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A NEW CRITERION FOR THE STABILITY OF COMPLETE SYNCHRONIZATION IN A CLASS OF UNIDIRECTIONAL COUPLED CHAOTIC SYSTEMS

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Abstract

In this paper, the stability of complete synchronization in a class of unidirectional coupled chaotic systems is investigated. A particular Lyapunov functional is constructed in terms of the master-slave system directly, instead of error systems. By means of some similar Lipschitz conditions, a new criterion for the asymptotically stability of complete synchronization motion is obtained. This result can be applied to some systems widely.

1. Introduction

As the sensitivity to the initial conditions, even two identical chaotic systems starting from slightly different initial conditions would evolve in time in an unsynchronized manner. Therefore, the setting of synchronized behavior in coupled

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chaotic systems has a great importance and interest [1]. Since the pioneering work of Pecora and Carroll [2], the synchronization of chaotic systems and its application have been a subject of active research in the field of chaos. Many research efforts have been focused on chaos control and chaos synchronization problems in physical chaotic systems [3-5]. For the complete synchronization in unidirectional coupling chaotic systems, several theoretical methods have been developed, such as the Pecora and Carroll method, linear and nonlinear feedback control, the adaptive coupling, impulsive control methods, time-delay feedback approach [6], and so on.

Meanwhile, stability of the synchronized motion is also a very relevant issue. Many criteria have been established for complete synchronization. One of the most popular and widely used criteria is the use of the Lyapunov exponents of the linearized system for the synchronization error [7], another method is the use of the Lyapunov functions [8], some further criteria for the stability of synchronized states are carried out from the analysis of eigenvectors [9], invariance principle [10], etc. Park [11] presented a master-slave synchronization scheme via a single controller. Such an adaptive backstepping control law is derived to make the error dynamics only depending on the error state. Thus, the stability of synchronization was easily translated to the stability of the trivial solution of error dynamics. By constructing a suitable Lyapunov-Krasovskii functional, [12] studied the exponential stability of synchronization in some particular unidirectional coupled chaotic systems. Applied such method, we often consider the linearization system instead of the error system itself, so some criteria of stability relate to the evolution of the drive systems. Chen et al. [10] developed some feasible method of chaos synchronization in systems dissatisfying global Lipschitz conditions. By means of the varying coupling strength techniques sufficient criteria for both complete synchronization and generalized synchronization are established.

In the study of the stability of synchronized motion, error systems are often discussed, and then obtain some criterion for stability. But, in general, there are some specific terms relating to the driving system in error systems. In these cases, it is difficult to judge the stability of error systems. Usually, methods of such as the linearized systems, global or locally Lipschitz condition are considered. Though various techniques are applied, most of the corresponding criteria for the synchronization include the evolution of the drive systems more or less. This paper attempts to study the stability of synchronized motion via some kind of similar

Lipschitz conditions, in which the error systems do not employed. Our method is constructing a Lyapunov functional in terms of the master-slave system directly. Sufficient criteria for complete synchronization are presented.

2. The General Theorem for Stability of Synchronization Motion

Consider the following coupled chaotic system:

$$\dot{x}(t) = f(t,\ x(t),\ x(t-\tau)), \tag{1}$$

$$\dot{y}(t) = f(t,\ y(t),\ y(t-\tau)) + K(x(t),\ y(t)), \tag{2}$$

where $f \in C(R \times R \times R,\ R),\ \ K \in C(R \times R,\ R),\ \ K(x,\ x) = 0.$

First, some notations must be introduced.

$C = C([-\tau,\ 0],\ R)$ is the space of continuous functions mapping the interval $[-\tau,\ 0]$ into R. For $\phi \in C,\ \|\phi\| = \sup_{-\tau \le \theta \le 0} |\phi(\theta)| \cdot x_t(\theta) = x(t+\theta),\ \ \theta \in [-\tau,\ 0].$

Theorem 1. *Suppose that there exists a continuous functional* $V(t,\ \phi,\ \psi)$, $\phi,\ \psi \in C$ *such that*

(i) $a(|\phi(0) - \psi(0)|) \le V(t,\ \phi,\ \psi) \le b(\|\phi - \psi\|),$

(ii) $\dot{V}(t,\ x_t,\ y_t) \le -c(|x(t) - y(t)|),$

where $a(x),\ \ b(x),\ \ c(x)$ *are Kamke-type functions,* $\dot{V}(t,\ x_t,\ y_t)$ *is time derivative along the solutions of equations* (1) *and* (2), *then the complete synchronization motion* $\{x(t) = y(t)\}$ *between equations* (1) *and* (2) *is uniformly asymptotically stable*.

The proof is similar to the Lyapunov stability theorem (Hale and Lunel [13]), here omitted.

3. Synchronization for Two Classes of Delay Systems

Consider the following coupled chaotic systems:

$$\dot{x}(t) = f(x(t)) + g(x(t-\tau)), \tag{3}$$

$$\dot{y}(t) = f(y(t)) + g(y(t-\tau)) + K(x(t) - y(t)), \tag{4}$$

where $f,\ g \in C(R,\ R),\ \ g(x)$ is nondecreasing, $K > 0.$

Theorem 2. *Assume that there exist positive constants* L *and* k *such that* $g(x) - g(y) < L(x - y)$, $-K(x - y) + f(x) - f(y) + g(x) - g(y) < -k(x - y)$, *for* $x \geq y$. *Then the complete synchronization between system* (3) *and* (4) *is uniformly asymptotically stable*.

Proof. Let $x = x(t)$ be the solution of the initial problem

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t-\tau)), \\ x(t_0 + \theta) = \phi(\theta),\ \theta \in [-\tau, 0], \end{cases} \tag{5}$$

and $y = y(t)$, $z = z(t)$ are the solutions of following initial problems, respectively,

$$\begin{cases} \dot{y}(t) = f(y(t)) + g(y(t-\tau)) + K(x(t) - y(t)), \\ y(t_0 + \theta) = \psi(\theta),\ \theta \in [-\tau, 0], \end{cases} \tag{6}$$

$$\begin{cases} \dot{z}(t) = f(z(t)) + g(z(t-\tau)) + K(x(t) - z(t)), \\ z(t_0 + \theta) = \max(\phi(\theta), \psi(\theta)),\ \theta \in [-\tau, 0]. \end{cases} \tag{7}$$

It is obvious that

$$x(t) \geq z(t), \quad y(t) \geq z(t), \quad t \geq t_0. \tag{8}$$

Define a functional as

$$V_1(t) = V_1(x_t, z_t) = x(t) - z(t) + \int_{t-\tau}^{t} (g(x(s)) - g(z(s)))ds.$$

Then $V_1(x_t, z_t)$ satisfies the condition (i) of Theorem 1, and the derivative of $V_1(t)$ is

$$\dot{V}_1(t) = -K(x(t) - z(t)) + f(x(t)) - f(z(t)) + g(x(t)) - g(z(t)) < -k(x(t) - z(t)).$$

Also, we define a similar functional as

$$V_2(t) = V_2(y_t, z_t) = y(t) - z(t) + \int_{t-\tau}^{t} (g(y(s)) - g(z(s)))ds,$$

$$\dot{V}_2(t) = -K(y(t) - z(t)) + f(y(t)) - f(z(t)) + g(y(t)) - g(z(t)) < -k(y(t) - z(t)).$$

By Theorem 1, synchronization motions $\{x(t) = z(t)\}$ and $\{y(t) = z(t)\}$ are uniformly asymptotically stable, therefore synchronization motion $\{x(t) = y(t)\}$ is also uniformly asymptotically stable.

Consider other coupled chaotic systems:

$$\dot{x}(t) = f(x(t)) + h(x(t))g(x(t-\tau)), \tag{9}$$

$$\dot{y}(t) = f(y(t)) + h(x(t))g(y(t-\tau)) + K(x(t) - y(t)), \tag{10}$$

where $f, g, h \in C(R, R)$, $K > 0$.

Denote a function $H(x)$ which satisfies $\dot{H}(x) = h^{-1}(x)$.

Theorem 3. *Assume that there exist Kamke-type functions* $a(x)$, $b(x)$ *and* $c(x)$, *positive scalars L and r such that*

$$| g(x) - g(y) | < L| x - y |, \quad a(| x - y |) \le | H(x) - H(y) | \le b(| x - y |),$$

$$(H(x) - H(y))(h^{-1}(x) f(x) - h^{-1}(y) f(y)) - K(H(x) - H(y))(x - y)$$

$$+ \frac{1}{4}(H(x) - H(y))^2 + (g(x) - g(y))^2 \le -c(| x(t) - y(t) |).$$

Then the complete synchronization between system (9) *and* (10) *is uniformly asymptotically stable*.

Proof. Define a functional as

$$V_3(t) = V_3(x_t, y_t) = \frac{1}{2}(H(x(t)) - H(y(t)))^2 + \int_{t-\tau}^{t} (g(x(s)) - g(y(s)))^2 ds.$$

Then $V_3(x_t, y_t)$ satisfies the condition (i) of Theorem 1, and the derivative of $V_3(t)$ is

$$\begin{aligned}
\dot{V}_3(t) = {} & (H(x(t)) - H(y(t)))(h^{-1}(x(t)) f(x(t)) - h^{-1}(y(t)) f(y(t))) \\
& + (H(x(t)) - H(y(t)))(g(x(t-\tau)) - g(y(t-\tau))) - K(H(x(t)) \\
& - H(y(t)))(x(t) - y(t)) + (g(x(t)) - g(y(t)))^2 - (g(x(t-\tau)) - g(y(t-\tau)))^2 \\
\le {} & (H(x(t)) - H(y(t)))(h^{-1}(x(t)) f(x(t)) - h^{-1}(y(t)) f(y(t))) - K(H(x(t)) \\
& - H(y(t)))(x(t) - y(t)) + \frac{1}{4}(H(x(t)) - H(y(t)))^2 + (g(x(t)) - g(y(t)))^2 \\
\le {} & -c(| x(t) - y(t) |).
\end{aligned}$$

Applied Theorem 1, synchronization motions $\{x(t) = y(t)\}$ is uniformly asymptotically stable.

4. Conclusion

In this paper, the issue of complete synchronization in a class of unidirectional coupled chaotic systems has been discussed. By means of some similar Lipschitz condition, a new criterion for the asymptotically stability of complete synchronization motion is obtained. Our method is constructing a Lyapunov functional in terms of the master-slave system directly, not with the error systems. Although the Lipschitz condition of the master system is not satisfied, complete synchronization in coupled chaotic systems can also be showed.

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