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A STUDY ON THE EFFECT OF VARIABLE SUCTION AND HEAT TRANSFER TO UNSTEADY MHD FREE CONVECTIVE FLOW ALONG A VERTICAL POROUS PLATE THROUGH B-SPLINE COLLOCATION

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Abstract

The aim of the present work is to study the free convection flow and heat transfer through a viscous incompressible electrically conducting fluid along an isothermal vertical porous non-conducting plate with time dependent suction and exponentially decaying heat generation in the presence of transverse magnetic field. The governing equations are solved numerically using cubic B-spline collocation method. The effects of various parameters on velocity and temperature distributions, skin-friction and Nusselt number are studied.

1. Introduction

Convective heat transfer and fluid along porous plate in the presence of magnetic field and internal heat generation have attracted the interest of scientific investigators as its useful applications in different branches of Science and Technology such as nuclear science, fire engineering, combustion modeling,

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geophysical etc. Similarity solutions for natural convection with internal heat generation, which decays exponentially, are derived by Crepeau and Clarksean [5]. Sattar et al. [17] obtained analytical and numerical solutions for free convection flow along a porous plate with variable suction in porous medium. Soundalgekar et al. [19, 20] examined viscous dissipation effect of unsteady free convection flow along vertical porous plate with different boundary conditions. Variable heat transfer with mass transfer effect on accelerated vertical plate is investigated by Raptis and Tzivanidis [15]. The effect of mass transfer and free convection past a vertical porous plate is studied by Hossain and Begum [8]. Ferdows et al. [7] analyzed free convection flow with variable suction in presence of thermal radiation. Alam et al. [1] studied Dufour and Soret effects with variable suction on unsteady MHD free convection flow along a porous plate.

In this paper, we consider the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non-conducting plate with variable suction and exponentially decaying heat generation in the presence of transverse magnetic field. In applications where the electromagnetic heating exists the exponentially decaying heat generation model can be utilized as heat source [5, 16].

2. Formulation of the Problem

Consider an unsteady incompressible electrically conducting two-dimensional laminar free convective boundary layer flow along a non-conducting vertical porous plate with constant temperature T_w, and Q the internal volumetric rate of heat generation of the fluid. Let the x-axis be taken along the plate and y-axis be taken normal to the plate. Intensity B_0 of the magnetic field is applied in y-direction. Also the external field is assumed to be zero with electrical field due to polarization of charges and Hall effects are neglected. Using the Boussinesq approximation within the boundary layer, the governing equations of continuity, momentum and energy [2, 11, 13, 18], respectively, are given by

$$\frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} - \nu\frac{\partial^2 u}{\partial y^2} = g\beta(T - T_\infty) - \frac{\sigma B_v^2}{\rho}u, \tag{2.2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q, \tag{2.3}$$

where u and v are the Darcy velocities in x, y directions, t is time variable, $\nu\left\{=\frac{\mu}{\rho}\right\}$ kinematic viscosity, μ is viscosity of fluid, ρ is fluid density, g being the acceleration due to gravity, σ is electrical conductivity, C_p is specific heat at constant pressure, T is temperature of fluid in boundary layer and k is thermal conductivity.

The appropriate boundary conditions are

$$\begin{aligned} &u=0,\ v=v(t),\ T=T_w, \quad \text{at } y=0\\ &u\to 0,\ T\to T_\infty, \quad \text{at } y\to\infty; \end{aligned} \tag{2.4}$$

where T_∞ is fluid temperature far away from the plate.

Using the following dimensionless variables

$$\eta=\frac{y}{h},\quad u=Uf(\eta),\quad \theta(y)=\frac{T-T_\infty}{T_w-T_\infty} \tag{2.5}$$

with length scale

$$h=h(t), \tag{2.6}$$

$$v=v(t)=-V_0\frac{\nu}{h(t)}, \tag{2.7}$$

where V_0 is suction parameter [18], into the equations (2.2) and (2.3), we obtain

$$f''+(\eta+V_0)f'-Mf=-Gr\theta \tag{2.8}$$

and

$$\theta''+\Pr(\eta+V_0)\theta'=-Se^{-\eta}, \tag{2.9}$$

where η is similarity variable, U is uniform characteristic velocity, f is dimensionless stream function, $Q\left\{=Sk\left(\frac{T_w-T_\infty}{h^2\mu C_p}\right)e^{-\eta}\right\}$, $Gr\left\{=\frac{g\beta h^2(T-T_\infty)}{\nu U}\right\}$ is Grashof number, $M\left\{=\frac{\sigma B_0^2 h^2}{\mu}\right\}$ is magnetic parameter, $\Pr\left\{=\frac{\mu C_p}{k}\right\}$ is Prandtl number and S (0 or 1) is heat generation parameter.

The boundary conditions are reduced to

$$f(0) = 0, \;\; f'(0) = 0, \;\; f'(\infty) = 0, \;\; \theta(0) = 1 \text{ and } \theta(\infty) = 0. \tag{2.10}$$

The skin-friction coefficient at the plate is given by

$$C_f = 2(\mathrm{Re})^{-1} f'(0), \tag{2.11}$$

where $\mathrm{Re} = \frac{Uh}{v}$ is the Reynolds number.

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = -\theta'(0). \tag{2.12}$$

The governing equations (2.8) and (2.9) are non-linear second order coupled differential equations and solved under the boundary conditions (2.10) using cubic spline collocation method.

3. Spline Collocation

In this section, cubic B-splines are used to construct numerical solutions to boundary value problems discussed in equation (2.8). A detailed description of B-spline functions generated by subdivision can be found in [6, 14].

Consider equally-spaced knots of a partition $\pi : a = x_0 < x_1 < \cdots < x_n = b$ on $[a, b]$. Let $S_3[\pi]$ be the space of continuously-differentiable, piecewise, fifth-degree polynomials on π. That is, $S_3[\pi]$ is the space of cubic polynomials on π. Prenter [14] defined the cubic spline $s(t)$ as

$$s(t) = \sum_{j=-1}^{n+1} a_j B_j(t), \tag{3.1}$$

where n is the number of subintervals of $[a, b]$, $a_{-1}, a_0, \ldots, a_{n+1}$ are $(n+3)$ unknowns and the functions $B_i(t)$, with additional knots $t_{-2} < t_{-1} < t_0$ and $t_{n+2} >$

$t_{n+1} > t_n$ is defined by

$$B_i(t) = \frac{1}{h^3}\begin{cases} (t - t_{i-2})_+^3, & \text{if } t \in [t_{i-2}, t_{i-1}], \\ h^3 + 3h^2(t - t_{i-1})_+ + 3h(t - t_{i-1})_+^2 - 3(t - t_{i-1})_+^3, & \text{if } t \in [t_{i-1}, t_i], \\ h^3 + 3h^2(t_{i+1} - t)_+ + 3h(t_{i+1} - t)_+^2 - 3(t_{i+1} - t)_+^3, & \text{if } t \in [t_i, t_{i+1}], \\ (t_{i+2} - t)_+^3, & \text{if } t \in [t_{i+1}, t_{i+2}], \\ 0, & \text{otherwise,} \end{cases}$$

$$(3.2)$$

$$B_i(t_j) = \begin{cases} 4 & \text{if } j = 1, \\ 1 & \text{if } j = i - 1 \text{ or } i + 1, \\ 0 & \text{if } j = i + 2 \text{ or } j = i - 2 \end{cases} \tag{3.3}$$

and

$$B_i(t) = 0 \text{ for } t \geq t_{i+2} \text{ and } t \leq t_{i-2}. \tag{3.4}$$

Here $B_i(t)$ is denoted as cubic B-spline with knots at $t_{-2} < t_{-1} < \cdots < t_n < t_{n+1} < t_{n+2}$. From equation (3.1) it is clear that $s(t)$ is a basis of B-splines at different knots.

To compute $s(t)$ we use equations (3.2) and (3.3) which give the values of $B_i(t)$ and its derivatives are evaluated with the help of the power function $(t - t_k)_+$ defined by

$$\begin{aligned} (t - t_k)_+ &= t - t_k \quad \text{if} \quad t > t_k \\ &= 0 \qquad \text{if } t \leq t_k. \end{aligned}$$

Consider a second-order linear BVP of the form

$$u''(x) = p(x)u'(x) + q(x)u(x) + f(x), \quad a \leq x \leq b \tag{3.5}$$

with boundary conditions

$$u(a) = K_1, \quad u(b) = K_2,$$

where $u(x)$, $p(x)$, $q(x)$ and $f(x)$ are continuous functions defined in the interval $x \in [a, b]$; K_1 and K_3 are finite real constants.

In this section, the spline solution of equation (3.5) is determined using a collocation method [6]. Let $S(t)$ given by (3.1) be an approximate solution of equation (3.5), where a_j are unknown real coefficients and $B_j(t)$ are cubic B-spline functions.

Let $x_0, x_1, ..., x_n$ be $n+1$ grid points in the interval $[a, b]$, so that

$$x_i = a + ih, \quad i = 0, 1, ..., n; \quad x_0 = a, \quad h = (b-a)/h. \tag{3.6}$$

It is required that the approximate solution (3.1) satisfies the differential equation at the points $x = x_i$. Putting (3.1) in (3.5), it follows that

$$\sum_{j=-1}^{n+1} a_j [B_j''(x_i) - p(x_i) B_j'(x_i) - q(x_i) B_j(x_i)] = f(x_i), \quad i = 0, 1, ..., n, \tag{3.7}$$

$$\left.\begin{aligned} \sum_{j=-1}^{n+1} a_j B_j(x_0) = k_1 \\ \sum_{j=-1}^{n+1} a_j B_j(x_n) = k_2 \end{aligned}\right\}. \tag{3.8}$$

The value of the B-spline functions at the knots $x_{-1}, x_0, ..., x_{n+1}$ are determined using (3.2), (3.3), (3.4). A system of $n+3$ linear equations in $n+3$ unknowns $a_{-1}, a_0, ..., a_{n+1}$ is thus obtained. This system can be written in matrix-vector form as follows:

$$AC = F, \tag{3.9}$$

where A is $(n+3) \times (n+3)$ dimensional band matrix,

$$C = [a_{-1}, ..., a_n, a_{n+1}]^T$$

and

$$F = [k_1, f(x_0), f(x_1), ..., f(x_n), f(x_{n+1}), k_2]^T,$$

T denoting transpose. Eliminating a_{-1}, a_{n+1} from this system we get a system of $n+1$ equations in $n+1$ unknowns a_j $(j = 0, 1, ..., n)$. The coefficient matrix of which is a diagonally dominant three-band non-singular matrix. Hence, we obtain the approximate solution (3.1). In case of non-linear boundary value problem, the equations can be converted into linear form by any known method like quasilinearization [3] or Newton's linearization [9, 10] and hence, this method can be used as iterative method. The procedure to obtain a spline approximation of u_i $(i = 0, 1, ..., j;$ where j denotes the number of iterations) by an iterative method starts with fitting a curve satisfying the end conditions and this curve is designated as u_i. We obtain the successive iterations u_i's with the help of an algorithm described as above till the desired accuracy.

4. Spline Solution

For the numerical study the outer boundary is set at $\eta_\infty = 5$ so that the domain of the problem is restricted to $[0, 5]$. Hence, the boundary conditions $f'(\infty) = 0$ and $\theta(\infty) = 0$ are considered as $f'(5) = 0$ and $\theta(5) = 0$. As discussed above the collocation equations for the corresponding problem (2.8), (2.9) with (2.10) are as follows:

From (2.9) we obtain

$$\sum_{j=-1}^{n+1} a_j [B_j''(\eta_i) + \Pr(\eta_i + V_0) B_j'(\eta_i)] = -Se^{-\eta_i}, \quad i = 0, 1, 2, ..., n. \tag{4.1}$$

The boundary condition $\theta(0) = 1$ and $\theta(5) = 0$ give

$$\sum_{j=-1}^{n+1} a_j B_j(\eta_0) = 1 \text{ and } \sum_{j=-1}^{n+1} a_j B_j(\eta_n = 5) = 0 \tag{4.2}$$

and from equation (2.8) we get

$$\sum_{j=-1}^{n+1} a_j [B_j''(\eta_i) + (\eta_i + V_0) B_j'(\eta_i) - MB_j(\eta_i)] = -Gr\theta_i, \quad i = 0, 1, 2, ..., n. \tag{4.3}$$

Boundary conditions $f'(0) = 0,\ f'(\infty) = 0$ give

$$\sum_{j=-1}^{n+1} a_j B'_j(\eta_0) = 0 \quad \text{and} \quad \sum_{j=-1}^{n+1} a_j B'_j(\eta_n = 5) = 0. \tag{4.4}$$

For the complete solution of the problem first we solve the collocation equations (4.1), (4.2) for $\theta(\eta)$ and using these values in (4.3), (4.4) we get an approximate solution $f(\eta)$. For various parameters $Gr\,(= 2, 3, 4)$, $M\,(= 0.0, 0.5, 1.0)$, $V_0\,(= 0.5, 1.0, 1.5)$, $S\,(= 0.0, 1.0)$, $\Pr\,(= 0.71, 1.0, 2.0)$ the solution profiles are obtained and presented in Tables 4.1, 4.2 and Figures 4.1 to 4.6.

Table 4.1. Values of $-\theta'(0)$ for various values of Pr and V_0

	$\Pr = 0.71$	$\Pr = 1.0$	$\Pr = 2.0$
$S = 0, V_0 = 0.5$	0.913524314	1.141186760	1.832815441
$S = 0, V_0 = 1.0$	1.180244765	1.525244265	2.638976523
$S = 0, V_0 = 1.5$	1.466544812	1.938767256	3.508910801
$S = 1, V_0 = 0.5$	0.224777787	0.477705467	1.218343625
$S = 1, V_0 = 1.0$	0.457776382	0.821353548	1.968740216
$S = 1, V_0 = 1.5$	0.715546351	1.201532301	2.794931312

Table 4.2. Values of $f'(0)$ for various values of Pr, V_0, Gr and M

	$S = 0.0, V_0 = 0.5, Gr = 2.0$			$S = 1.0, V_0 = 0.5, Gr = 2.0$		
M	$\Pr = 0.71$	$\Pr = 1.0$	$\Pr = 2.0$	$\Pr = 0.71$	$\Pr = 1.0$	$\Pr = 2.0$
0.00	1.5142565	0.9839964	0.6902434	1.6419118	1.3743923	0.9306154
0.5	1.0418941	0.9009161	0.6478582	1.4397677	1.2243343	0.8523637
1.0	0.9579268	0.8376916	0.6145156	1.2939565	1.1142520	0.7933431
	$S = 0.0, V_0 = 1.0, Gr = 2.0$			$S = 1.0, V_0 = 1.0, Gr = 2.0$		
M	$\Pr = 0.71$	$\Pr = 1.0$	$\Pr = 2.0$	$\Pr = 0.71$	$\Pr = 1.0$	$\Pr = 2.0$
0.00	1.0487361	0.8647590	0.5614893	1.5321837	1.2442210	0.7854535
0.5	0.9638364	0.8055893	0.5352573	1.3679737	1.1272801	0.7295907
1.0	0.8973283	0.7583828	0.5136594	1.2442872	1.0377857	0.6858875

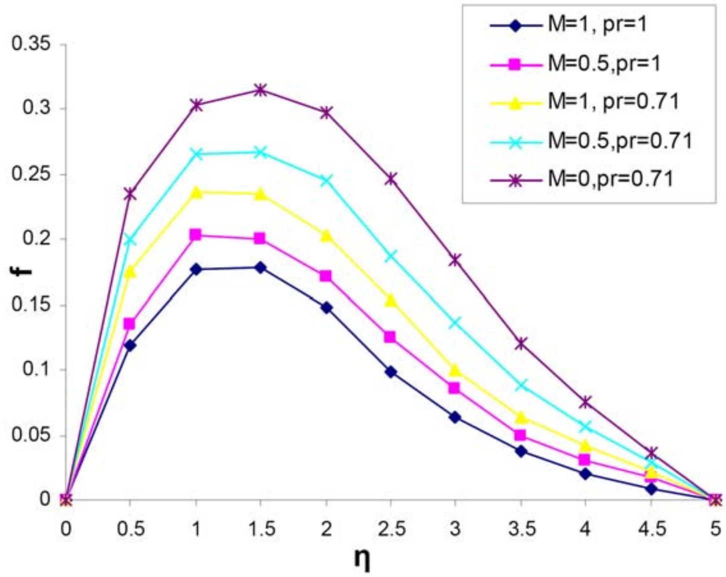

Figure 4.1. Velocity profiles $(V_0 = 0.5,\ Gr = 2.0,\ S = 0)$.

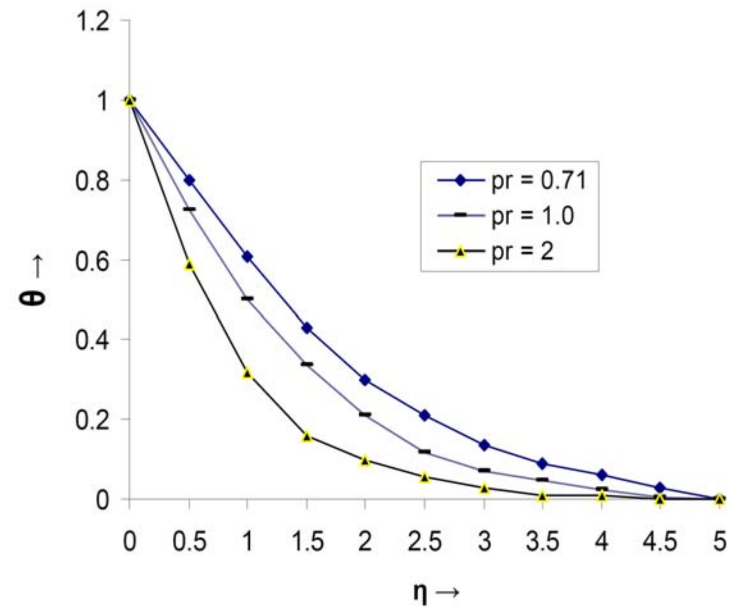

Figure 4.2. Temperature distribution $(V_0 = 0.5,\ Gr = 2.0,\ S = 0.0)$.

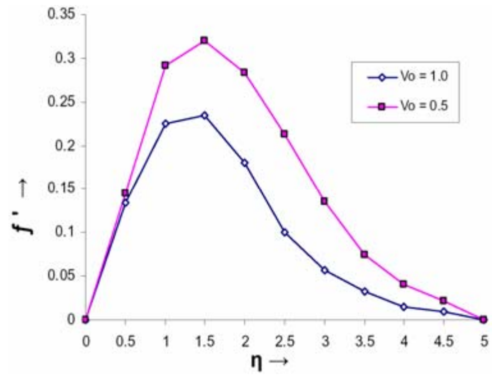

Figure 4.3. Velocity distribution $(\mathrm{Pr} = 0.71,\ Gr = 2.0,\ M = 1.0,\ S = 0.0)$.

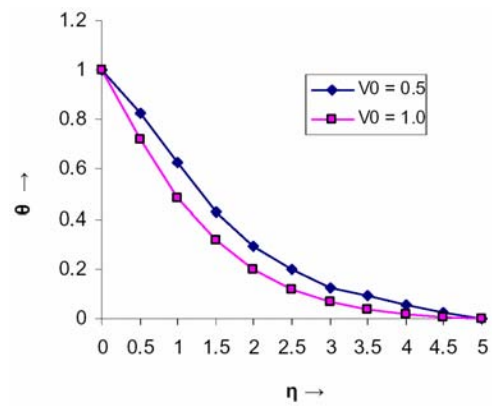

Figure 4.4. Temperature distribution $(\mathrm{Pr} = 0.71,\ Gr = 2.0,\ M = 1.0,\ S = 0.0)$.

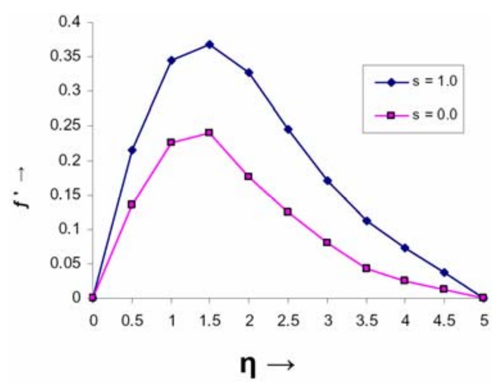

Figure 4.5. Velocity distribution $(\mathrm{Pr} = 0.71,\ Gr = 2.0,\ M = 1.0,\ V_0 = 0.5)$.

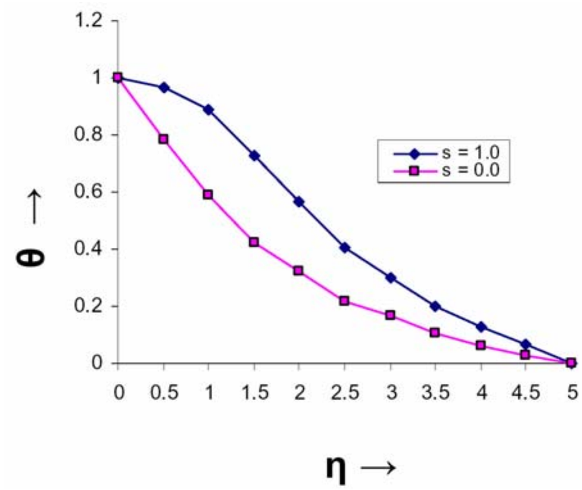

Figure 4.6. Temperature distribution $(\Pr = 0.71, Gr = 2.0, M = 1.0, V_0 = 0.5)$.

5. Results and Discussion

Table 4.1 gives the estimation of $-\theta'(0)$ (rate of heat transfer) for $S = 1, 0$. This shows that the rate of heat transfer increase with increase in Prandtl number and suction parameter irrespective of absence (i.e., $S = 1.0$) or presence $(S = 0)$ of heat generation. This is attributed to the fact that with the increase in Prandtl number or suction, the thermal boundary layer thickness reduces which consequently increases the temperature gradient.

Table 4.2 shows that with the increase in magnetic parameter, suction parameter and Prandtl number the value of $f'(0)$ decreases, but it increases with the increase in the Grashof number. It is observed that $f'(0)$ is increased in the presence of heat generation $(S = 1.0)$, with comparison to absence of heat generation $(S = 0.0)$. Also it is found that some of the results are in good agreement with those obtained by Sattar et al. [17].

Since as magnetic parameter increases, the Lorentz force, which opposes the flow, also increases which leads to deceleration of the flow which is observed from Figure 4.1 that fluid velocity decreases with the increase in the magnetic parameter M. It is also observed from Figure 4.2 that the velocity profiles and temperature profiles decrease with the increase in the Prandtl number.

From Figures 4.3 and 4.4, it is observed that when the suction parameter V_0 increases, fluid velocity and fluid temperature decrease, i.e., more fluid reserved through the plate. Both, boundary layer and thermal boundary layer thicknesses decrease with the increase in suction parameter V_0. As velocity profiles in the presence of heat generation are higher in comparison to absence of heat generation it is observed from Figure 4.5 that heat generation assists the flow significantly. It is found from Figure 4.6 that fluid temperature increases in the presence of heat generation hence the magnitude of temperature profiles are higher in presence of heat generation. Boundary layer and thermal boundary layer thicknesses increased well in presence of heat generation.

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