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AN IMAGE DENOISING METHOD USING BIVARIATE RATIONAL INTERPOLATION FUNCTION

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Abstract

A simple and effective image denoising method which can keep image detail is presented in this paper. The novel procedure for the image noise removal based on the image edge detail preserving is proposed. A bivariate rational interpolation with parameters is used in the algorithm. An interpolation surface is constructed using an image data as the interpolation data. According to the maximum and minimum membrane energy value of the interpolation surface, the noise pixel is detected. If it is a noise point, the value is replaced by the rank-ordered mean of the filter window. Thus, the image noise is reduced by re-interpolation.

1. Introduction

Image noise is extraneous visible artifacts that degrade image quality. It not only can effect the visual perception but also hinder the correct identification. Image denoising is the most fundamental technique of image processing [1, 2].

Popular methods, as commonly used in image/video software and hardware products, are neighborhood averaging, gradient inverse weighting, linear interpolation, median filter. Teboul et al. [3] used edge-preserving function to keep

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the edge and texture information of the original image. However, image noise filtering still remains a difficult challenge because noise removal introduces artifacts and causes blurring of images. In order to improve the filtering effect, some improved median filtering algorithms are proposed, such as adaptive median filter [4], center weighted median filter, max-minimum method [5]. Compared with the original median filter, these methods increase the noise detection process. Relatively the later is better to protect the image details, but the filtering compute slowly. Windyga [6] proposed peak-valley filtering method. If the gray value of pixels to be detected is the maximum or minimum value within the gray of the filter window, then the point is the noise point. The gray noise detection and estimation are completed within one step. The filtering speed improves greatly, but the denoising effect is not very good and loses image detail easily. Alajlan et al. [7] proposed a new method. The noise points are detected by the peak-valley method and dealt with by the maximum-minimum value. Compared with the maximum-minimum method, the filtering speed is increased. But the effect is not satisfactory for higher density noise. Chan et al. [8] proposed a new denoising algorithm which can obtain better filtering effect for higher density noise. However, this algorithm is based on variational and iterative method of solving partial differential equations. The computation is complex.

According to the integrated analysis of above method, they have merits and demerits, respectively. The major factors of denoising are detail-preserving of the image, efficiency of the filter and computational complexity. Therefore, we propose a method based on a rational function for solving these problems. A bivariate rational interpolation has been constructed and studied by Zhang et al. [9-10]. The bivariate rational interpolating function which based on function values has piecewise explicit rational mathematical representation. The expression is piece wised. Based on the nonlinear and simple forms of the function, it can be used for image processing [12]. We propose an algorithm using the rational interpolation function as noise filter. First, the membrane energy extreme value of the interpolation surface is used to identify possible noise pixels in a size of 3×3 filter window. Second, the noise pixels replaced by the gray filter window based on the membrane energy. Last, the new gray values as interpolation data are used for re-interpolation. Compared with the traditional algorithm, this algorithm can maintain the details of the image. The average PSNR is improved.

The paper is arranged as follows: In Section 2, the bivariate rational function is introduced. Section 3 deals with the algorithm design principle of image denoising.

2. Bivariate Rational Interpolation Function

2.1. Established interpolation function

Let $\Omega : [a, b; c, d]$ be the plane region, and let $f(x, y)$ be a bivariate function defined in the region Ω and let $a = x_1 < x_2 < \cdots < x_{n+1} = b$ and $c = y_1 < y_2 < \cdots < y_{m+1} = d$ be the knot sequences. Denote $f(x_i, y_j)$ by $f_{i,j}$, then $\{(x_i, y_i, f_{i,j}), i = 1, 2, ..., n+1; j = 1, 2, ..., m+1\}$ is the given set of data points. For any point $(x, y) \in [x_i, x_{i+1}; y_j, y_{j+1}]$ in the (x, y)-plane. Let $h_i = x_{i+1} - x_i$, $l_j = y_{j+1} - y_j$, and let $\theta = \frac{x - x_i}{h_i}$ and $\eta = \frac{y - y_j}{l_j}$. The function $P_{i,j}(x, y)$ is defined in the subregion $[x_i, x_{i+1}; y_j, y_{j+1}]$ and depends on the data at nine points $\{(x_r, y_s, f_{r,s}, r = i, i+1, i+2, s = j, j+1, j+2)\}$. It is called the *bivariate rational interpolating function* based on function values and the bivariate rational interpolating function $P_{i,j}(x, y)$ can be expressed as follows:

$$P_{i,j}(x, y) = \sum_{r=0}^{2}\sum_{s=0}^{2}\omega_{r,s}(\theta, \alpha_i; \eta, \beta_j) f_{i+r, j+s}. \tag{1}$$

2.2. Energy of the interpolation function

The interpolating surface varies as the parameters vary. Based on this, not only the membrane energy of the interpolation surface but also the maxima and minima of the energy can be derived. Thus, the membrane energy is introduced

$$E = \frac{1}{2}\iint_S (X_u^2 + X_v^2) dudv. \tag{2}$$

The membrane energy of the interpolation function can be derived by (1) and (2). Namely

$$E = \frac{1}{2}\iint_\sigma \left(\left(\frac{\partial P_{i,j}(x, y)}{\partial x} + \frac{\partial P_{i,j}(x, y)}{\partial y}\right)\right) dxdy. \tag{3}$$

According to the parameters of the interpolation function, the maxima and minima of the energy are derived by suitable parameters $\max E(\alpha, \beta)$ and $\min E(\alpha, \beta)$, respectively.

3. Image Denoising Based on Bivariate Rational Interpolation Function

Given an $m \times n$ image $I_{m,n}$, let $f_{i,j} (0 \leq i \leq m-1, 0 \leq j \leq n-1)$ be the gray value of the *i* line and the *j* row of $I_{m,n}$. The pixel coordinate is (i, j). So $I_{m,n}$ can be denoted as a two-dimensional discrete signal which samples at integer points. The gray value of each pixel can be denoted as data point and continuous interpolating surface can be constructed based on bivariate rational interpolation method for discrete image. The surface $P_{i,j}(x, y)$ can be constructed using the points $f_{i,j}, i, j = 0, 1, \ldots, n-1$. First, matrix $f_1 = \{f_{i,j}, 0 \leq i \leq m-1, 0 \leq j \leq n-1\}$ is expanded into $f_2 = \{f_{i,j}, 0 \leq i \leq m, 0 \leq j \leq n\}$, so the values of f_{ij} $(i = m$ or $j = n)$ are added. Outer-interpolation method is used, let $f_{m,j} = 2f_{m-1,j} - f_{m-2,j}$ $(0 \leq j \leq n-1)$, $f_{i,n} = 2f_{i,n-1} - f_{i,n-2}$ $(0 \leq i \leq m-1)$. Then expand f_2 to $f_3 = \{f_{i,j}, -1 \leq i \leq m, -1 \leq j \leq n\}$ which is similar as to expanding f_1 and the last number of $M \times N (3 \times 3)$ interpolation surface $P_{i,j}(x, y)$ are constructed based on f_3. On each interval $[x_i, x_{i+1}] \times [y_i, y_{i+1}]$, $(i, j = 0, 1, \ldots, n-2)$, a bivariate rational interpolating spline surface $P_{i,j}(x, y)$ is constructed.

Based on noise characteristics, the noise test conditions are limited strictly. The pixel to be processed is located in the center of a size of 3×3 filter window. According to the membrane energy of the interpolation surface, the filtered value *m* and *M* are obtained $m = \frac{\min E}{S_\sigma}$ and $M = \frac{\max E}{S_\sigma}$.

Then followed is to determination whether the pixel (gray scale) value is between *m* and *M*. If it is in the interval, then the point is the signal point and can be as the filtered output value with the gray value directly. If the centre pixel of the gray value is greater than *M* or less than *m*, then the point is noise point. For the size of 3×3 filter window, the ascending sequence is obtained after statistics sorting. For example, n_1 is the minimum value in the filter window, n_9 is the maximum value in the filter window. n_2, n_3, n_4 are most close to the minimum gray value of n_1, similarly, n_6, n_7, n_8 are most close to the maximum gray value of n_9. The mean gray value of the filter window n^* is obtained through calculation. If the detected point is a noise point, the value is replaced. Through the noise point detection value M and m, the substitute L is obtained, where $L = \omega_1 m + \omega_2 M$. Then the noise point is replaced as following:

$$\begin{cases} |\, n^* - n_1 \,| > |\, n_9 - n^* \,| L = \omega_1 m + \omega_2 M, & 0 < \omega_1 < 1/2,\ 1/2 < \omega_2 < 1, \\ |\, n^* - n_1 \,| < |\, n_9 - n^* \,| L = \omega_1 m + \omega_2 M, & 1/2 < \omega_1 < 1,\ 0 < \omega_2 < 1/2, \\ |\, n^* - n_1 \,| < |\, n_9 - n^* \,| L = \omega_1 m + \omega_2 M, & \omega_1 = \omega_2 = 1/2. \end{cases}$$

The images will appear small spots when the noise densities reach a higher level. In this case, the detection conditions can be strictly and the filtering window increases appropriately. The followed is re-interpolation using the new gray value as the interpolation data.

Conclusions

In this paper, an image noise removal algorithm based on the image edge detail preserving was proposed, which used the membrane energy value of a bivariate rational interpolation with parameters for noise point detection. Comparing with the traditional algorithms, this algorithm has small quantity of calculation and strong robustness. The algorithm has shown its effectiveness.

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