with Lebesgue measurable elements satisfying

$$F_i(t)^T F_i(t) \leq I, \quad i = 1, 2, \dots, n. \quad (3)$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\rho_i(z(t))$, i.e.,

$$\mu_i(z(t)) = \frac{\rho_i(z(t))}{\sum_{i=1}^n \rho_i(z(t))},$$

where

$$z(t) = [z_1(t), z_2(t), \dots, z_g(t)], \quad \rho_i(z(t)) = \prod_{j=1}^g \lambda_{ij}(z_j(t)).$$

$\lambda_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in λ_{ij} . It is assumed that

$$\rho_i(z(t)) \geq 0, \quad i = 1, 2, \dots, n, \quad \sum_{i=1}^n \rho_i(z(t)) > 0, \quad \forall t \geq 0.$$

Then, it can be seen that

$$\mu_i(z(t)) \geq 0, \quad i = 1, 2, \dots, n, \quad \sum_{i=1}^n \mu_i(z(t)) = 1, \quad \forall t \geq 0.$$

By using the center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy model (1) can be expressed by the following model:

$$\begin{cases} x(t+1) = \sum_{i=1}^n \mu_i \{ \tilde{A}_{i1}x(t) + \tilde{A}_{i2}x(t-d) + \tilde{B}_i u(t) \}, \\ y(t) = \sum_{i=1}^n \mu_i \{ C_{i1}x(t) + C_{i2}x(t-d) \}, \end{cases} \quad (4)$$

where $\mu_i \triangleq \mu_i(z(t))$.

Similar to the continuous system in [4], we design the fuzzy state observer as follows:

R^i : If $z_1(t)$ is λ_{i1} and $z_2(t)$ is λ_{i2} and \dots and $z_g(t)$ is λ_{ig} , then

$$\begin{cases} \hat{x}(t+1) = A_{i1}\hat{x}(t) + A_{i2}\hat{x}(t-d) + B_i u(t) + G_i(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C_{i1}\hat{x}(t) + C_{i2}\hat{x}(t-d), \quad i = 1, 2, \dots, n, \end{cases} \quad (5)$$

where $G_i \in R^{q \times l}$ is the observer gain to be determined. Then the overall fuzzy observer is given by

$$\begin{cases} \hat{x}(t+1) = \sum_{i=1}^n \mu_i \{ A_{i1}\hat{x}(t) + A_{i2}\hat{x}(t-d) + B_i u(t) + G_i(y(t) - \hat{y}(t)) \}, \\ \hat{y}(t) = \sum_{i=1}^n \mu_i \{ C_{i1}\hat{x}(t) + C_{i2}\hat{x}(t-d) \}. \end{cases} \quad (6)$$

Based on fuzzy observer, we can design the overall output feedback controller:

$$u(t) = \sum_{i=1}^n \mu_i K_i \hat{x}(t), \quad (7)$$

where $K_i \in R^{m \times q}$ is the controller gain to be determined.

Define the observation error as follows:

$$e(t) = x(t) - \hat{x}(t). \quad (8)$$

Combining (4), (6), (7) and (8), we can obtain the global model:

$$\begin{cases} x(t+1) = \sum_{i=1}^n \sum_{j=1}^n \mu_i \mu_j \{(\tilde{A}_{i1} + \tilde{B}_i K_j) x(t) + \tilde{A}_{i2} x(t-d) - \tilde{B}_i K_j e(t)\}, \\ e(t+1) = \sum_{i=1}^n \sum_{j=1}^n \mu_i \mu_j \{R_{ij} x(t) + \Delta A_{i2}(t) x(t-d) + S_{ij} e(t) + T_{ij} e(t-d)\}, \end{cases} \quad (9)$$

where $R_{ij} = \Delta A_{i1}(t) + \Delta B_i(t) K_j$, $S_{ij} = A_{i1} - G_i C_{j1} - \Delta B_i(t) K_j$, $T_{ij} = A_{i2} - G_i C_{j2}$.

3. Main Result

Two important lemmas should be introduced because they are the key to prove the main theorem.

Lemma 1 [2]. For any two matrices $X \in R^{m \times n}$, $Y \in R^{m \times n}$, we have

$$X^T Y + Y^T X \leq X^T S X + Y^T S^{-1} Y,$$

where $S \in R^{m \times m}$ and $S > 0$.

Lemma 2 [18]. Y , U and E are the matrices of appropriate dimensions, and $Y = Y^T$, then for any matrix F satisfying $F^T F \leq I$, we have the following equivalent condition:

$$Y + UFE + E^T F^T U^T < 0$$

if and only if there exists a constant $\varepsilon > 0$ satisfying

$$Y + \varepsilon U U^T + \varepsilon^{-1} E^T E < 0.$$

When the states are unmeasurable, based on the Lyapunov functional approach, the delay-independent stabilization result of T-S fuzzy discrete system (9) is summarized in the following theorem:

Theorem 1. For the prescribed scalars $\varepsilon_{ij} > 0$ and $\eta_{ij} > 0$ ($1 \leq i \leq j \leq n$), if there exist matrices $X > 0$, $\tilde{P}_2 > 0$, $P_3 > 0$, $P_4 > 0$, Y_i and N_i ($i, j = 1, 2, \dots, n$) satisfying the following LMIs:

$$\begin{bmatrix} \Omega_{11}^{ii} & * \\ \Omega_{21}^{ii} & \Omega_{22}^{ii} \end{bmatrix} < 0, \quad 1 \leq i \leq n, \quad (10)$$

$$\begin{bmatrix} \Omega_{11}^{ij} + \Omega_{11}^{ji} & * & * \\ \Omega_{21}^{ij} & \Omega_{22}^{ij} & * \\ \Omega_{21}^{ji} & 0 & \Omega_{22}^{ji} \end{bmatrix} < 0, \quad 1 \leq i < j \leq n, \quad (11)$$

$$\Psi_{ii} < 0, \quad 1 \leq i \leq n, \quad (12)$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad 1 \leq i < j \leq n, \quad (13)$$

then the closed-loop fuzzy system (9) is asymptotically stable. Moreover, the controller gains are given by

$$K_i = Y_i X^{-1}, \quad (14)$$

where

$$\begin{aligned} \Omega_{11}^{ij} &= \begin{bmatrix} -X + \tilde{P}_2 & * & * & * \\ 0 & -\tilde{P}_2 & * & * \\ A_{i1}X + B_iY_j & A_{i2}X & -\frac{1}{3}X + \varepsilon_{ij}U_iU_i^T & * \\ 0 & 0 & \varepsilon_{ij}P_3U_iU_i^T & -\frac{1}{4}P_3 \end{bmatrix}, \\ \Omega_{21}^{ij} &= \begin{bmatrix} E_{i1}X + E_iY_j & E_{i2}X & 0 & 0 \\ 0 & 0 & 0 & U_i^T P_3 \end{bmatrix}, \quad \Omega_{22}^{ij} = \begin{bmatrix} -\varepsilon_{ij}I & * \\ 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix}, \\ \Psi_{ij} &= \begin{bmatrix} \Lambda_{ij} & * & * & * \\ 0 & -P_4 & * & * \\ B_iK_j & 0 & -\frac{1}{3}X + \eta_{ij}U_iU_i^T & * \\ A_{i1} - G_iC_{j1} & A_{i2} - G_iC_{j2} & -\eta_{ij}U_iU_i^T & -\frac{1}{4}P_3^{-1} + \eta_{ij}U_iU_i^T \end{bmatrix}, \end{aligned}$$

$$\Lambda_{ij} = -P_3 + P_4 + \eta_{ij}^{-1}K_j^T E_i^T E_i K_j.$$

Proof. Choose the Lyapunov function as

$$\begin{aligned} V(t) &= x^T(t)P_1x(t) + \sum_{w=1}^d x^T(t-w)P_2x(t-w) \\ &\quad + e^T(t)P_3e(t) + \sum_{w=1}^d e^T(t-w)P_4e(t-w), \end{aligned} \quad (15)$$

where $P_k > 0$ ($k = 1, 2, 3, 4$). Let

$$V_1(x(t)) = x^T(t)P_1x(t) + \sum_{w=1}^d x^T(t-w)P_2x(t-w),$$

$$V_2(e(t)) = e^T(t)P_3e(t) + \sum_{w=1}^d e^T(t-w)P_4e(t-w),$$

$$H_{ij} = A_{i1} + \Delta A_{i1}(t) + (B_i + \Delta B_i(t))K_j.$$

Then we have

$$\begin{aligned} \Delta V_1(x(t)) &= V_1(x(t+1)) - V_1(x(t)) \\ &= x^T(t+1)P_1x(t+1) - x^T(t)P_1x(t) + x^T(t)P_2x(t) - x^T(t-d)P_2x(t-d) \\ &= \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \mu_i \mu_j \mu_k \mu_l \\ &\quad \cdot \left\{ x^T(t) \left[\left(\frac{H_{ij} + H_{ji}}{2} \right)^T P_1 \left(\frac{H_{kl} + H_{lk}}{2} \right) - P_1 + P_2 \right] x(t) \right. \\ &\quad + x^T(t-d) \left[\left(\frac{\tilde{A}_{i2} + \tilde{A}_{j2}}{2} \right)^T P_1 \left(\frac{\tilde{A}_{k2} + \tilde{A}_{l2}}{2} \right) - P_2 \right] x(t-d) \\ &\quad + e^T(t) \left[\left(\frac{\tilde{B}_i K_j + \tilde{B}_j K_i}{2} \right)^T P_1 \left(\frac{\tilde{B}_k K_l + \tilde{B}_l K_k}{2} \right) \right] e(t) \\ &\quad + 2x^T(t) \left[\left(\frac{H_{ij} + H_{ji}}{2} \right)^T P_1 \left(\frac{\tilde{A}_{k2} + \tilde{A}_{l2}}{2} \right) \right] x(t-d) \\ &\quad - 2x^T(t) \left[\left(\frac{H_{ij} + H_{ji}}{2} \right)^T P_1 \left(\frac{\tilde{B}_k K_l + \tilde{B}_l K_k}{2} \right) \right] e(t) \\ &\quad \left. - 2x^T(t-d) \left[\left(\frac{\tilde{A}_{i2} + \tilde{A}_{j2}}{2} \right)^T P_1 \left(\frac{\tilde{B}_k K_l + \tilde{B}_l K_k}{2} \right) \right] e(t) \right\}, \end{aligned}$$

$$\begin{aligned}
\Delta V_2(e(t)) = & \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \mu_i \mu_j \mu_k \mu_l \left\{ x^T(t) \left[\left(\frac{R_{ij} + R_{ji}}{2} \right)^T P_3 \left(\frac{R_{kl} + R_{lk}}{2} \right) \right] x(t) \right. \\
& + x^T(t-d) \left[\left(\frac{\Delta A_{i2}(t) + \Delta A_{j2}(t)}{2} \right)^T P_3 \left(\frac{\Delta A_{k2}(t) + \Delta A_{l2}(t)}{2} \right) \right] x(t-d) \\
& + e^T(t) \left[\left(\frac{S_{ij} + S_{ji}}{2} \right)^T P_3 \left(\frac{S_{kl} + S_{lk}}{2} \right) - P_3 + P_4 \right] e(t) \\
& + e^T(t-d) \left[\left(\frac{T_{ij} + T_{ji}}{2} \right)^T P_3 \left(\frac{T_{kl} + T_{lk}}{2} \right) - P_4 \right] e(t-d) \\
& + 2x^T(t) \left[\left(\frac{R_{ij} + R_{ji}}{2} \right)^T P_3 \left(\frac{\Delta A_{k2}(t) + \Delta A_{l2}(t)}{2} \right) \right] x(t-d) \\
& + 2x^T(t) \left[\left(\frac{R_{ij} + R_{ji}}{2} \right)^T P_3 \left(\frac{S_{kl} + S_{lk}}{2} \right) \right] e(t) \\
& + 2x^T(t) \left[\left(\frac{R_{ij} + R_{ji}}{2} \right)^T P_3 \left(\frac{T_{kl} + T_{lk}}{2} \right) \right] e(t-d) \\
& + 2x^T(t-d) \left[\left(\frac{\Delta A_{i2}(t) + \Delta A_{j2}(t)}{2} \right)^T P_3 \left(\frac{S_{kl} + S_{lk}}{2} \right) \right] e(t) \\
& + 2x^T(t-d) \left[\left(\frac{\Delta A_{i2}(t) + \Delta A_{j2}(t)}{2} \right)^T P_3 \left(\frac{T_{kl} + T_{lk}}{2} \right) \right] e(t-d) \\
& \left. + 2e^T(t) \left[\left(\frac{S_{ij} + S_{ji}}{2} \right)^T P_3 \left(\frac{T_{kl} + T_{lk}}{2} \right) \right] e(t-d) \right\}.
\end{aligned}$$

By Lemma 1, we obtain that

$$\begin{aligned}
\Delta V(t) \leq & \sum_{i=1}^n \mu_i^2 \{ x^T(t) [H_{ii}^T(3P_1)(H_{ii}) + R_{ii}^T(4P_3)R_{ii} - P_1 + P_2] x(t) \\
& + x^T(t-d) [\tilde{A}_{i2}^T(3P_1)\tilde{A}_{i2} + \Delta A_{i2}^T(t)(4P_3)\Delta A_{i2}(t) - P_2] x(t-d) \}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^{n-1} \sum_{j>i}^n \mu_i \mu_j \left\{ x^T(t) \left[(H_{ij} + H_{ji})^T \left(\frac{3}{2} P_1 \right) (H_{ij} + H_{ji}) - 2P_1 + 2P_2 \right. \right. \\
& + (R_{ij} + R_{ji})^T (2P_3) (R_{ij} + R_{ji}) \left. \right] x(t) \\
& + x^T(t-d) \left[(\tilde{A}_{i2} + \tilde{A}_{j2})^T \left(\frac{3}{2} P_1 \right) (\tilde{A}_{i2} + \tilde{A}_{j2}) - 2P_2 \right. \\
& + (\Delta A_{i2}(t) + \Delta A_{j2}(t))^T (2P_3) (\Delta A_{i2}(t) + \Delta A_{j2}(t)) \left. \right] x(t-d) \left. \right\} \\
& + \sum_{i=1}^n \mu_i^2 \{ e^T(t) [K_i^T \tilde{B}_i^T (3P_1) \tilde{B}_i K_i + S_{ii}^T (4P_3) S_{ii} - P_3 + P_4] e(t) \\
& + e^T(t-d) [T_{ii}^T (4P_3) T_{ii} - P_4] e(t-d) \} \\
& + \sum_{i=1}^{n-1} \sum_{j>i}^n \mu_i \mu_j \left\{ e^T(t) \left[(\tilde{B}_i K_j + \tilde{B}_j K_i)^T \left(\frac{3}{2} P_1 \right) (\tilde{B}_i K_j + \tilde{B}_j K_i) - 2P_3 + 2P_4 \right. \right. \\
& + (S_{ij} + S_{ji})^T (2P_3) (S_{ij} + S_{ji}) \left. \right] e(t) \\
& + e^T(t-d) [(T_{ij} + T_{ji})^T (2P_3) (T_{ij} + T_{ji}) - 2P_4] e(t-d) \left. \right\}. \tag{16}
\end{aligned}$$

If (16) < 0 while $\tilde{x}(t) \neq 0$ and $\tilde{e}(t) \neq 0$, then the discrete system (9) is asymptotically stable. Similar to the method proposed in [4], suppose each sum in (16) is negative definite, we can complete the proof of Theorem 1 by using Lemma 1, Lemma 2 and Schur complement.

Remark 1. With the following steps:

Step 1. Solving LMIs (10) and (11), we can obtain X , Y_i and P_3 ;

Step 2. By equation (14), we can have K_i ;

Step 3. Put the above results into equations (12) and (13), then calculate P_4 and G_i by solving the LMIs (12) and (13);

we can easily have the controller gains K_i and the observer gains G_i ($i = 1, 2, \dots, n$).

4. A Numerical Example

In this section, an example is used to illustrate the proposed method.

Example. Assuming an uncertain T-S fuzzy discrete system with time-delay (4) has the following parameters:

$$\begin{aligned} A_{11} &= \begin{bmatrix} -0.13 & -0.39 \\ 1 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.57 & -0.83 \\ 0 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ A_{21} &= \begin{bmatrix} -1.24 & -7.62 \\ 1 & 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -1.33 & -8.72 \\ 0 & 0 \end{bmatrix}, \quad C_{11} = C_{21} = [0 \quad 0.8], \\ C_{12} &= C_{22} = [0.4 \quad 0], \quad U_1 = U_2 = \begin{bmatrix} 0.1 & 0.08 \\ 0.02 & 0 \end{bmatrix}, \quad E_{11} = E_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \\ E_{12} &= E_{22} = \begin{bmatrix} 0.4 & 0 \\ 0.3 & 0 \end{bmatrix}, \quad E_1 = E_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{aligned}$$

Let $\varepsilon_{11} = 12.16$, $\varepsilon_{22} = 6.55$ and $\varepsilon_{12} = 8.73$, solving LMIs (10) and (11), by Step 2 in Remark 1, we have

$$K_1 = [-13.8391 \quad -29.1143], \quad K_2 = [-11.2369 \quad -25.3347].$$

Then put them into (12) and (13), let $\eta_{ij} = \varepsilon_{ij}$, finally, we can obtain

$$G_1 = \begin{bmatrix} 2.3781 \\ 5.2596 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 2.1024 \\ 4.9815 \end{bmatrix}$$

by Step 3 in Remark 1.

5. Conclusion

In this paper, under the unmeasurable states environment, we proposed a new method to design an observer-based output feedback controller for a class of uncertain T-S fuzzy discrete systems with time-delay. The effectiveness of this method can be verified by the given numerical example.

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