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AN APPLICATION OF THE PERTURBATION METHOD AND THE ADOMIAN DECOMPOSITION METHOD (ADM) IN SOLVING FITZHUGH-NAGUMO (FHN) EQUATIONS

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Abstract

In this paper, the regular perturbation method and the Adomian decomposition method are used to construct the solution of the FHN equations.

1. Introduction

The FitzHugh-Nagumo (FHN) equations [13] form a reaction-diffusion model. The FHN equations are used to model the electric waves of the heart: the excitation wave and the recuperation wave. Because of nonlinearity, they do not have an exact analytic solution. Solutions have been constructed by the uniform fixed "grid method". Recently, Longin Some of Ouagadougou University, in his Ph.D. thesis [12], [14], has constructed a numerical solution using the mobile grid method. The Adomian decomposition method (ADM) and the regular perturbation method are very useful to get an approximation of a solution of an equation. Here, we use both methods to study the FHN equations.

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2. An Application of the Regular Perturbation Method

General properties of the perturbation method and its applications can be found in [11]. The general form of the FHN equations is given by

$$\begin{cases} \dfrac{\partial u(z,\, t)}{\partial t} = u(z,\, t) - u^3(z;\, t) - v(z,\, t) + \dfrac{\partial^2 u(z,\, t)}{\partial z^2}, \; -\infty < z < +\infty,\; t > 0, \\ \dfrac{\partial v(z,\, t)}{\partial t} = \varepsilon(u(z,\, t) - \gamma_1 v(z,\, t) - \gamma_2) + \delta\dfrac{\partial^2 u(z,\, t)}{\partial z^2}, \; -\infty < z < +\infty,\; t > 0, \end{cases} \tag{2.1}$$

where $\varepsilon, \gamma_1, \gamma_2, \delta$ are given real constants, $u(z, t)$ is the excitation wave, $v(z, t)$ is the recuperation wave, z is a space variable and t stands for time.

We examine the following initial and boundary value problem:

$$\begin{cases} \dfrac{\partial u(z,\, t)}{\partial t} = u(z,\, t) - u^3(z;\, t) - v(z,\, t) + \dfrac{\partial^2 u(z,\, t)}{\partial z^2}, \; -50 < z < 50,\; t > 0, \\ \dfrac{\partial v(z,\, t)}{\partial t} = \varepsilon(u(z,\, t) - \gamma_1 v(z,\, t)) + \dfrac{\partial^2 v(z,\, t)}{\partial z^2}, \; -50 < z < 50,\; t > 0 \end{cases} \tag{2.2}$$

with the initial conditions

$$u(z;\, 0) = -0.8 + 2.0\exp\left[-\left(\frac{z}{10}\right)^2\right], \quad -50 < z < 50, \tag{2.3}$$

$$v(z,\, 0) = -0.1, \tag{2.4}$$

and the Neumann boundary conditions

$$\frac{\partial u(-50,\, t)}{\partial z} = \frac{\partial u(50,\, t)}{\partial z} = 0, \quad t \geq 0, \tag{2.5}$$

$$\frac{\partial v(-50,\, t)}{\partial z} = \frac{\partial v(50,\, t)}{\partial z} = 0, \quad t \geq 0. \tag{2.6}$$

Suppose that the solutions $u(z, t)$ and $v(z, t)$ of problems (2.2)-(2.6) have the following form [1]:

$$u(z, t) = \sum_{i=0}^{N} \varepsilon^i u_i(z, t) + R_{1N}(z, t, \varepsilon), \quad v(z, t) = \sum_{i=0}^{N} \varepsilon^i v_i(z, t) + R_{2N}(z, t, \varepsilon), \tag{2.7}$$

where $R_{1N}(z, t, \varepsilon)$ and $R_{2N}(z, t, \varepsilon)$ are the remainder terms of these two series.

Taking (2.7) into (2.2), (2.3), (2.4), (2.5), (2.6), and collecting equal powers of ε, we obtain systems of recurrent initial and boundary value problems for $u_i(z, t)$ and $v_i(z, t)$, $i = 0, 1, 2, \ldots,$

$$\begin{cases} \dfrac{\partial u_0(z, t)}{\partial t} = u_0(z, t) - u_0^3(z; t) - v_0(z, t) + \dfrac{\partial^2 u_0(z, t)}{\partial z^2}, \\ \dfrac{\partial v_0(z, t)}{\partial t} = \dfrac{\partial^2 v_0(z, t)}{\partial z^2}, \\ u_0(z; 0) = -0.8 + 2.0 \exp\left[-\left(\dfrac{z}{10}\right)^2\right], \\ v_0(z, 0) = -0.1, \\ \dfrac{\partial u_0(-50, t)}{\partial z} = \dfrac{\partial u_0(50, t)}{\partial z} = 0, \\ \dfrac{\partial v_0(-50, t)}{\partial z} = \dfrac{\partial v_0(50, t)}{\partial z} = 0; \end{cases} \tag{2.8}$$

$$\begin{cases} \dfrac{\partial u_1(z, t)}{\partial t} = u_1(z, t) - 3u_0^2(z; t)u_1(z, t) - v_1(z, t) + \dfrac{\partial^2 u_1(z, t)}{\partial z^2}, \\ \dfrac{\partial v_1(z, t)}{\partial t} = \dfrac{\partial^2 v_1(z, t)}{\partial z^2} + u_0(z, t) - \gamma_1 v_0(z, t), \\ u_1(z; 0) = 0, \\ v_1(z, 0) = 0, \\ \dfrac{\partial u_1(-50, t)}{\partial z} = \dfrac{\partial u_1(50, t)}{\partial z} = 0, \\ \dfrac{\partial v_1(-50, t)}{\partial z} = \dfrac{\partial v_1(50, t)}{\partial z} = 0; \end{cases} \tag{2.9}$$

$$\begin{cases} \dfrac{\partial u_2(z, t)}{\partial t} = u_2(z, t) - 3u_1^2(z; t)u_0(z, t) - 3u_0^2(z; t)u_2(z, t) \\ \qquad\qquad - v_2(z, t) + \dfrac{\partial^2 u_2(z, t)}{\partial z^2}, \\ \dfrac{\partial v_2(z, t)}{\partial t} = \dfrac{\partial^2 v_2(z, t)}{\partial z^2} + u_1(z, t) - \gamma_1 v_1(z, t), \\ u_2(z; 0) = 0, \\ v_2(z, 0) = 0, \\ \dfrac{\partial u_2(-50, t)}{\partial z} = \dfrac{\partial u_2(50, t)}{\partial z} = 0, \\ \dfrac{\partial v_2(-50, t)}{\partial z} = \dfrac{\partial v_2(50, t)}{\partial z} = 0; \end{cases} \tag{2.10}$$

etc., and further systems for higher order terms of the series (2.7) can easily be calculated using a symbolic computation package, such as Scientific Workplace, Maple or Mathematica.

3. Application of ADM to Find the Terms of Series (2.7)

General properties of ADM and its applications can be found in [1-10]. To find $u_0(z, t)$ and $v_0(z, t)$, we consider system (2.8). We first get $v_0(z, t)$ using the following initial and boundary value problem:

$$\begin{cases} \dfrac{\partial v_0(z, t)}{\partial t} = \dfrac{\partial^2 v_0(z, t)}{\partial z^2}, \\ v_0(z, 0) = -0.1, \\ \dfrac{\partial v_0(-50, t)}{\partial z} = \dfrac{\partial v_0(50, t)}{\partial z} = 0. \end{cases} \tag{3.1}$$

According to the ADM, we suppose that the solution of (3.1) has the following form:

$$v_0(z, t) = \sum_{i=0}^{\infty} v_{0,i}(z, t). \tag{3.2}$$

From (3.1), we have

$$v_0(z,\ t) = v_0(z,\ 0) + \int_0^t \frac{\partial^2 v_0(z,\ s) ds}{\partial z^2}, \tag{3.3}$$

$$\frac{\partial v_0(50,\ t)}{\partial z} - \frac{\partial v_0(-50,\ t)}{\partial z} - \int_{-50}^{50} \frac{\partial v_0(z,\ t)}{\partial t} dz = 0. \tag{3.4}$$

From (3.3)-(3.4), we have

$$v_0(z, t) = v_0(z, 0) + \frac{\partial v_0(50, t)}{\partial z} - \frac{\partial v_0(-50, t)}{\partial z} + \int_0^t \frac{\partial^2 v_0(z, s) ds}{\partial z^2} - \int_{-50}^{50} \frac{\partial v_0(z, t)}{\partial t} dz.$$

We thus obtain the following Adomian algorithm:

$$\begin{cases} v_{0,0}(z,\ t) = v_0(z,\ 0) + \dfrac{\partial v_0(50,\ t)}{\partial z} - \dfrac{\partial v_0(-50,\ t)}{\partial z} = -0.1, \\ v_{0,n+1}(z,\ t) = \displaystyle\int_0^t \frac{\partial^2 v_{0,n}(z,\ s) ds}{\partial z^2} - \int_{-50}^{50} \frac{\partial v_{0,n}(z,\ t)}{\partial t} dz, \quad \forall n \geq 0, \end{cases} \tag{3.5}$$

which gives us

$$v_{0,0}(z,\ t) = -0.1;\ v_{0,1}(z,\ t) = 0;\ v_{0,2}(z,\ t) = 0;\ \cdots;\ v_{0,n}(z,\ t) = 0,$$

so that

$$v_0(z,\ t) = -0.1. \tag{3.6}$$

To find $u_0(z,\ t)$, we consider the following initial and boundary value problem:

$$\begin{cases} \dfrac{\partial u_0(z,\ t)}{\partial t} = u_0(z,\ t) - u_0^3(z;\ t) - v_0(z,\ t) + \dfrac{\partial^2 u_0(z,\ t)}{\partial z^2}, \\ u_0(z;\ 0) = \varphi_1(z) = -0.8 + 2.0 \exp\left[-\left(\dfrac{z}{10}\right)^2\right], \\ \dfrac{\partial u_0(-50,\ t)}{\partial z} = \dfrac{\partial u_0(50,\ t)}{\partial z} = 0. \end{cases} \tag{3.7}$$

We modify problem (3.7) and examine the following equivalent problem:

$$\begin{cases} \dfrac{\partial u_0(z,\, t)}{\partial t} = u_0(z,\, t) - u_0^3(z;\, t) - v_0(z,\, t) + \dfrac{\partial^2 u_0(z,\, t)}{\partial z^2}, \\ u_0(z;\, 0) = \varphi_2(z) = -0.8 + 2.0\exp\left[-\left(\dfrac{z}{10}\right)^2\right] + \dfrac{z^2}{50}\exp(-25), \\ \dfrac{\partial u_0(-50,\, t)}{\partial z} = \dfrac{\partial u_0(50,\, t)}{\partial z} = 0. \end{cases} \tag{3.8}$$

Thus, we have

$$\frac{d\varphi_2(-50)}{dz} = \frac{d\varphi_2(50)}{dz}, \tag{3.9}$$

and note that, in the interval $[-50,\, 50]$, $\varphi_1(z)$ and $\varphi_2(z)$ have the same graph.

We suppose that the solution of (3.8) has the following form:

$$u_0(z,\, t) = \sum_{i=0}^{\infty} u_{0,\, i}(z,\, t). \tag{3.10}$$

From (3.8), we have

$$u_0(z,\, t) = u_0(z,\, 0) + \int_0^t u_0(z,\, s)ds - \int_0^t u_0^3(z,\, s)ds + 0.1t + \int_0^t \frac{\partial^2 u_0(z,\, s)}{\partial z^2} ds, \tag{3.11}$$

$$\frac{\partial u_0(50,\, t)}{\partial z} - \frac{\partial u_0(-50,\, t)}{\partial z} + \int_{-50}^{50} u_0(z,\, t)dz$$

$$- \int_{-50}^{50} u_0^3(z,\, t)dz + 10 - \int_{-50}^{50} \frac{\partial u_0(z,\, t)}{\partial t} dz = 0. \tag{3.12}$$

From (3.11)-(3.12), we have

$$u_0(z,\, t) = u_0(z,\, 0) + \frac{\partial u_0(50,\, t)}{\partial z} - \frac{\partial u_0(-50,\, t)}{\partial z} + 0.1t + 10$$

$$+ \int_0^t u_0(z,\, s)ds - \int_0^t u_0^3(z,\, s)ds + \int_0^t \frac{\partial^2 u_0(z,\, s)}{\partial z^2} ds$$

$$+ \int_{-50}^{50} u_0(z,\, t)dz - \int_{-50}^{50} u_0^3(z,\, t)dz - \int_{-50}^{50} \frac{\partial u_0(z,\, t)}{\partial t} dz, \tag{3.13}$$

and obtain the following Adomian algorithm:

$$\begin{cases} u_0(z,\ t) = -0.8 + 2.0\exp\left[-\left(\dfrac{z}{10}\right)^2\right] + \dfrac{z^2}{50}\exp(-25) + 0.1t + 10, \\ u_{n+1}(z,\ t) = \displaystyle\int_0^t u_0(z,\ s)ds - \int_0^t A_n(z,\ s)ds + \int_0^t \frac{\partial^2 u_0(z,\ s)}{\partial z^2}ds \\ \qquad + \displaystyle\int_{-50}^{50} u_0(z,\ t)dz - \int_{-50}^{50} A_n(z,\ t)dz,\ \forall n \geq 0, \end{cases} \tag{3.14}$$

where

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{+\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \qquad n = 0, 1, 2, ...,$$

we have

$$\begin{aligned} u_{0,1} = &-3509.4t - 41.31t^2 - 0.192t^3 - 0.00025t^4 - 87391. \\ &- (505.8t + 5.52t^2 + 0.02t^3)e^{-0.01z^2} - (110.4t + 0.6t^2)e^{-0.02z^2} \\ &- 8.0te^{-0.03z^2} - (7.0251\times 10^{-11}t + 7.666\times 10^{-13}t^2 \\ &+ 2.7776\times 10^{-15}t^3)z^2 - (2.1293\times 10^{-24}t + 1.1573\times 10^{-26}t^2)z^4 \\ &- 2.1429\times 10^{-38}tz^6 - (-8.0000\times 10^{-4}t \\ &+ 1.6666\times 10^{-13}t^2)z^2e^{-0.01z^2} - 3.3331\times 10^{-12}tz^2e^{-0.02z^2} \\ &- 4.629\times 10^{-25}tz^4e^{-0.01z^2}, \end{aligned}$$

$$\begin{aligned} u_{0,2} = &\ 1.7191\times 10^8 t + 4.1934\times 10^6 t^2 + 51391.t^3 + 341.74t^4 \\ &+ 1.2024t^5 + 0.00194t^6 + 1.0714\times 10^{-6}t^7 + 2.3949\times 10^9 \\ &+ (1.3399\times 10^{-6}t + 4.3071\times 10^{-8}t^2 + 5.9997\times 10^{-10}t^3 \\ &+ 4.2176\times 10^{-12}t^4 + 1.4832\times 10^{-14}t^5 + 2.0832\times 10^{-17}t^6)z^2 \end{aligned}$$

$$
\begin{aligned}
&+ (2.0227 \times 10^{-20} t + 1.214 \times 10^{-21} t^2 + 1.5901 \times 10^{-23} t^3 \\
&+ 8.5635 \times 10^{-26} t^4 + 1.7359 \times 10^{-28} t^5) z^4 \\
&+ (9.0473 \times 10^{-36} t^2 + 9.2003 \times 10^{-38} t^3 + 2.6788 \times 10^{-40} t^4) z^6 \\
&+ (4.107 \times 10^{-49} t^2 + 2.0833 \times 10^{-51} t^3) z^8 \\
&+ 2.4800 \times 10^{-63} t^2 z^{10} + (9.648 \times 10^6 t + 3.1012 \times 10^5 t^2 \\
&+ 4320.2 t^3 + 30.373 t^4 + 0.1068 t^5 + 0.00015 t^6) e^{-0.01 z^2} \\
&+ (1.0487 \times 10^6 t + 62942. t^2 + 824.44 t^3 + 4.44 t^4 \\
&+ 0.009 t^5) e^{-0.02 z^2} + (10141. t^2 + 103.04 t^3 + 0.3 t^4) e^{-0.03 z^2} \\
&+ (1104.0 t^2 + 5.6 t^3) e^{-0.04 z^2} + 48.0 t^2 e^{-0.05 z^2} + (2.9127 \times 10^{-7} t \\
&- 0.20237 t^2 - 2.208 \times 10^{-3} t^3 - 8.0 \times 10^{-6} t^4 \\
&+ 2.4998 \times 10^{-15} t^5) z^2 e^{-0.01 z^2} + (-0.13248 t^2 - 0.00064 t^3 \\
&+ 1.2500 \times 10^{-13} t^4) z^2 e^{-0.02 z^2} + (-0.0192 t^2 \\
&+ 3.1109 \times 10^{-12} t^3) z^2 e^{-0.03 z^2} + 3.333 \times 10^{-11} t^2 z^2 e^{-0.04 z^2} \\
&+ (1.6000 \times 10^{-7} t^2 - 6.6664 \times 10^{-17} t^3 \\
&+ 1.7359 \times 10^{-26} t^4) z^4 e^{-0.01 z^2} + (-3.9998 \times 10^{-15} t^2 \\
&+ 6.4807 \times 10^{-25} t^3) z^4 e^{-0.02 z^2} + 9.258 \times 10^{-24} t^2 z^4 e^{-0.03 z^2} \\
&+ (-1.8516 \times 10^{-28} t^2 + 6.0003 \times 10^{-38} t^3) z^6 e^{-0.01 z^2} \\
&+ 1.2858 \times 10^{-36} t^2 z^6 e^{-0.02 z^2} + 8.928 \times 10^{-50} t^2 z^8 e^{-0.01 z^2},
\end{aligned}
$$

etc. Using (3.14), we can calculate each term of the series in (3.10), and obtain the solution of (3.8):

$$u_0(z,\, t) = u_{0,\,0}(z,\, t) + u_{0,1}(z,\, t) + u_{0,\,2}(z,\, t) + \cdots,$$

i.e.,

$$\begin{aligned}
u_0(z,\, t) = {} & 2.3948 \times 10^9 + 1.7191 \times 10^8 t + 4.1934 \times 10^6 t^2 + 51391.t^3 \\
& + 341.74t^4 + 1.2024t^5 + 0.00194t^6 + 1.0714 \times 10^{-6} t^7 \\
& + (2.7776 \times 10^{-13} + 1.3398 \times 10^{-6} t + 4.307 \times 10^{-8} t^2 \\
& + 5.9997 \times 10^{-10} t^3 + 4.2176 \times 10^{-12} t^4 + 1.4832 \times 10^{-14} t^5 \\
& + 2.0832 \times 10^{-17} t^6) z^2 + (2.0225 \times 10^{-20} t + 1.2140 \times 10^{-21} t^2 \\
& + 1.5901 \times 10^{-23} t^3 + 8.5635 \times 10^{-26} t^4 + 1.7359 \times 10^{-28} t^5) z^4 \\
& + (-2.1429 \times 10^{-38} t + 9.0473 \times 10^{-36} t^2 + 9.2003 \times 10^{-38} t^3 \\
& + 2.6788 \times 10^{-40} t^4) z^6 + (4.107 \times 10^{-49} t^2 + 2.0833 \times 10^{-51} t^3) z^8 \\
& + 2.4800 \times 10^{-63} t^2 z^{10} + (2.0 + 9.6475 \times 10^6 t + 3.1011 \times 10^5 t^2 \\
& + 4320.2t^3 + 30.373t^4 + 0.1068t^5 + 0.00015t^6) e^{-0.01z^2} \\
& + (1.0486 \times 10^6 t + 62941.t^2 + 824.44t^3 + 4.44t^4 + 0.009t^5) e^{-0.02z^2} \\
& + (-8.0t + 10141.t^2 + 103.04t^3 + 0.3t^4) e^{-0.03z^2} + (1104.0t^2 \\
& + 5.6t^3) e^{-0.04z^2} + 48.0t^2 e^{-0.05z^2} + (8.0029 \times 10^{-4} t - 0.20237t^2 \\
& - 2.208 \times 10^{-3} t^3 - 8.0 \times 10^{-6} t^4 + 2.4998 \times 10^{-15} t^5) z^2 e^{-0.01z^2} \\
& + (-3.3331 \times 10^{-12} t - 0.13248t^2 - 0.00064t^3 \\
& + 1.2500 \times 10^{-13} t^4) z^2 e^{-0.02z^2} + (-4.629 \times 10^{-25} t - 0.0192t^2 \\
& + 3.1109 \times 10^{-12} t^3) z^2 e^{-0.03z^2} + 3.333 \times 10^{-11} t^2 z^2 e^{-0.04z^2}
\end{aligned}$$

$$+ (1.6000 \times 10^{-7} t^2 - 6.6664 \times 10^{-17} t^3 + 1.7359 \times 10^{-26} t^4) z^4 e^{-0.01z^2}$$

$$+ (-3.9998 \times 10^{-15} t^2 + 6.4807 \times 10^{-25} t^3) z^4 e^{-0.02z^2}$$

$$+ 9.258 \times 10^{-24} t^2 z^4 e^{-0.03z^2} + (-1.8516 \times 10^{-28} t^2$$

$$+ 6.0003 \times 10^{-38} t^3) z^6 e^{-0.01z^2} + (-1.8516 \times 10^{-28} t^2$$

$$+ 6.0003 \times 10^{-38} t^3) z^6 e^{-0.01z^2} + 1.2858 \times 10^{-36} t^2 z^6 e^{-0.02z^2}$$

$$+ 8.928 \times 10^{-50} t^2 z^8 e^{-0.01z^2} + \cdots.$$

To find $u_1(z, t)$ and $v_1(z, t)$, we consider the system (2.9). We first get $v_1(z, t)$ using the following initial and boundary value problem:

$$\begin{cases} \dfrac{\partial v_1(z, t)}{\partial t} = \dfrac{\partial^2 v_1(z, t)}{\partial z^2} + u_0(z, t) - \gamma_1 v_0(z, t), \\ v_1(z, 0) = 0, \\ \dfrac{\partial v_1(-50, t)}{\partial z} = \dfrac{\partial v_1(50, t)}{\partial z} = 0, \end{cases} \tag{3.15}$$

and we suppose that the solution of (3.15) has the following form:

$$v_1(z, t) = \sum_{i=0}^{\infty} v_{1,i}(z, t). \tag{3.16}$$

As in the previous system, we have for $v_1(z, t)$ the following Adomian algorithm:

$$\begin{cases} v_{1,0}(z, t) = v_1(z, 0) + \dfrac{\partial v_1(50, t)}{\partial z} - \dfrac{\partial v_1(-50, t)}{\partial z} + \displaystyle\int_0^t u_0(z, s) ds \\ \qquad - \gamma_1 \displaystyle\int_0^t v_0(z, s) ds + \int_{-50}^{50} u_0(z, t) dz - \gamma_1 \int_{-50}^{50} v_0(z, s) ds, \\ v_{1,n+1}(z, t) = \displaystyle\int_0^t \dfrac{\partial^2 v_{1,n}(z, s)}{\partial z^2} ds - \int_{-50}^{50} \dfrac{\partial v_{1,n}(z, t)}{\partial t} dz, \end{cases} \tag{3.17}$$

which gives us

$$v_{1,0}(z,\ t) = 1.9770 \times 10^{10} t + 10.0\gamma_1 + 0.1t\gamma_1 + 5.1170 \times 10^{8} t^{2}$$

$$+ 6.6249 \times 10^{6} t^{3} + 47619.t^{4} + 190.60t^{5} + 0.39706t^{6}$$

$$+ 2.7714 \times 10^{-4} t^{7} + 1.3393 \times 10^{-7} t^{8} + 2.3948 \times 10^{11}$$

$$+ (2.7776 \times 10^{-13} t + 6.699 \times 10^{-7} t^{2} + 1.4357 \times 10^{-8} t^{3}$$

$$+ 1.4999 \times 10^{-10} t^{4} + 8.4352 \times 10^{-13} t^{5} + 2.472 \times 10^{-15} t^{6}$$

$$+ 2.976 \times 10^{-18} t^{7}) z^{2} + (1.0113 \times 10^{-20} t^{2} + 4.0467 \times 10^{-22} t^{3}$$

$$+ 3.9753 \times 10^{-24} t^{4} + 1.7127 \times 10^{-26} t^{5} + 2.8932 \times 10^{-29} t^{6}) z^{4}$$

$$+ (3.0158 \times 10^{-36} t^{3} - 1.0715 \times 10^{-38} t^{2} + 2.3001 \times 10^{-38} t^{4}$$

$$+ 5.3576 \times 10^{-41} t^{5}) z^{6} + (1.369 \times 10^{-49} t^{3} + 5.2083 \times 10^{-52} t^{4}) z^{8}$$

$$+ 8.2667 \times 10^{-64} t^{3} z^{10} + (2.0t + 4.8238 \times 10^{6} t^{2}$$

$$+ 1.0337 \times 10^{5} t^{3} + 1080.1t^{4} + 6.0746t^{5} + 0.0178t^{6}$$

$$+ 2.1429 \times 10^{-5} t^{7}) e^{-0.01z^{2}} + (5.243 \times 10^{5} t^{2} + 20980.0t^{3}$$

$$+ 206.11t^{4} + 0.888t^{5} + 0.0015t^{6}) e^{-0.02z^{2}}$$

$$+ (3380.3t^{3} - 4.0t^{2} + 25.76t^{4} + 0.06t^{5}) e^{-0.03z^{2}}$$

$$+ (368.0t^{3} + 1.4t^{4}) e^{-0.04z^{2}} + 16.0t^{3} e^{-0.05z^{2}}$$

$$+ (4.0015 \times 10^{-4} t^{2} - 6.7457 \times 10^{-2} t^{3}$$

$$- 5.52 \times 10^{-4} t^{4} - 1.6 \times 10^{-6} t^{5} + 4.1663 \times 10^{-16} t^{6}) z^{2} e^{-0.01z^{2}}$$

$$+ (2.5 \times 10^{-14} t^{5} - 0.04416t^{3} - 0.00016t^{4}$$

$$- 1.6666 \times 10^{-12} t^{2}) z^{2} e^{-0.02z^{2}} + (7.7773 \times 10^{-13} t^{4}$$

$$
\begin{aligned}
&- 0.0064t^3 - 2.3145 \times 10^{-25} t^2) z^2 e^{-0.03z^2} \\
&+ 1.111 \times 10^{-11} t^3 z^2 e^{-0.04z^2} + (5.3333 \times 10^{-8} t^3 \\
&- 1.6666 \times 10^{-17} t^4 + 3.4718 \times 10^{-27} t^5) z^4 e^{-0.01z^2} \\
&+ (1.6202 \times 10^{-25} t^4 - 1.3333 \times 10^{-15} t^3) z^4 e^{-0.02z^2} \\
&+ 3.086 \times 10^{-24} t^3 z^4 e^{-0.03z^2} + (1.5001 \times 10^{-38} t^4 \\
&- 6.172 \times 10^{-29} t^3) z^6 e^{-0.01z^2} + 4.286 \times 10^{-37} t^3 z^6 e^{-0.02z^2} \\
&+ 2.976 \times 10^{-50} t^3 z^8 e^{-0.01z^2} + \cdots,
\end{aligned}
$$

$$
\begin{aligned}
v_{1,1}(z, t) = {} & -1.0252 \times 10^{11} t - 10.0\gamma_1 - 1.9939 \times 10^9 t^2 \\
& - 1.9136 \times 10^7 t^3 - 95897.t^4 - 240.25t^5 \\
& - 0.19666t^6 - 1.0714 \times 10^{-4} t^7 - 1.977 \times 10^{12} \\
& + 7.44 \times 10^{-19} t^8 + (4.0453 \times 10^{-20} t^3 \\
& + 1.214 \times 10^{-21} t^4 + 9.5408 \times 10^{-24} t^5 \\
& + 3.4253 \times 10^{-26} t^6 + 4.9597 \times 10^{-29} t^7) z^2 \\
& + (2.2619 \times 10^{-35} t^4 - 1.0715 \times 10^{-37} t^3 \\
& + 1.3801 \times 10^{-37} t^5 + 2.6788 \times 10^{-40} t^6) z^4 \\
& + (1.9166 \times 10^{-48} t^4 + 5.8332 \times 10^{-51} t^5) z^6 \\
& + 1.86 \times 10^{-62} t^4 z^8 + (-0.02t^2 - 32159.t^3 \\
& - 516.88t^4 - 4.3206t^5 - 2.0248 \times 10^{-2} t^6 \\
& - 5.0857 \times 10^{-5} t^7 - 5.3573 \times 10^{-8} t^8) e^{-0.01z^2}
\end{aligned}
$$

$$+ (-7.36t^4 - 0.0224t^5)e^{-0.04z^2}$$

$$+ (-6990.7t^3 - 209.82t^4 - 1.6489t^5$$

$$- 0.00592t^6 - 8.5714 \times 10^{-6} t^7)e^{-0.02z^2} + (0.08t^3$$

$$- 50.708t^4 - 0.30912t^5 - 0.0006t^6)e^{-0.03z^2}$$

$$- 0.4t^4 e^{-0.05z^2} + (0.0004t^2 + 643.17t^3$$

$$+ 10.339t^4 + 0.08642t^5 + 4.05 \times 10^{-4} t^6$$

$$+ 1.0171 \times 10^{-6} t^7 + 1.0715 \times 10^{-9} t^8) z^2 e^{-0.01z^2}$$

$$+ (279.63t^3 + 8.3943t^4 + 6.5962 \times 10^{-2} t^5$$

$$+ 2.368 \times 10^{-4} t^6 + 3.4286 \times 10^{-7} t^7) z^2 e^{-0.02z^2}$$

$$+ (3.0428t^4 - 0.0144t^3 + 1.8547 \times 10^{-2} t^5$$

$$+ 3.6t^6) z^2 e^{-0.03z^2} + (0.5888t^4$$

$$+ 1.792 \times 10^{-3} t^5) z^2 e^{-0.04z^2} + 0.04t^4 z^2 e^{-0.05z^2}$$

$$+ (6.6667 \times 10^{-18} t^6 - 1.7664 \times 10^{-5} t^4$$

$$- 5.12 \times 10^{-8} t^5 - 8.8887 \times 10^{-16} t^3) z^4 e^{-0.02z^2}$$

$$+ (5.3353 \times 10^{-8} t^3 - 6.7483 \times 10^{-6} t^4$$

$$- 4.416 \times 10^{-8} t^5 - 1.0667 \times 10^{-10} t^6$$

$$+ 2.3807 \times 10^{-20} t^7) z^4 e^{-0.01z^2} + (5.5996 \times 10^{-16} t^5$$

$$- 5.76 \times 10^{-6} t^4 - 2.7774 \times 10^{-28} t^3) z^4 e^{-0.03z^2}$$

$$+ 1.7776 \times 10^{-14} t^4 z^4 e^{-0.04z^2} + (5.3333 \times 10^{-12} t^4$$

$$- 1.3333 \times 10^{-21} t^5 + 2.3145 \times 10^{-31} t^6) z^6 e^{-0.01z^2}$$

$$+ 1.7144 \times 10^{-40} t^4 z^8 e^{-0.02z^2} + 2.976 \times 10^{-54} t^4 z^{10} e^{-0.01z^2}$$

$$+ (5.1846 \times 10^{-29} t^5 - 5.3333 \times 10^{-19} t^4) z^6 e^{-0.02z^2}$$

$$+ 2.7775 \times 10^{-27} t^4 z^6 e^{-0.03z^2}$$

$$+ (1.2001 \times 10^{-42} t^5 - 6.172 \times 10^{-33} t^4) z^8 e^{-0.01z^2} + \cdots.$$

Thus using (3.17), we can calculate each term of (3.16), and, by the same process, we can calculate $u_1(z, t)$; and deduce solution of system (2.2) from this.

4. Conclusion

1. We remark that using the regular perturbation method and the ADM, with a symbolic computation package, we can construct a solution of the FHN equations which is valid in a bounded domain.

2. Our future preoccupation is to study the convergence of these solutions and draw a comparison between these results and those of Some [12].

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