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TWO MULTIVARIATE STOCHASTIC VOLATILITY MODELS APPLIED TO AIR POLLUTION DATA FROM SÃO PAULO, BRAZIL

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Abstract

In this paper, we use some recently introduced multivariate stochastic volatility models to problems related to air pollution. The models considered here are commonly used in studies of financial time series. In this paper, they are used to analyse the weekly average volatility of five

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pollutants affecting the inhabitants of the city of São Paulo, Brazil. Two different models are proposed to explain the behaviour of the weekly average measurements of those pollutants. Those models depend upon some parameters that are estimated using a Bayesian formulation via Markov chain Monte Carlo (MCMC) methods.

1. Introduction

Air pollution is a common problem affecting the inhabitants of large cities throughout the world and São Paulo-Brazil, is not an exception to that. Having an area of over than one million and a half square meters (IBGE [26]) and a population of over than 10 millions inhabitants (IBGE [27]), São Paulo is one of the largest cities in the world. Besides being the financial centre of the country, it has a large concentration of industries and an excessive number of cars and trucks circulating in the city. In São Paulo, as in Mexico City, motor vehicles are considered to be the main reason for the high levels of pollution.

It is a well known fact that ambient levels of air pollution have been consistently associated with adverse health outcomes (see for instance, Ito and Thurston [28] and Ostro et al. [37]). Those problems are mostly due to respiratory and cardiovascular events (WHO [52]). Dose-response functions, relating increases in air pollutants to morbidity as well as mortality have been established on solid epidemiological grounds in studies performed in several locations (WHO [52]). Unfortunately, there is a lack of uniformity in the area of coverage of the air pollution monitoring networks across the globe. That hinders the possibility of performing epidemiological studies in regions such as Latin America, Asia and Africa, where the process of industrialization and the automotive fleet is growing fast, in a scenario of inadequate air pollution monitoring. Even in areas of the developed world, the proper determination of exposure is problematic. That is so, because conventional air pollution systems may experience periods of malfunctioning or inadequate spatial coverage, leading to a situation of exposure misclassification either in terms of time or spatial resolution (Ryan and LeMasters [46] and Chen et al. [12]). In this scenario, adequate pollution modeling is of pivotal importance in reducing exposure bias. That contributes to elaborate more robust dose-response functions and, thus, helps to establish public policies of air pollution control in more solid basis.

Among the several pollutants affecting the air quality in São Paulo, we have sulfur dioxide (SO_2), nitrogen dioxide (NO_2), particulate matter with diameter smaller than 10 microns (PM_{10}), ozone (O_3) and carbon monoxide (CO). Those pollutants have different effects on the health of the population and on the environment. For instance, it is a well known fact that for ozone concentration above 0.11 parts per million (0.11ppm), a very sensitive part of the population (e.g., elderly and newborn) in that environment may experience serious health deterioration (see for example, Bell et al. [8], Cifuentes et al. [13], Dockery et al. [16], Gauderman et al. [19], Gouveia and Fletcher [22] and Martins et al. [34]). The sulfur dioxide and the nitrogen oxides (NO_x) are considered the main responsible for acid rain (see for instance, Mohnen [36] and Likens [33]). Those pollutants may suffer oxidation in the atmosphere and in combination with appropriate humidity they may produce acid rain. For instance, SO_2 may react with water in the atmosphere to produce sulfuric acid and the nitrogen oxidises and in conjunction with water, may produce a reaction whose result is nitric acid (see for example, Likens [33]). We also have that exposure to carbon monoxide and PM_{10} during pregnancy may produce adverse effects on newborn (Ritz and Yu [41], Ritz et al. [42] and Ritz et al. [43]).

The pollutants considered here are taken into account because, in general, they are the ones used by the environmental authorities in São Paulo to report the air quality in the city. Additionally, they are the ones that are emitted or whose precursors are emitted mainly by the burning of fossil fuel (see for instance, Likens [33]). However, there are other types of sources such as nitrogen fertilisers, confined animal feedlots, lightning and soil microbes (Likens [33]).

There are many works using different methods to study problems related to air pollution. Among those works, we may quote Álvarez et al. [7], Brown et al. [10], Flaum et al. [18], Guardani et al. [23], Horowitz [25], Lanfredi and Macchiato [31], Leadbetter [32], Pan and Chen [38], Piegorsch et al. [39], Roberts [44, 45], Seigneur [47], Smith [48] and Zolghadri and Henry [55]. When the interest is centred in modeling the number of times, a pollutant concentration surpasses a given threshold, an alternative methodology is the use of Poisson models (see for example, Achcar et al. [1], Achcar et al. [3], Achcar et al. [4] and Raftery [40]).

In this paper, the interest resides in analysing the behaviour of the variability of the weekly average measurements of pollutants instead of analysing their time series. Hence, we are going to use stochastic volatility (SV) models (see for example, Ghysels et al. [21], Kim et al. [29] and Meyer and Yu [35]). This family of models has been extensively used to analyse financial time series (see for example, Danielsson [15] and Yu [53]), as a powerful alternative for the usual existing ARCH type models (autoregressive conditional heteroscedastic) introduced by Engle [17] and the generalized autoregressive conditional heteroscedastic (GARCH) models introduced by Bollerslev [9].

In the context of environmental problems, SV models have been used by Holan et al. [24] to predict spawning of shovelnose sturgeon; and in Achcar et al. [2, 5, 6], we have the use of SV models to study the weekly averaged ozone measurements in the different regions of Mexico City. In those works, either a univariate or bivariate SV model was used to analyse the weekly averaged ozone measurements in the five regions in which the Mexico City is divided. In here, we use two multivariate SV models to study the behaviour of five pollutants that affect the city of São Paulo, Brazil. The advantages of using SV-type models to analyse time series are that they assume the existence of two processes to model the series: one process describing the behaviour of the observations and another describing the latent volatility.

This paper is organised as follows: Section 2 presents the multivariate stochastic volatility models considered here. In Section 3, a Bayesian formulation of the problems is given as well as the criterion used to select the best model to explain the data used. In Section 4, an application of the models is made to the data from the city of São Paulo. Finally, in Section 5, we discuss some of the results obtained.

2. Multivariate Stochastic Volatility Models

In this section, we consider some multivariate stochastic volatility models to study the behaviour of weekly average measurements of five pollutants present in the city of São Paulo. Several types of multivariate stochastic volatility models may be found in the literature (see for example, Yu and Meyer [54]). The ones considered here may be described as follows: Let $N \geq 1$ be a fixed known integer number that records the amount of data used. (In here, N will represent the number of weeks in which we have the weekly average measurements of a pollutant of interest.) Let $Z_j(t)$, $j = 1, 2, \ldots, K$, $t = 1, 2, \ldots, N$, be $K\ (\geq 1)$ times series recording the results of K events. (In our case, $Z_j(t)$ is the weekly average measurement of a given pollutant j in the tth week, $t = 1, 2, \ldots, N$, $j = 1, 2, \ldots, K$.) Denote by $Y_j(t)$, $t = 1, 2, \ldots, N$ the log-return series defined by $\log[Z_j(t)/Z_j(t-1)]$, $j = 1, 2, \ldots, K$.

Let $\mathbf{Y}(t) = (Y_1(t), Y_2(t), ..., Y_K(t))'$, $t = 1, 2, ..., N$ be a K-dimensional vector whose coordinates are the K times series $Y_i(t)$, $i = 1, 2, ..., K$. The set $\mathbf{Y} = \{\mathbf{Y}(1), \mathbf{Y}(2), ..., \mathbf{Y}(N)\}$ is the set of observed data.

We assume that $\mathbf{Y}(t)$ may be written in the following form:

$$\mathbf{Y}(t) = H(t)\boldsymbol{\varepsilon}(t), \tag{1}$$

where $\boldsymbol{\varepsilon}(t) = (\varepsilon_1(t), \varepsilon_2(t), ..., \varepsilon_K(t))'$ is an error vector and $H(t)$ is a $K \times K$ diagonal matrix given by

$$H(t) = \operatorname{diag}(e^{h_1(t)/2}, e^{h_2(t)/2}, ..., e^{h_K(t)/2}), \tag{2}$$

where $\mathbf{h}(t) = (h_1(t), h_2(t), ..., h_K(t))$, $t = 1, 2, ..., N$ is a vector of latent variables given as follows. We assume that $\mathbf{h}(t)$, $t = 1, 2, ..., N$ is such that its coordinates follow an $\mathrm{AR}(1)$ model, i.e., for $i = 1, 2, ..., K$,

$$\begin{cases} h_i(1) = \mu_i + \eta_i(1), \\ h_i(t) = \mu_i + \phi_{ii}[h_i(t-1) - \mu_i] + \eta_i(t), \; t = 2, 3, ..., N, \end{cases} \tag{3}$$

where $-1 < \phi_{ii} < 1$, $i = 1, 2, ..., K$, and $\boldsymbol{\eta}(t) = (\eta_1(t), \eta_2(t), ..., \eta_K(t))$ is assumed to have a multivariate Normal distribution with mean vector $\mathbf{0} = (0, 0, ..., 0)$ and variance-covariance matrix the diagonal matrix $\operatorname{diag}(\sigma^2_{\eta_1}, \sigma^2_{\eta_2}, ..., \sigma^2_{\eta_K})$.

Remark. Note that by definition, we have for $i = 1, 2, ..., K$, that $h_i(1)$ has Normal distribution $N(\mu_i, \sigma^2_{\eta_i})$ and that given $h_i(t-1)$, $t = 2, 3, ..., N$, we have that $h_i(t)$ has Normal distribution $N(\mu_i + \phi_{ii}[h_i(t-1) + \mu_i], \sigma^2_{\eta_i})$. Also, note that $Y_i(t)$, $i = 1, 2, ..., K$, may be written as

$$Y_i(t) = e^{h_i(t)/2}\varepsilon_i(t), \quad t = 1, 2, ..., N. \tag{4}$$

We also assume that $\boldsymbol{\varepsilon}(t)$ has a multivariate Normal distribution with mean vector $\mathbf{0} = (0, 0, ..., 0)$ and variance-covariance matrix $\Sigma_{\boldsymbol{\varepsilon}}$ given by

$$\Sigma_{\boldsymbol{\varepsilon}} = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \cdot & \cdot & \cdot & \rho_{1K} \\ & 1 & \rho_{23} & \cdot & \cdot & \cdot & \rho_{2K} \\ & & \cdot & & & & \cdot \\ & & & \cdot & & & \cdot \\ & & & & \cdot & & \cdot \\ & & & & & 1 & \rho_{K-1,K} \\ & & & & & & 1 \end{pmatrix}, \tag{5}$$

where ρ_{ij} is the covariance of $\varepsilon_i(t)$ and $\varepsilon_j(t)$, $i, j = 1, 2, ..., K$, $i \neq j$, $t = 1, 2, ..., N$. Note that the variance of $\varepsilon_i(t)$, $i = 1, 2, ..., K$ is assumed to be equal to one, $t = 1, 2, ..., N$.

Therefore, by definition, we have that given $\mathbf{h}(t)$, the vector $\mathbf{Y}(t)$ has a multivariate Normal distribution with mean vector **0** and variance-covariance matrix given by

$$\Sigma_{\mathbf{Y}} = \mathrm{Var}(\mathbf{Y}(t)) = \begin{pmatrix} e^{h_1(t)} & \rho_{12}e^{h_1(t)/2}e^{h_2(t)/2} & \cdot & \cdot & \cdot & \rho_{1K}e^{h_1(t)/2}e^{h_K(t)/2} \\ & e^{h_2(t)} & \cdot & \cdot & \cdot & \rho_{2k}e^{h_2(t)/2}e^{h_K(t)/2} \\ & & \cdot & & & \cdot \\ & & & \cdot & & \cdot \\ & & & & \cdot & \cdot \\ & & & & & e^{h_K(t)} \end{pmatrix}. \tag{6}$$

Remark. Note that $\mathbf{h}(t) = (h_1(t), h_2(t), ..., h_K(t))$ gives the volatility of the series studied.

Two different models are going to be considered here to represent the volatility of the series. They are described as follows:

1. **Model I.** In this model, we assume that the coordinates of the error vector $\boldsymbol{\varepsilon}(t)$ are independent, i.e., $\rho_{ij} = 0$, $i, j = 1, 2, ..., K$, $i \neq j$. Therefore, in this case, $\mathbf{Y}(t)$ will have a multivariate Normal distribution with mean vector **0** and variance-covariance matrix given by (2).

2. **Model II.** In this version of the model, we assume that the covariance functions ρ_{ij}, $i, j = 1, 2, ..., K$, $i \neq j$ are unknown quantities that need to be estimated. Hence, $\mathbf{Y}(t)$ will have a multivariate Normal distribution with mean vector **0** and variance-covariance matrix given by (6).

3. A Bayesian Analysis of the Problem

The parameters of the model will be estimated using a Bayesian point of view via Markov chain Monte Carlo (MCMC) methods. Bayesian inference approach using MCMC methods (see for example, Gelfand and Smith [20] and Smith and Roberts [49]) has been used to estimate parameters of models applied to a wide range of areas. In here, we are going to use it to estimate parameters in SV models. The use of MCMC methods for analysing problems involving SV models applied to problems in ecology and environmental sciences can also been found in Achcar et al. [2, 5, 6], where an application to air pollution in Mexico City is given; and in Holan et al. [24], where an application to predicting spawning of shovelnose sturgeon is shown. The use of MCMC methods in this type of problems is justified by the presence of great difficulties such as high dimensionality of the vector of parameters, likelihood function with no closed form and high computational costs that make the use of standard classical inference approach very difficult.

Inference will be performed using a sample drawn from the posterior distribution of the parameters of interest. Each model will be analysed separately.

1. **Model I.** In this case, we assume that $\boldsymbol{\varepsilon}(t) = (\varepsilon_1(t), \varepsilon_2(t), \ldots, \varepsilon_K(t))'$ has a Normal distribution with mean vector $\mathbf{0}$. We also assume that the correlation between two coordinates of the error vector is zero and that the variance of each coordinate is one. The vector of parameters in this model is $\boldsymbol{\theta}_I = (\boldsymbol{\phi}, \boldsymbol{\sigma}_\eta^2, \boldsymbol{\mu})$, where $\boldsymbol{\phi} = (\phi_{11}, \phi_{22}, \ldots, \phi_{KK})$, $\boldsymbol{\sigma}_\eta^2 = (\sigma_{\eta_1}^2, \sigma_{\eta_2}^2, \ldots, \sigma_{\eta_K}^2)$ and $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_K)$. We assume that ϕ_{ii}, σ_{η_i} and μ_i have as prior distributions a Normal, an Inverse Gamma and a Normal distributions, respectively, $i = 1, 2, \ldots, K$. That is, ϕ_{ii}, σ_{η_i} and μ_i have prior distributions $N(a_{ii}, b_{ii})$, $IG(c_i, d_i)$ and $N(e_i, f_i)$, respectively, $i = 1, 2, \ldots, K$, where the hyperparameters a_{ii}, b_{ii}, c_i, d_i, e_i, f_i are considered to be known and will be specified later. (In here, $IG(c, d)$ is the Inverse Gamma distribution with mean $d/(c-1)$ and variance $d^2/[(c-1)^2(c-2)]$, with $c > 2$.)

2. **Model II.** When this model is taken into account, we have that the vector of parameters is $\boldsymbol{\theta}_{II} = (\boldsymbol{\theta}_I, \boldsymbol{\rho})$, where $\boldsymbol{\rho} = (\rho_{12}, \rho_{13}, \ldots, \rho_{1K}, \ldots, \rho_{K-11}, \ldots, \rho_{K-1K})$. We assume that ρ_{ij} has as prior distribution a Normal distribution $N(g_{ij}, h_{ij})$ and that

$\boldsymbol{\theta}_I$ will have the same prior distributions as in Model I with possibly different hyperparameters.

Note that, by hypothesis, in both models, the latent variables $\mathbf{h}(t) = (h_1(t), h_2(t), ..., h_k(t)), \quad t = 1, 2, ..., N$ are such that their prior distributions are given by

$$g(\mathbf{h}(1) | \boldsymbol{\theta}) \propto \prod_{i=1}^{K} (\sigma_{\eta_i}^2)^{-1/2} \exp\left[-\frac{1}{2\sigma_{\eta i}^2}(h_i(1) - \mu_i)^2\right], \; t = 1 \tag{7}$$

and, for $t = 2, 3, ..., N$,

$$g(\mathbf{h}(t) | \mathbf{h}(t-1), \boldsymbol{\theta}) \propto \prod_{i=1}^{K} \prod_{t=2}^{N} (\sigma_{\eta_i}^2)^{-1/2}$$

$$\cdot \exp\left(-\frac{1}{2\sigma_{\eta i}^2}[h_i(t) - \mu_i - \phi_{ii}(h_i(t-1) - \mu_i)]^2\right), \tag{8}$$

for $\boldsymbol{\theta} = \boldsymbol{\theta}_I, \boldsymbol{\theta}_{II}$.

Set $\boldsymbol{\theta} = \boldsymbol{\theta}_{II}$ and take $\boldsymbol{\varphi} = (\boldsymbol{\theta}, \mathbf{h})$, where $\mathbf{h} = (\mathbf{h}(1), \mathbf{h}(2), ..., \mathbf{h}(N))$. Hence, we have that the joint likelihood function of $\boldsymbol{\theta}$ and $\mathbf{h}$, in Model II, is given by

$$L(\mathbf{Y} | \boldsymbol{\varphi}) \propto \prod_{t=1}^{N} P(\mathbf{Y}(t) | \mathbf{h}(t), \boldsymbol{\theta}), \tag{9}$$

where $P(\mathbf{Y}(t) | \mathbf{h}(t), \boldsymbol{\theta})$ is the multivariate Normal distribution with mean vector zero and variance-covariance matrix $\sum_{\mathbf{Y}}$ given by (6). When Model I is considered, we just take $\boldsymbol{\theta} = \boldsymbol{\theta}_I$ and set $\rho_{ij} = 0$ in (6).

Considering either $\boldsymbol{\varphi} = (\boldsymbol{\theta}_I, \mathbf{h})$ or $\boldsymbol{\varphi} = (\boldsymbol{\theta}_{II}, \mathbf{h})$, we have that the joint posterior distribution of the vector of parameters and the latent variables is given by

$$P(\boldsymbol{\varphi} | \mathbf{Y}(t)) \propto g(\mathbf{h}(1) | \boldsymbol{\theta}) P(\mathbf{Y}(1) | \mathbf{h}(1), \boldsymbol{\theta})$$

$$\cdot \left(\prod_{t=2}^{N} g(\mathbf{h}(t) | \mathbf{h}(t-1), \boldsymbol{\theta}) P(\mathbf{Y}(t) | \mathbf{h}(t), \boldsymbol{\theta})\right) \mathbf{P}(\boldsymbol{\theta}), \tag{10}$$

where $P(\boldsymbol{\theta})$ is the prior distribution of the vector of parameters with $\boldsymbol{\theta} = \boldsymbol{\theta}_I, \boldsymbol{\theta}_{II}$, $P(\mathbf{Y}(t) | \mathbf{h}(t), \boldsymbol{\theta})$ is as in (9) and $g(\mathbf{h}(1) | \boldsymbol{\theta})$, $g(\mathbf{h}(t) | \mathbf{h}(t-1), \boldsymbol{\theta})$, $t = 2, 3, ..., N$ are given by the set of equations (7) and (8).

A sample of the joint posterior distribution is generated using MCMC methods such as the Gibbs sampling and the Metropolis-Hastings algorithm (see for instance, Gelfand and Smith [20] and Smith and Roberts [49]). This task is simplified by using the software WinBugs (see Spiegelhalter et al. [50]).

Different Bayesian discrimination methods are introduced in the literature to choose the best model to explain the behaviour of a given data set. In this paper, we are going to work with the Deviance Information Criterion (DIC). The DIC (see Spiegelhalter et al. [51]) is given by

$$DIC = \hat{D} + 2p_D, \tag{11}$$

where $\hat{D}$ is the deviance evaluated in the posterior mean and p_D is the effective number of parameters in the model, given by $p_D = \overline{D} - \hat{D}$, with $\overline{D}$ the posterior mean deviance. Smaller values of DIC indicate the best models. (Note that these values could be negative.)

4. An Application to the Data of the City of São Paulo

The data set used in this work is the weekly average measurements of five pollutants $(SO_2, NO_2, PM_{10}, O_3 \text{ and } CO)$ obtained by the Instituto de Astronomia, Geofísica e Ciências Atmosférica of the Universidade de São Paulo (http://www.iag.usp.br/) during the period ranging from May 1996 until December 2006. Hence, we have that $N = 555$ and $K = 5$.

Figure 1 presents the plots of the weekly average measurements of each pollutants considered here versus time measured in weeks.

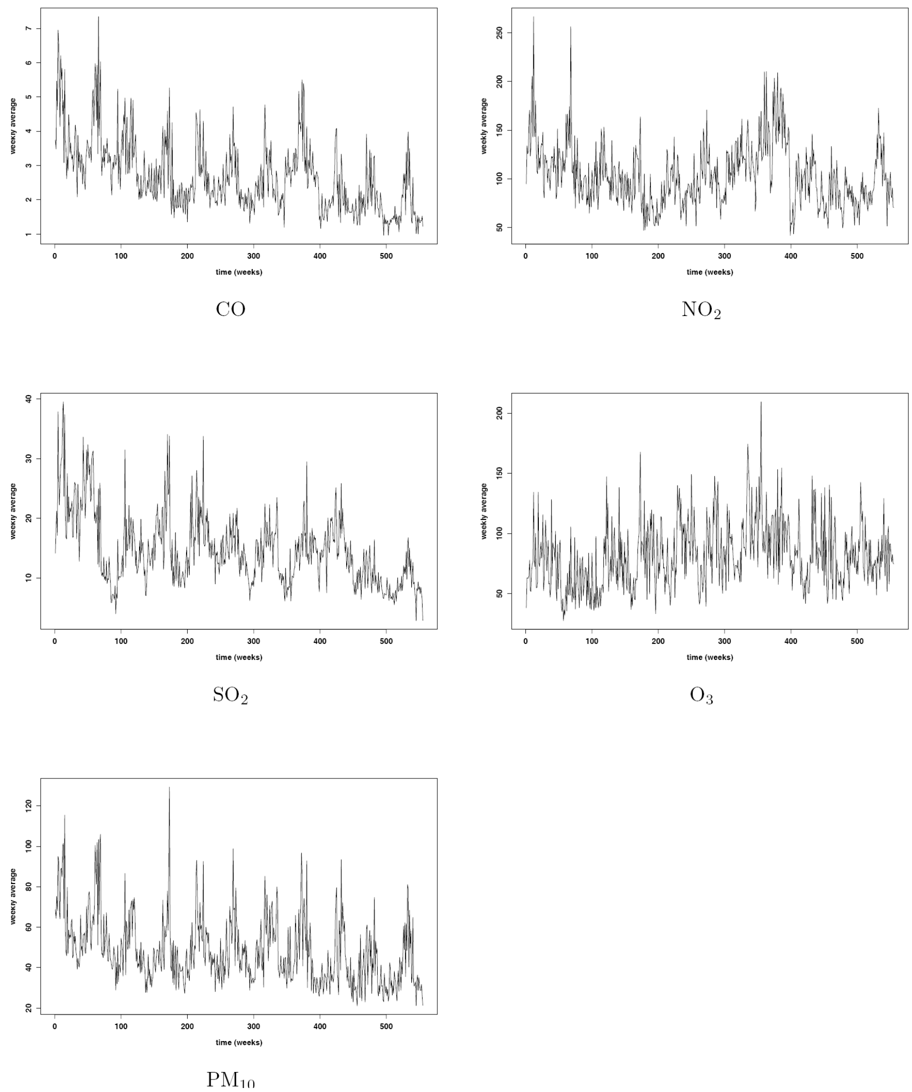

Figure 1. Weekly average measurement of the pollutants CO, NO_2, SO_2, O_3 and PM_{10} versus time, measured in weeks, for the entire observational period.

We may observe from Figure 1, that there is a consistent decrease of the weekly average measurements of CO, SO_2 and PM_{10} during the period at which the data was collected. We may also observe that there is a cyclic behaviour of the plots. That could be an indication that the behaviour of those pollutants is different during

Summer and Winter, but are very similar from Summer to Summer and from Winter to Winter. When we consider the pollutant NO_2, we may also observe a decreasing behaviour throughout the observational period with the exception of the period ranging from the 300th to the 400th and at the last weeks, where we observe a slight increase of the weekly average measurements. Regarding the pollutant O_3, we may observe that throughout the observational period, there is not an indication of a decreasing behaviour in its measurements.

It is worth mentioning that in 1996, the environmental authorities in the city of São Paulo implemented an environmental law similar to the one implemented in Mexico City in 1990. That law restricted the number of cars allowed to circulate in the Metropolitan Area. The use of cars during weekdays was controlled by the last number on the car's registration number. It is possible to see that after the implementation of such measure, around the 150th week (i.e., a week around 1999/2000), there was a decrease in the weekly average measurements of pollutants, mainly CO, NO_2 and PM_{10}. We may also observe from Figure 1, that after the 400th week (i.e., a week in the beginning of 2004), there is a stabilisation in a lower level of the weekly averages of the pollutants CO, NO_2 and PM_{10}. In the case of the pollutant SO_2, that stabilisation occurs around the 460th week. We would like to call attention to the fact that in the year 2004, another environmental law was implemented. That law (CONAMA [14]), establishes the guidelines for the regulation of motorcycles and similar vehicles. However, it seems that in the case of ozone, some stronger measures should be taken in order to reduce its level.

In Figure 2, we have the plots of the log-returns, $Y_i(t)$, centred at their means, $t = 1, 2, ..., N$, $i \in \{\mathrm{CO}, \mathrm{NO}_2, \mathrm{SO}_2, \mathrm{O}_3, \mathrm{PM}_{10}\}$.

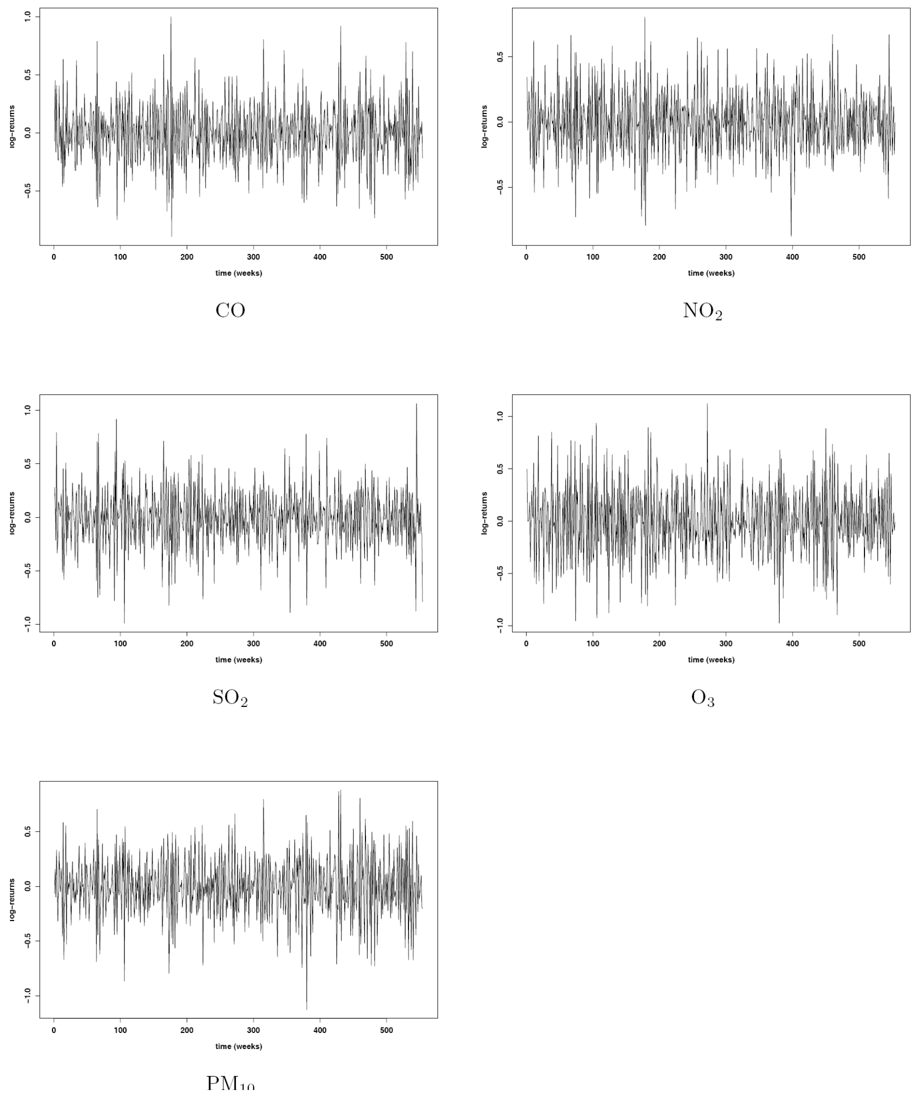

Figure 2. Log-returns centred at their means for the pollutants CO, NO_2, SO_2, O_3 and PM_{10} versus time, measured in weeks, for the entire observational period.

It is possible to observe from Figure 2 that there is a larger variability in the measurements from one week to the next for the pollutants CO, SO_2 and O_3. However, in the case of ozone, there is a decrease in the variability after a week around the 470th week.

In order to perform the estimation of the parameters of the models, we need to specify the values of their hyperparameters.

Remark. We are going to use the following association between the pollutants and indices. The pollutants O_3, CO, SO_2, PM_{10} and NO_2 are associated to the indices 1, 2, 3, 4 and 5, respectively. We also report the estimates of $\tau_{\eta_i} = 1/\sigma^2_{\eta_i}$ instead of reporting those of $\sigma^2_{\eta_i}$, $i = 1, 2, \ldots, K$.

1. **Model I.** The hyperparameters of the prior distributions of ϕ_{ii}, $\sigma^2_{\eta_i}$ and μ_i are $a_{ii} = 0$, $b_{ii} = 0.1$, $c_i = d_i = 1$, $e_i = 0$ and $f_i = 100$, $i = 1, 2, 3, 4, 5$.

Remark. The choice of the hyperparameters was made in order to have non-informative prior distributions and also to improve the convergence of the Gibbs sampling algorithm that is internally implemented in the software WinBugs.

In Table 1, we have the summary of the estimated mean, standard deviation (indicated by SD) and the 95% credible interval of the quantities of interest when Model I is considered.

Table 1. Estimated posterior mean, standard deviation (SD) and 95% credible interval of the quantities of interest when Model I is used

Pollutant	Parameter	Mean	SD	95% Credible Interval
O_3	μ_1	–2.17	0.08	(–2.33; –2.03)
	ϕ_1	0.41	0.20	(0.01; 0.76)
	τ_{η_1}	4.97	1.47	(2.71; 8.50)
CO	μ_2	–2.78	0.11	(–3.00; –2.58)
	ϕ_2	0.70	0.10	(0.46; 0.85)
	τ_{η_2}	4.19	1.30	(2.13; 7.26)
SO_2	μ_3	–2.71	0.09	(–2.89; –2.55)
	ϕ_3	0.46	0.13	(0.19; 0.68)
	τ_{η_3}	2.90	0.79	(1.67; 4.80)
PM_{10}	μ_4	–2.65	0.10	(–2.84; –2.46)
	ϕ_4	0.66	0.13	(0.36; 0.84)
	τ_{η_4}	4.36	1.39	(2.08; 7.57)
NO_2	μ_5	–2.81	0.08	(–2.96; –2.66)
	ϕ_5	0.31	0.17	(–0.03; 0.61)
	τ_{η_5}	4.26	1.34	(2.29; 7.37)

In Figure 3, we have the plots of the square roots of the estimated volatility of each pollutant when Model I is considered.

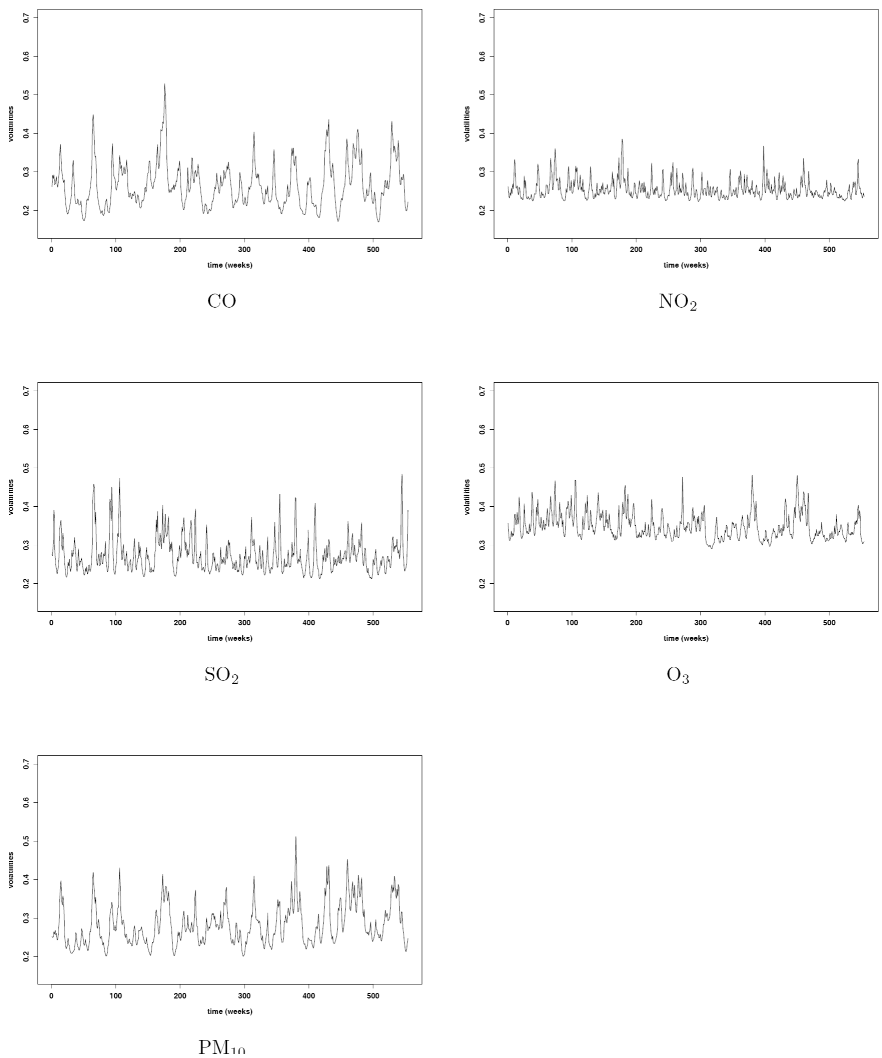

Figure 3. Square roots of the volatility of the pollutants CO, NO_2, SO_2, O_3 and PM_{10}, when Model I is used.

We may observe from Figure 3 that the volatility has more variation when we consider the pollutants CO and PM_{10}. It has smaller range and similar behaviour for NO_2 and O_3. The case of SO_2 is a middle term between those cases.

2. **Model II.** When we consider Model II, the hyperparameters of the prior distributions of ϕ_{ii}, $\sigma^2_{\eta_i}$ and μ_i, $i = 1, 2, 3, 4, 5,$ are given in Table 2.

Table 2. Hyperparameters of the prior distributions of the parameters ϕ_{ii}, $\sigma^2_{\eta_i}$ and μ_i, $i = 1, 2, 3, 4, 5,$ when Model II is considered

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
a_{ii}	0.41	0.70	0.46	0.66	0.31
b_{ii}	0.04	0.01	0.02	0.02	0.03
c_i	11.42	10.38	13.38	9.79	10.18
d_i	2.30	2.48	4.61	2.24	2.39
e_i	–2.17	–2.78	–2.71	–2.65	–2.81
f_i	0.01	0.01	0.01	0.01	0.01

The hyperparameters of the prior distributions of ρ_{ij} are $g_{ij} = 0$ and $h_{ij} = 0.1$, $i, j = 1, 2, 3, 4, 5,$ $i \neq j$.

Remark. It is important to call attention to the fact that the change in the values of the hyperparameters of the prior distributions of the parameters ϕ_{ii}, $\sigma^2_{\eta_i}$ and μ_i from Model I to Model II is due to the fact that information provided by Model I was used when considering the values for the case of Model II. Therefore, we are using an empirical Bayesian approach (see Carlin and Louis [11]).

In Table 3, we give the summary of the estimates of the quantities of interest.

Table 3. Estimated posterior mean, standard deviation (SD) and 95% credible interval of the quantities of interest when Model II is used

Pollutant	Parameter	Media	SD	95% Credible Interval
O_3	μ_1	–2.18	0.06	(–2.30; –2.06)
	ϕ_1	0.56	0.10	(0.36; 0.74)
	τ_{η_1}	1.55	0.26	(1.12; 2.16)
CO	μ_2	–2.76	0.07	(–2.90; –2.64)
	ϕ_2	0.66	0.07	(0.52; 0.79)
	τ_{η_2}	5.48	1.15	(3.57; 7.92)
SO_2	μ_3	–2.72	0.07	(–2.85; –2.59)
	ϕ_3	0.53	0.08	(0.37; 0.67)
	τ_{η_3}	1.31	0.19	(0.98; 1.76)
PM_{10}	μ_4	–2.63	0.07	(–2.77; –2.50)
	ϕ_4	0.62	0.11	(0.39; 0.81)
	τ_{η_4}	2.01	0.40	(1.40; 2.94)
NO_2	μ_5	–2.82	0.05	(–2.92; –2.72)
	ϕ_5	0.40	0.12	(0.16; 0.62)
	τ_{η_5}	5.58	1.20	(3.56; 8.26)
	ρ_{12}	0.24	0.04	(0.15; 0.32)
	ρ_{13}	0.59	0.03	(0.53; 0.64)
	ρ_{14}	0.76	0.02	(0.72; 0.80)
	ρ_{15}	0.69	0.02	(0.64; 0.73)
	ρ_{23}	0.53	0.03	(0.46; 0.59)
	ρ_{24}	0.66	0.03	(0.60; 0.71)
	ρ_{25}	0.54	0.03	(0.47; 0.60)
	ρ_{34}	0.79	0.02	(0.75; 0.83)
	ρ_{35}	0.68	0.02	(0.63; 0.73)
	ρ_{45}	0.72	0.02	(0.68; 0.76)

In Figure 4, we have similar plots of those shown in Figure 3, but now considering Model II.

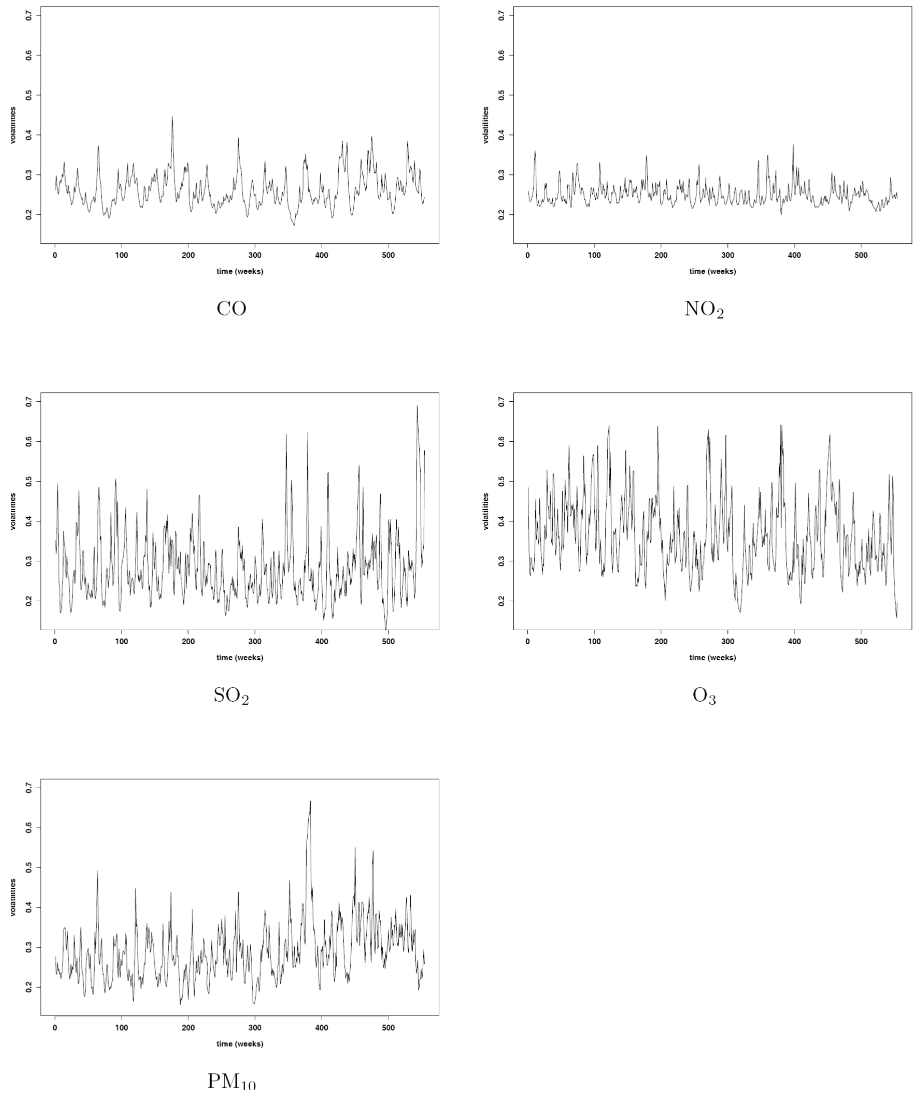

Figure 4. Square roots of the volatility of the pollutants CO, NO_2, SO_2, O_3 and PM_{10}, when Model II is used.

It is possible to observe from Figure 4 that the behaviour of the volatility of the pollutant NO_2 is very similar to that given by Model I. In the case of the pollutant

CO, we have a similar shape of the plot to the one given by Model I. However, the range at which the plot obtained by Model II varies, is smaller. When looking at the plots of the volatility for the remaining pollutants, we may see that the variability is very large and its behaviour is not as controlled as in the case of Model I.

Model I and Model II have DIC equal to 928.3 and –873.54, respectively. Hence, Model II is the one that best explains the behaviour of the pollutants CO, SO_2, NO_2, PM_{10} and O_3, present in the city of São Paulo.

5. Conclusion

Observing Figures 3 and 4, it is important to point out that there is no conclusive evidence of a consistent decrease and stabilisation of the levels of the volatility of the five pollutants throughout the observational period. However, we can observe a slight decrease in the volatility after the 470th week in the case of O_3 and NO_2 in both models and in the case of PM_{10} when Model II is considered. As mentioned earlier, in 1996, there was an environmental law regulating the amount of vehicles circulating in the Metropolitan Area of São Paulo. Additionally, in 2004, further measures were taken (in this case, regulating the use of motorcycles and similar vehicles). Hence, from the results presented here, we may notice that there is an indication that the environmental laws implemented in São Paulo by the environmental authorities have helped to decrease the levels of the volatility of some of the pollutants considered here.

As we have seen, the model that best explains the pollution data of the city of São Paulo is Model II. In that model, we assume the presence of a non-zero correlation between two distinct coordinates of the error vector $\boldsymbol{\varepsilon}(t),\ t = 1, 2, \ldots, N$. That also produce an effect on the covariance between two different pollutants. It is possible to see that the error coordinate associated to the pollutant ozone has the largest correlation with the error coordinate associated to the pollutant PM_{10}, followed by the pollutants NO_2 and SO_2. The correlation between ozone and PM_{10} may be explained by the fact that when there is a large concentration of particulate matter in the atmosphere, we have that the concentration of ozone will be smaller. That may be explained by the fact that in the presence of particulate matter, ozone oxidises and therefore there might be a decrease in its concentration. We may also see that the error coordinate associated to PM_{10} has the largest correlation with that related to SO_2.

The non-zero correlation of the error coordinates provides a perturbation in the contribution given by the square roots of the volatility of the pollutants to the covariance between those pollutants. We may also see that the largest perturbation occurs when we take into account pollutants PM_{10} and SO_2, followed by the case of O_3 and PM_{10}. The third largest perturbation occurs when we consider the possible interaction between PM_{10} and NO_2. The fourth largest correlation is that related to O_3 and NO_2. The latter could be explained by the fact that O_3 forms in the atmosphere due to photolysis of NO_2 (Kumar et al. [30]).

Observing the values of μ, ϕ and σ^2 given by Tables 1 and 3, we have that both models produce very similar values for the estimated mean μ and the parameter ϕ. However, estimated values of the variance σ^2 are substantially different. The estimated value of σ^2 determines the variance associated to the latent variables related to the volatility of the series of log-returns. In this way, we observe that Model I produces a larger variance in the volatility of the pollutants O_3 and PM_{10} and produce smaller variance in the remaining pollutants. This is reflected in the plots given in Figures 3 and 4.

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