Advances in Fuzzy Sets and Systems
Volume 8, Number 1, 2011, Pages 1-11; Issue Date: June 2011
Published Online: July 22, 2011
This paper is available online at http://pphmj.com/journals/afss.htm

POSSIBILISTIC OPTIMISTIC CRITERIA IN A FUZZY ENVIRONMENT

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Abstract

In this paper, we consider decision making under uncertainty, when the probability of the states of nature is not known a priori and the outcomes of each alternative are characterized only approximately. In the literature, there are many criteria used for ranking alternatives in such cases as mentioned in this paper, but these criteria have many drawbacks. The fuzzy approach is very useful to handle such situations. Here we construct a new fuzzy optimistic criterion, namely the 2-Fuzzy Optimistic Criterion (2-FOC) using the fuzzy aspiration degree that has been defined. This criterion is a modification of the existing possibilistic optimistic criterion that has been widely used in such situations for ranking of alternatives. The criterion constructed is more realistic and it has been proved that when ranking of alternatives is not possible using the existing criterion, then it is possible to clearly rank the alternatives using the fuzzy criterion constructed. An example has also been given to illustrate the same.

2010 Mathematics Subject Classification: 03E72.

Keywords and phrases: fuzzy set, intuitionistic fuzzy set, fuzzy aspiration degree, fuzzy aspiration class, fuzzy non-aspiration class, 1-fuzzy optimistic criterion, 2-fuzzy optimistic criterion.

Received June 6, 2011

1. Introduction

In the field of decision making under uncertainty in the crisp environment, there are many criteria in the literature for ranking of alternatives. The most commonly used criterion is the expected utility criterion axiomatized by Savage [10], despite early criticisms formulated by Allais [1], Ellsberg [8] and later by Kahneman and Tversky [9]. Here the subjective values attached to each consequence as well as the degree of confidence of the possible outcomes commensurate and are specifically quantified. However, in most problems which generally deal with practical life situations, this may not be always possible. Thus, there are many models designed for preference modeling in the presence of poor information. The most famous decision rule of this kind is the max-min rule of Wald [13] axiomatized by Arrow and Hurwicz [2]. Here the set X of consequences is ranked by means of some utility function 'u' valued on any ordinal scale. If no information is assumed about the states of nature, if the decision maker is a pessimist, then the acts are ranked on the merits of the worst consequences and if the DM is an optimist, then the acts are ranked on the merit of the best consequence. If plausibility ordering on states is available, then the above criterion can be refined [3, 4]. In such cases, acts are ranked according to the merits of their worst consequences restricted to the most plausible states.

Another refinement of the Wald criterion is the possibilistic criterion [5-7] based on a utility function 'u' on X and a possibility distribution π on S representing the relative plausibility of states both mapping on the same totally ordered scale and which can be compared.

In this paper, we use the fuzzy approach, which is very useful to handle decision problems in real life situations where the probability of the states of nature is not known a priori and outcomes of each alternatives are characterized only approximately. Here instead of the utility function, we use the fuzzy membership function and also the fuzzy aspiration degree that has been defined, to modify the existing possibilistic optimistic criterion which is commonly used in such cases, so that the drawbacks seen in this criterion can be rectified.

2. Decision Making Under Uncertainty

This section presents the basic setting of a decision making under uncertainty. Decision making under uncertainty here implies a choice among a set of potential alternatives, the consequences of which are not known perfectly. Such a decision problem is characterized by a set S of states representing the possible situations, a set X of possible consequences, and a set D of alternatives which are taken as elements of X^S. Thus, the elements of D are mappings $d_i : S \to X$, where $d_i(s)$ represents the consequences of an alternative d_i for any state $s \in S$. Here S and X are taken to be finite. Thus, if $S = \{s_1, s_2, ..., s_n\}$ and $X = \{x_1, x_2, ..., x_m\}$, then $d_i \in X^S$ is completely characterized by the vector consequences $\{d_i(s_1), d_i(s_2), ..., d_i(s_n)\}$.

Most of the rules used for decision making under uncertainty involve a real valued function 'u' on X encoding the utility of the consequences and a numerical set – function $\prod$ on 2^S representing the confidence of events. The representation of uncertainty about the state of nature is completely determined by the knowledge of the degree of plausibility of states (here in this paper, encoded by possibility distribution π on S). The alternatives are then ranked using a valuation function v on D, where $v : X^S \to L$, where L is an ordered set and $v(d_i)$ measures the subjective attractiveness of d_i for the decision maker. Here $v(d_i)$ is a function of the values $u(d_i(s_1)), ..., u(d_i(s_n))$ and the set function which represents the DM's knowledge about the state of nature. The alternatives are then ranked according to the values of $v(d_i)$.

3. Important Results

Definition 3.1. A *fuzzy set* A^1 in X is given by Zadeh [14] as

$$A^1 = \{(x, \mu_A(x))/x \in X\},$$

where $\mu_A : X \to [0, 1]$ is the membership function of the fuzzy set A^1, $\mu_A(x) \in [0, 1]$.

Definition 3.2. The *Possibilistic Optimistic Criterion* (*POC*) [5-7] is given by

$$d_i \geq d_j \Leftrightarrow v^+(d_i) \geq v^+(d_j), \quad \text{where} \quad v^+(d_i) = \max_{s \in S}\{\min[\pi(s), u(d_i(s))]\}.$$

Though this criterion is a refinement of many criteria in the crisp environment, this criterion has many drawbacks, some major drawbacks being

(1) An alternative d_1 can be ranked equal to another alternative d_2 even if d_1 is at least as satisfying as d_2 in all states and better in some states (including most plausible ones), which is not true in real life situations.

(2) If for a given alternative there exists only one consequence with high utility and high plausibility of state, then there is a great chance for this alternative to be selected irrespective of the very low utility values of the other consequences with high plausibility of states. This is not realistic, since the decision maker in such a case is overoptimistic.

Thus, we see whether we can modify this criterion such that these drawbacks can be rectified. For this, we construct the 1-Fuzzy Optimistic Criterion (1-FOC) as follows.

4. 1-Fuzzy Optimistic Criterion (1-FOC)

This criterion is based on a membership function μ_A on *X*, where $\mu_A(x_{ij})$ represents the fuzzy set membership of a consequence $x_{ij} \in X$ in the fuzzy set 'satisfaction', and the possibility distribution π on *S* representing the relative plausibility of states, both mapping on the same totally ordered scale and which can be compared. Here *X* represents the set of consequences and *S* the states of nature for the decision problem.

Definition 4.1. 1-*Fuzzy Optimistic Criterion* (1-*FOC*) is given by

$$d_i \geq d_j \Leftrightarrow S_{11}^{+}(d_i) \geq S_{11}^{+}(d_j), \quad S_{11}^{+}(d_i) = V_j[J_{11}^{+}(s_j, x_{ij})],$$

where $J_{11}^{+}(s_j, x_{ij}) = \wedge[\pi(s_j), \mu_A(x_{ij})] \varepsilon [0, 1]$.

Here $S_{11}^{+}(d_i)$ represents the degree of satisfaction of the alternative d_i for the DM, where $d_i(s_j) = x_{ij}$. Thus, the alternatives can be ranked using this criterion.

Note 4.2. 1-FOC is same as POC except that the utility function being replaced by the fuzzy membership function in the fuzzy set 'satisfaction'. Thus, the drawbacks seen in POC are also seen in 1-FOC. Hence, in order to verify whether the drawbacks in POC are rectified, it is sufficient to verify whether the drawbacks are rectified in 1-FOC.

An optimist always expects the best to occur. In all the above mentioned criteria for an optimist, the best consequences restricted to most plausible states are given high score, but the best consequences restricted to non-plausible states are given very low score. This is not true for an optimist, since the best consequences restricted to non-plausible states are also considered by the decision maker and high score is given in such cases also though not as high as the score given to the best consequences restricted to the most plausible states. We now construct a fuzzy optimistic criterion, namely 2-FOC using the fuzzy aspiration degree which specifies the best consequences and give high score values in this criterion for the best consequences depending on the plausibility of the states.

5. Fuzzy Aspiration Degree

Here we introduce the fuzzy aspiration degree and also the fuzzy aspiration class, which is used to define the 'best consequences' of the Decision Maker (DM) for the given decision problem.

Definition 5.1. If a DM is 'totally satisfied' with a consequence x_{ij}, then the fuzzy set membership of x_{ij} in the fuzzy set 'satisfaction' is 1, i.e., $\mu_A(x_{ij}) = 1$. Such consequences are called the *efficient consequences* of the decision problem.

Now in this section, we define the 'best consequences' as follows.

Definition 5.2. In a decision problem, there may be consequences which are highly (very) satisfying to the decision maker, i.e., consequences whose membership values in the fuzzy set 'satisfaction' are very close or equal to 1. Thus, if $\alpha_D \in [0, 1]$ is the minimum of the membership values of the consequence from which onwards the DM is 'highly satisfied', then α_D is called the *fuzzy aspiration degree* of the decision problem for the DM. All consequences $x_{ij} \in X$ which satisfy the property $\mu_A(x_{ij}) \geq \alpha_D$, are called the '*best consequences*' of the decision problem.

Example 5.3. If $\alpha_D = .9$, then the best consequences of the decision problem are all consequences whose membership values are greater than or equal to .9.

Note 5.4. (1) The fuzzy aspiration degree is the minimum degree of satisfaction aspired by the DM for the best consequences of the decision problem.

(2) The fuzzy aspiration degree varies according to the choice of the decision maker and the decision problem.

(3) The fuzzy aspiration degree specifies the best consequences of the decision problem for DM.

(4) When $\alpha_D = 1,$ the best consequence becomes the efficient consequence.

(5) By Definition 5.2, clearly, $\alpha_D > .5$ (even though in most cases its value is even higher then .5 and close to 1).

Definition 5.5. The *fuzzy aspiration class* of α_D denoted by $\mathrm{FAC}[\alpha_D],$ for the decision problems, is defined as $\mathrm{FAC}[\alpha_D] = \{x_{ij} \in X : \mu_A(x_{ij}) \geq \alpha_D\},$ i.e., $\mathrm{FAC}[\alpha_D]$ is the set of all best consequences of the decision problem.

Definition 5.6. The *fuzzy non-aspiration class* of α_D denoted by $\mathrm{FNAC}[\alpha_D],$ for the decision problem is defined as $\mathrm{FNAC}[\alpha_D] = \{x_{ij} \in X : \mu_A(x_{ij}) < \alpha_D\}.$

Result 5.7. (1) $\mathrm{FAC}[\alpha_D] \cup \mathrm{FNAC}[\alpha_D] = X.$

(2) $\mathrm{FAC}[\alpha_D] \cap \mathrm{FNAC}[\alpha_D] = \Phi.$

6. The Construction of 2-Fuzzy Optimistic Criterion (2-FOC)

We now construct the 2-*FOC* using the fuzzy aspiration degree.

Definition 6.1. We define the 2-FOC as follows.

Define

$$J_{21}^{+}(s_j, x_{ij}) = \begin{cases} \frac{1}{2}[1 + \alpha_D(\mu_A(x_{ij}) \wedge \pi(s_j)) + (1 - \alpha_D)(\mu_A(x_{ij}) \vee \pi(s_j))] & \text{if } \mu_A(x_{ij}) \geq \alpha_D, \\ \alpha_D(\pi(s_j) \wedge \mu_A(x_{ij})) + (1 - \alpha_D)(\pi(s_j) \vee \mu_A(x_{ij})) & \text{if } \mu_A(x_{ij}) < \alpha_D. \end{cases}$$

Then $J_{21}^{+}(s_j, x_{ij}) \varepsilon [0, 1]$ is the score of the alternative d_i corresponding to the pair $(s_j, x_{ij}),$ where $d_i(s_j) = x_{ij}$ and α_D is the fuzzy aspiration degree of the decision maker for the given decision problem.

Here a high weightage 'α_D' is given to the optimistic factor $\mu(x_{ij}) \wedge \pi(s_j)$ and a low weightage of $(1 - \alpha_D)$ is given to the factor $\mu_A(x_{ij}) \vee \pi(s_j).$ For the best consequences $\mu_A(x_{ij}) \geq \alpha_D$, the scores are constructed such that they are given high values. Also, introduction of the term $\mu_A(x_{ij}) \vee \pi(s_j)$ does not affect the optimistic attitude of the decision maker but at the same time it helps in clear ranking of the alternatives.

Also, if we define $S_{21}^{+}(d_i) = \max_j J_{21}^{+}(s_j, x_{ij})$, then this would be non-realistic, since the decision maker in such a case would be over optimistic. So taking into consideration all these $S_{21}^{+}(d_i) = \frac{1}{n}\sum_{j=1}^{n} J_{21}^{+}(s_j, x_{ij})$, where the score $J_{21}^{+}(s_j, x_{ij})$ is high for the best consequences, which is the attitude of an optimist.

The 2-*Fuzzy Optimistic Criterion* (2-FOC) is then given by

$$d_i \geq d_j \Leftrightarrow S_{21}^{+}(d_i) \geq S_{21}^{+}(d_j).$$

Note 6.2. If $\alpha_D = 1$ in 2-FOC, then $J_{21}^{+}(s_j, x_{ij}) = J_{11}^{+}(s_j, x_{ij})$, $\forall s_j \in S$, $x_{ij} \in X$, except when $\mu_A(x_{ij}) = 1$ in which case $J_{21}^{+}(s_j, x_{ij}) = .5 + .5\pi(s_j)$ which is slightly higher than $J_{11}^{+}(s_j, x_{ij}) = \pi(s_j)$. Thus, 2-FOC can be considered as a modification of 1-FOC.

We now give an example to show that the drawback seen in 1-FOC is rectified using 2-FOC.

Example 6.3. Let $D = \{d_1, d_2, d_3, d_4\}$ and $S = \{s_1, s_2, s_3, s_4\}$ with the possibility distribution of the states given by $\pi(s_1) = .3$, $\pi(s_2) = .5$, $\pi(s_3) = 1$, $\pi(s_4) = .6$. Let the decision table be given by

Table 6.3.1

D \ Ω	s_1	s_2	s_3	s_4
d_1	1	.3	.6	.8
d_2	.1	.3	.6	.7
d_3	.6	.3	.8	.7
d_4	.8	.7	.7	.4

Then by 1-FOC,

$$S_{11}^{+}(d_1) = .6, \quad S_{11}^{+}(d_2) = .6, \quad S_{11}^{+}(d_3) = .8, \quad S_{11}^{+}(d_4) = .7.$$

Thus $d_3 > d_4 > d_1 = d_2$. Here d_1 is as satisfying as d_2 in states s_1, s_2, s_3 and d_1 is better than d_2 in s_4. Yet using 1-FOC $d_1 = d_2$. Similarly, this is the same drawback seen in POC also.

But using 2-FOC for $\alpha_D = .8$ (say), we get

$$S_{21}^{+}(d_1) = .47, \quad S_{21}^{+}(d_2) = .42, \quad S_{21}^{+}(d_3) = .56, \quad S_{21}^{+}(d_4) = .61.$$

Thus $d_4 > d_3 > d_1 > d_2$.

We now prove that this defect is, in general, rectified using 2-FOC.

Theorem 6.4. *Let d_1 be as satisfying as d_2 in all states and better in some states and suppose in* 1-*FOC, $d_1 = d_2$. Then by* 2-*FOC, d_1 will be strictly better than d_2, i.e., $d_1 > d_2$.*

Proof. Let us suppose that d_1 is as satisfying as d_2 in all states and there is only one state, say s_r, where d_1 is better than d_2 and let the corresponding consequences be x_{1r} and x_{2r}, respectively. Let α_D be the aspiration degree of the DM, for the given decision problem. Let $\mathrm{FAC}[\alpha_D]$, $\mathrm{FNAC}[\alpha_D]$, denote, respectively, the fuzzy aspiration and fuzzy non-aspiration classes w.r.t. α_D. Then clearly, $\mu_A(x_{1r}) > \mu_A(x_{2r})$. Also, it is given that in 1-FOC, $d_1 = d_2$.

Then by the above given conditions,

(1) If $x_{1r} \in \mathrm{FAC}[\alpha_D]$ and $x_{2r} \in \mathrm{FNAC}[\alpha_D]$.

(2) If $x_{1r}, x_{2r} \in \mathrm{FAC}[\alpha_D]$.

(3) If $x_{1r}, x_{2r} \in \mathrm{FNAC}[\alpha_D]$.

In all cases except when $\alpha_D = 1$, it follows that $J_{21}^{+}(s_r, x_{1r}) > J_{21}^{+}(s_r, x_{2r})$ and when $\alpha_D = 1$ and $x_{1r} \in \mathrm{FAC}[\alpha_D]$, $x_{2r} \in \mathrm{FNAC}[\alpha_D]$, then also

$$J_{21}^{+}(s_r, x_{1r}) > J_{21}^{+}(s_r, x_{2r}).$$

Thus by 2-FOC, we get

$$S_{21}^{+}(d_1) = \frac{1}{n}\sum_{j=1}^{n} J_{21}^{+}(s_j,\ x_{ij})$$

$$= \frac{1}{n}\left[\sum_{j\neq r} J_{21}^{+}(s_j,\ x_{1j}) + J_{21}^{+}(s_r,\ x_{1r})\right]$$

$$= \frac{1}{n}\left[\sum_{j\neq r} J_{21}^{+}(s_j,\ x_{2j}) + J_{22}^{+}(s_r,\ x_{1r})\right]$$

$$> \frac{1}{n}\left[\sum_{j\neq r} J_{21}^{+}(s_j,\ x_{2j}) + J_{21}^{+}(s_r,\ x_{2r})\right]$$

$$= \frac{1}{n}\left[\sum_{j=1}^{n} J_{21}^{+}(s_j,\ x_{2j})\right] = S_{21}^{+}(d_2),$$

$\therefore\ S_{21}^{+}(d_1) > S_{21}^{+}(d_2)$ and hence $d_1 > d_2$.

Similarly, if d_1 is as satisfying as d_2 in all states and better in more than one state, then proceeding as above, we get $d_1 > d_2$. Hence the theorem.

Note 6.5. If $\alpha_D = 1$ in 2-FOC, and if $x_{1r},\ x_{2r} \in \text{FAC}[\alpha_D]$ or $x_{1r},\ x_{2r} \in \text{FNAC}[\alpha_D]$, then $d_1 = d_2$. In such cases, we can take $\alpha_D = .999$ (say) which approximates to 1. Then proceeding as in the proof of Theorem 6.4, $J_{21}^{+}(s_r,\ x_{1r}) > J_{21}^{+}(s_r,\ x_{2r})$ and thus $d_1 > d2$.

Hence in all the cases, clear ranking of alternatives is possible using the 2-FOC constructed.

7. Conclusion

In this paper, we have constructed a fuzzy criterion 2-FOC for an optimistic decision maker for ranking of alternatives in a fuzzy environment. This criterion is a modification of 1-FOC which is the fuzzy version of the possibilistic optimistic criterion in the literature. The fuzzy criterion 2-FOC constructed makes use of the ‘fuzzy aspiration degree’ which we have defined to specify the best consequences of the decision maker. This criterion is more realistic, and is an optimistic criterion which according to the attitude of the decision maker gives high values for the best consequences. Also, one of the major drawbacks seen in the existing criterion, namely that an alternative d_1 can be ranked equal to another alternative d_2 even if d_1 is atleast as satisfying as d_2 in all states and better in some states (including the most plausible ones) has been rectified using this criterion. This has been illustrated by an example and also proved in Theorem 6.4. If non-membership values of the consequences are available, then we have shown in our papers that in an intuitionistic fuzzy environment, these drawbacks can be rectified, depending on the attitude of the decision maker whether a pessimist [11] or an optimist [12].

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