Far East Journal of Applied Mathematics
Volume 49, Number 2, 2010, Pages 131-137
Published Online: February 9, 2011
This paper is available online at http://pphmj.com/journals/fjam.htm

ON *n*-TOTAL EDGE DOMINATION NUMBER IN GRAPHS

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Abstract

A set D of edges is a total edge dominating set of G, if every edge in G is adjacent to at least one edge in D. The total edge domination number $\gamma'_t(G)$ is defined to be the minimum number of edges in a total edge dominating set of G. A set D of edges is an n-total edge dominating set of G if every edge in G is adjacent to at least n-edges in D. The n-total edge domination number $\gamma'_{tn}(G)$ is defined to be the minimum number of edges in an n-total edge dominating set of G. In this paper, we initiate a study of n-total edge dominating set and in particular, we establish some results concerning the n-total edge domination number of G and also we establish Nordhaus-Gaddum type result for an n-total edge domination number of a graph.

1. Introduction

The graphs considered here are finite, nonempty, connected, undirected without

2010 Mathematics Subject Classification: 05C69.

Keywords and phrases: edge domination, total edge domination, n-total domination, n-total edge domination.

Received September 13, 2010

loops and multiple edges and without isolates. Any undefined terms in this paper may be found in Harary [3].

A set F of edges is an *edge dominating set* of G if every edge not in F is adjacent to at least one edge in F. The *edge domination number* $\gamma'(G)$ is defined to be the cardinality of a minimum edge dominating set in G. This parameter was introduced by Mitchell and Hedetniemi [4].

A set D of edges is a *total edge dominating set* of G if every edge in G adjacent to at least one edge in D. The *total edge domination number* $\gamma'_t(G)$ is defined to be the cardinality of a minimum total edge dominating set in G. This parameter was introduced by Kulli and Patwari [6].

A set D of edges is an *n-total edge dominating set* of G if every edge in G is adjacent to at least n-edges in D. The *n-total edge domination number* $\gamma'_{tn}(G)$ is defined to be the cardinality of a minimum n-total edge dominating set in G.

The paper contains the following notations and terminology:

$\Delta'(G)$ - the maximum degree among the lines of G (the degree of a line is the number of lines adjacent to it).

$\gamma_{tn}(G)$ - n-total domination number in graphs. This parameter was introduced by Kulli [5].

Results.

2. *n*-total Edge Domination Number

We note the following:

(2.1) Every n-total edge dominating set is a total edge dominating set and hence for every graph G, we have $\gamma'_t(G) \le \gamma'_{tn}(G)$.

(2.2) $\gamma'_{tn}(G)$ does not exist for graphs with less than two edges.

(2.3) For any connected graph G with $p \ge 3$ vertices

$$\gamma'_{tn}(G) = \gamma_{tn}(L(G)).$$

In the following proposition, we list the $\gamma'_{tn}(G)$ for some standard graphs, which are simple to observe, hence, we omit the proof.

Proposition 1. (1) $\gamma'_{tn}(K_{m1, n1}) = m1,$ *where* $2 \leq m1 \leq n1,$

(2) $\gamma'_{tn}(W_p) = \left\lfloor \frac{p+2}{3} \right\rfloor,$

(3) $\gamma'_{tn}(P_p) = \left\lfloor \frac{p}{3} \right\rfloor,$

(4) $\gamma'_{tn}(C_p) = \left\lfloor \frac{p}{2} \right\rfloor.$

The following result is easy to prove, hence we omit the proof.

Theorem 1. *For any graph G*,

$$\gamma'_{tn}(G) \leq \delta(G).$$

Theorem 2. *Let* K_p *be the complete graph*. *Then*

$$\gamma'_{tn}(K_p) = n + 1, \quad \textit{where } 1 \leq n \leq q,$$

where q is number of edges in G.

Proof. Let S be a minimum n-total edge dominating set in K_p, and for each edge e_1 in E, e_1 is adjacent with at least n edges of S. Then in particular, if e_2 is an edge in S, then e_2 is adjacent with at least n edges of S. This implies that S has at least $n + 1$ edges. Also, since n-total edge dominating set is minimum, we have $\gamma'_{tn}(K_p) = n + 1.$

Theorem 3. *Let G be a graph with minimum degree at least n*. *Let D be a minimal n-total edge dominating set*. *Then G contains a minimal edge dominating set*.

Proof. Let D be a minimal n-total edge dominating set of G. Suppose there is an edge $e \in D$ which is not adjacent to any edges in G. Then e is adjacent to at least n edges in D itself. Therefore, $D - \{e\}$ is an n-total edge dominating set, a contradiction. Thus every edge in D must be adjacent to at least one edge in G. Hence, G is an edge dominating set.

Theorem 4. *For any graph G*,

$$\gamma'_{tn}(G) \geq m.$$

Proof. Let D be a minimum n-total edge dominating set of G. Then every edge in G is dominated by at least m edges in D.

Therefore, $D \geq m$,

$$\gamma'_{tn}(G) \geq m.$$

Theorem 5. *Let G be a tree. Then*

$$\gamma'_{tn}(G) \leq \gamma'_t(G).$$

Proof. Let G be a tree and D be a total edge dominating set with minimum number of edges. Let E' denote the edge set of $\langle D \rangle$ the subgraph induced by D.

Let E' be an edge dominating set of G and F be the set of edges which are not adjacent to any edge in E'.

For every edge $e \in F$, we take exactly one edge adjacent to e and form a set F' of edges. Then we have

$D' = E'UF'$ is an n-total edge dominating set of G and

$$\gamma'_{tn}(G) \leq D' \leq D$$

$$\leq \gamma'_t(G).$$

Hence the proof.

Proposition 2. *For any connected graph*

$$\gamma'_{tn}(G) \leq q - \Delta'(G),$$

q is the number of edges in G and $q \geq 4$.

Proof. Let D be an n-total edge dominating set of G with minimum number of edges. Then every edge in D is adjacent to utmost $q - \Delta'(G)$ edges.

Hence, $\gamma'_{tn}(G) \leq q - \Delta'(G)$.

Proposition 3. *Let F be a minimum edge dominating set of G such that the induced subgraph* $\langle E - F \rangle$ *is* mK_2, $m \geq 1$. *Thus*

$$\gamma'_{tn}(G) + \gamma'(G) = q.$$

Proof. Since for a γ'_t set F of G, the induced subgraph $\langle E - F\rangle$ is mK_2, $m \geq 1$, $E - F$ itself is a γ'_{tn}-set of G, thus

$$\gamma'_{tn}(G) + \gamma'(G) = |E - F| + |F|$$

$$= q.$$

Theorem 6. *For any graph G,*

$$\gamma'_t(G) + q \geq \gamma'_{tn}(G).$$

Proof. Let S be a minimum n-total edge dominating set of G.

Let $e \in E - S$. Let e be adjacent with at least n edges of S, say $e_1, e_2, ..., e_n$. Suppose

$$D = S - \{e_1, e_2, ..., e_n\}.$$

Since S is an n-total edge dominating set, e is adjacent with at least one edge of D.

Therefore, we conclude that every edge of G is adjacent with at least one edge of D.

Thus, D is a total edge dominating set of G,

$$\gamma'_t(G) \geq \gamma'_{tn}(G) - |E|$$

$$\geq \gamma'_{tn}(G) - |\{e_1, e_2, ..., e_n\}|$$

$$\geq \gamma'_{tn}(G) - q,$$

q denotes the number of edges in S,

$$\gamma'_t(G) + q \geq \gamma'_{tn}(G).$$

Theorem 7. *For any graph G,*

$$\gamma'_{tn}(G) \leq \beta_1(G),$$

where $\beta_1(G)$ *is the line independence number of G.*

Proof. Let $D = \{e_1, e_2, e_3, ..., e_n\}$ be the maximal line independent set of G and it is also n-total edge dominating set of G.

Suppose e is an edge of G which is not adjacent to any edge of D. Then $E(G) - D$ forms n-total edge dominating set of G and

$$|E(G) - D| \leq |D|,$$

$$\gamma'_{tn}(G) \leq \beta_1(G).$$

Theorem 8. *For any nontrivial tree T,*

$$\gamma'_{tn}(T) \leq c,$$

where c is the number of cutvertices.

Proof. This follows from the fact that each edge is incident with a cutvertex.

We use the following result to prove our next result.

Theorem A [2]**.** *For any graph G of order* $p \geq 3$ *vertices,*

(i) $\beta_1(G) + \beta_1(\overline{G}) \leq 2\left\lceil \frac{p}{2} \right\rceil,$

(ii) $\beta_1(G) \cdot \beta_1(\overline{G}) \leq \left\lceil \frac{p}{2} \right\rceil^2.$

We establish Nordhaus-Gaddum [1] type result for an n-total edge domination number of a graph.

Theorem 9. *Let G be a graph such that both G and* $\overline{G}$ *have no isolated edges. Then*

(i) $\gamma'_{tn}(G) + \gamma'_{tn}(\overline{G}) \leq 2\left\lceil \frac{p}{2} \right\rceil,$

(ii) $\gamma'_{tn}(G) \cdot \gamma'_{tn}(\overline{G}) \leq \left\lceil \frac{p}{2} \right\rceil^2.$

Proof. This follows from Theorem A and Theorem 7.

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