JP Journal of Algebra, Number Theory and Applications
Volume 20, Number 2, 2011, Pages 231-233
Published Online: April 1, 2011
This paper is available online at http://pphmj.com/journals/jpanta.htm

ERRATA TO: A NOTE ON THE EQUALITY OF THE HILBERT POLYNOMIAL AND FUNCTION OF A MODULE WITH RESPECT TO AN IDEAL

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The following corrections need to be made to my paper with the above title, that appeared in JP Journal of Algebra, Number Theory and Applications, Volume 15, Number 2, 2009, pp. 157-162.

1. We will use $\widetilde{IM} = \bigcup_{k \geq 1} (I^{k+1}M :_M I^k)$, the Ratliff-Rush submodule of M associated with I and not $r(I, M) = \bigcup_{k \geq 1} (I^{k+1}M : I^k M)$, as is stated in the introduction.

2. Lemma 2.1 (Fundamental Lemma)**.** *Let (R, m) be a Noetherian local ring with maximal ideal m, M be a 2-dimensional Cohen Macaulay module and let (x, y) be a system of parameters with respect to M. Let I be any ideal such that (x, y) is a reduction of I and let $u_n = P_{I, M}(n) - H_{I, M}(n)$. Then*

$$\lambda(I^{n+1} M/(x, y) I^n M) - \lambda((I^n M :_M (x, y))/I^{n-1}M) = u_{n+1} + u_{n-1} - 2u_n$$

for all $n \geq 1$.

- Replace the first paragraph of the proof by the following:

2010 Mathematics Subject Classification: 13A30, 13C14, 13B20.

Keywords and phrases: Cohen Macaulay, Rees-graded modules, Briancon-Skoda theorem.

Received May 10, 2010

Consider the exact sequence

$$0 \to K \to (M/I^nM)^2 \to (x,\ y)M/I^n(x,\ y)M \to 0,$$

where $\alpha : (M/I^nM)^2 \to (x,\ y)M/I^n(x,\ y)M$ is defined by $\alpha(m + I^nM, n + I^nM)$ $= mx + ny + I^n(x,\ y)M$. Since $(x,\ y)$ is a M-regular sequence, $K = \{(ty + I^nM,$ $-tx + I^nM) | t \in M\}$. Define $\beta : K \to M/(I^nM :_M (x,\ y))$ by $\beta(ty + I^nM, -tx + I^nM)$ $= t + (I^nM :_M (x,\ y))$. It is clear that β is an isomorhism.

3. Replace Theorem 2.2 by the following:

Theorem 2.2. *Let* $(R,\ m)$ *be a local ring with infinite residue field*, *M be a* 2-*dimensional Cohen Macaulay module and let I be an ideal of definition with respect to M such that* $I^nM = \widetilde{I^nM}$ *for all* $n \geq 1$. *Then the following are equivalent*:

(a) $P_{I,M}(n) = H_{I,M}(n)$ *for all* $n \geq 1$,

(b) *there exist* $x,\ y \in I$ *such that* $I^{n+1}M = (x,\ y)I^nM$ *for all* $n \geq 2$ *and grade* $(G_I(R)_+,\ G_I(M)) > 0$.

Proof: (a)$\Rightarrow$(b): Set $u'_n = u_{n+1} + u_{n-1} - 2u_n$ and let $(x,\ y)$ be a reduction of *I* with respect to *M*. Then it follows from Lemma 2.1 and (a) that $u'_n = 0$ and thus $\lambda(I^{n+1}M/(x,\ y)I^nM) = \lambda(I^nM :_M (x,\ y))/I^{n-1}M)$ for all $n \geq 2$. We now claim that $(I^2M :_M (x,\ y)) = IM$; then it follows from the assumption that $I^3M =$ $(x,\ y)I^2M$ and thus also $I^{n+1}M = (x,\ y)I^nM$ for all $n \geq 2$ and thus grade $(G_I(R)_+,\ G_I(M)) > 0$.

Suppose that $a \in (I^2M :_M (x,\ y))$. That is $a(x,\ y) \subseteq I^2M$. But $I^jM = (x,\ y)I^{j-1}M$ for all $j >> 0$, since $(x,\ y)$ is a reduction of *I* with respect to *M* and therefore also $\overline{I^j} = \overline{(x,\ y)I^{j-1}}$, where "-" denotes mod *annM*. Thus $\overline{aI^j} = \overline{a(x,\ y)I^{j-1}} \subseteq \overline{I^{j-1}I^2M} = \overline{I^{j+1}M}$ and therefore $aI^j \subseteq I^{j+1}M$, i.e., $a \in \widetilde{IM}$. We now have $a \in IM$ and the statement follows.

Conversely, suppose that (b) is true. Let $a \in (I^n M :_M (x, y))$. Then also $\overline{a(x, y)} \subseteq \overline{I^n} M$. But as above, $\overline{I^{j+1}} = \overline{(x, y) I^j}$ for all $j >> 0$, so that $\overline{aI^{j+1}} = \overline{a(x, y) I^j} \subseteq \overline{I^n I^j} M$. Thus $aI^{j+1} \subseteq I^{n+j} M$, so that $a \in \left(I^{n+j} M :_M I^{j-1}\right) \subseteq \widetilde{I^{n-1} M} = I^{n-1} M$ and we have $(I^n M :_M (x, y))/I^{n-1} M = 0$. Thus $\lambda(I^{n+1} M/(x, y) I^n M) = u_n'$ for all $n \geq 1$. But it follows from our assumption that $\lambda(I^{n+1} M/(x, y) I^n M) = 0$ for $n \geq 2$, so that $u_n' = 0$ for all $n \geq 2$. But $u_m = 0$ for $m >> 0$, so let n be the largest index such that $u_n \neq 0$. If $n \geq 1$, then $u_{n+2}, u_{n+1} = 0$ and from $u_n' = 0$ it follows that $u_n = 0$, a contradiction, and (a) follows.

4. In Lemma 2.3, replace $(I^{n+1} M : IM) M$ with $(I^{n+1} M :_M I)$.

5. Replace the proof of Lemma 2.3 by the following:

Proof. Let $x \in I$ be an M regular element. By the Artin Rees lemma, there exists a number c such that for all $n > c$ we have $I^{n+1} M \cap xM = I^{n+1-c}(I^c M \cap xM)$. Let now $m \in (I^{n+1} M :_M I)$. Then $xm \subseteq I^{n+1} M \cap xM = I^{n+1-c}(I^c M \cap xM) \subseteq xI^{n+1-c} M$. Thus $m \in I^{n+1-c} M$, since x is not a zero divisor. But there exists a number I such that for all $n \geq l$, $(I^{n+1} M :_M I) \cap I^l M = I^n M$ (cf. [2, 1.1]) and thus for n large enough it follows from this that $m \in I^n M$.

6. In Proposition 2.4, replace $(I^J M : IM) M$ with $(I^J M :_M I)$.

7. In Proposition 2.5, replace $r(I, M)$ with $\widetilde{IM}$.