Advances and Applications in Statistics
Volume 19, Number 1, 2010, Pages 65-79
Published Online: February 9, 2011
This paper is available online at http://pphmj.com/journals/adas.htm

FOURTH-ORDER KERNEL METHOD USING LINE TRANSECT SAMPLING

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Abstract

In this paper, we introduce a nonparametric fourth-order kernel method for line transect sampling. This method produces a new estimator for the density of objects using line transect data. The asymptotic properties of the proposed estimator are derived under some mild assumptions. Moreover, an explicit formula for the smoothing parameter h is obtained based on minimizing the asymptotic mean square error (AMSE). Further, another estimator is suggested when there is no information whether the shoulder condition (i.e., $f'(0) = 0$) is valid or not. The performances of the proposed estimators are studied and compared with some existing estimators by simulation technique. As the results demonstrated, the fourth-order kernel method has overall better performance than the traditional kernel method, and in many cases is much more effective.

1. Introduction

Line transect method is commonly used by biologists to estimate population density. In addition to its logical framework with intuitive reasoning, sampling using

2010 Mathematics Subject Classification: 62F25.

Keywords and phrases: line transect method, shoulder condition, fourth-order kernel method.

Received March 22, 2010; Revised May 24, 2010

line transect has been a very convenient, easy and relatively cheaper method to obtain density of any living or non-living object in an ecosystem. To achieve the experiment, at least one observer moves across the population following a specific line with length L, looking to the right and to the left of the line. It is not sufficient just to record the number of observed objects, n; instead an observer must take the perpendicular distance (x) from the centerline to a detected object. When objects are observed from a line transect with a detection function $g(x)$, the distance x to observed object from randomly placed transect will tend to have a probability density $f(x)$ of the same shape as $g(x)$ but scaled so that the area under $f(x)$ equals to the unity. Buckland et al. [4] and Burnham et al. [6] constitute the key references for this distance sampling procedure.

Logical considerations deriving from the analysis of the physical sighting process suggest that $g(x)$ may usually be assumed monotonically decreasing and satisfying the shoulder condition (i.e., $g'(x) = 0$). Accordingly, $f(x)$ is in turn monotonically decreasing with $f'(0) = 0$. However, recent studies have shown that the shoulder condition may not hold for many wildlife lines transect data such as whales, jack rabbits and small animals in tall grass (Buckland [2]; Mack and Quang [13]). The basic model for line transect sampling is introduced in the key paper by Burnham and Anderson [5] who obtain the fundamental relation for estimating the density of objects in a specific area which can be expressed as

$$D = \frac{E(n) f(0)}{2L}.$$

Accordingly, D can be estimated by

$$\hat{D} = \frac{n\hat{f}(0)}{2L},$$

where $\hat{f}(0)$ represents a sample estimator of $f(0)$ based on the n observed perpendicular distances $x_1, x_2, ..., x_n$ which is usually supposed to be random sample (Buckland et al. [4]. Hence, the key aspects in line transect sampling turns out to be the modeling of $f(x)$ as well as the estimation of $f(0)$.

In a parametric approach, let $f(x)$ be the unknown probability density function of perpendicular distance. Then a parametric method assumes a model $f(x, \theta)$

which is a member of a family of proper probability density functions of known functional form but depends on an unknown parameter θ, where θ may take a vector value and should be estimated by using the perpendicular distances. A variety of approaches to estimate θ will lead to $\hat{f}(0) = f(0, \hat{\theta}).$ A number of parametric models have been proposed for $f(x),$ and there is extensive literature on the use of the maximum likelihood techniques for estimation of $f(0).$ See, for example, Burnham and Anderson [5]; Pollock [15]; Burnham et al. [6] and Buckland [2].

The parametric methods are very powerful, but they are highly dependent on the specification of the model. As an alternative method to parametric approach, recent works have focused on employing the nonparametric traditional kernel method which can be considered as the second-order kernel method to estimate $f(0)$ (see Chen [7]; Mack et al. [14]). Eidous [9] proposed some methods to improve the performance of the kernel estimator using line transect data. As a nonparametric method Eidous [10] introduced the histogram method. He derived the asymptotic properties of the histogram estimator with bias correction term using line transect data. On the other hand, Buckland [3] introduced a semiparametric estimator for $f(0)$ based on a key half-normal with Hermite polynomial correction. Also, Barabesi [1] proposed a new semiparametric estimator based on the local parametric method. While the goal of nonparametric methods is to remove the model-dependence of the estimator, the semiparametric methods are applied to attain the advantages of the parametric and nonparametric models by combining them in one estimator.

This paper suggested two new estimators for $f(0)$ based on the fourth-order kernel method. The first estimator is developed and its asymptotic properties are derived under the assumption that the first and third derivatives of the underlying probability density function are zero at the origin, that is, $f'(0) = 0$ (the shoulder condition) and $f'''(0) = 0.$ On the other hand, if we are not sure whether the shoulder condition is valid or not, then another estimator is proposed based on a combination between the negative exponential which does not have a shoulder at the origin and fourth-order kernel models. The simulation results and the real numerical example show that the performances of the proposed estimators are highly likely in line transect sampling.

2. The Fourth-order Kernel Estimator

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a probability density function $f(x)$. Define $r_j = \int_{-\infty}^{\infty} u^j K_{(4)}(u)\,du$, where $K_{(4)}(u)$ is the fourth-order kernel defined to have

$$r_0 = 1, \quad r_1 = r_2 = r_3 = 0 \quad \text{and} \quad r_4 \neq 0. \tag{1}$$

An attractive approach is to obtain $K_{(4)}(u)$ as a function of $K(u)$, where $K(u)$ is a symmetric traditional kernel function satisfying the conditions

$$\int_{-\infty}^{\infty} K(u)\,du = 1, \quad \int_{-\infty}^{\infty} uK(u)\,du = 0, \quad \int_{-\infty}^{\infty} u^2 K(u)\,du \neq 0. \tag{2}$$

Define also

$$K_{(4)}(u) = \frac{(s_4 - s_2 u^2)K(u)}{s_4 - s_2^2}, \tag{3}$$

where

$$s_j = \int_{-\infty}^{\infty} u^j K(u)\,du. \tag{4}$$

The fourth-order kernel density estimator $\hat{f}(x)$ of $f(x)$ is (Wand and Jones [16])

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K_{(4)}\left(\frac{x - X_i}{h}\right), \quad -\infty < x < \infty, \tag{5}$$

where h is a positive number controling the smoothness of the fitted density curve, usually called the *bandwidth*. To apply equation (5) to line transect data, some modifications have to be made. Usually, all distances x_i are nonnegative. This implies that

$$f(x) = 0 \quad \text{if} \quad x < 0. \tag{6}$$

However, the fourth-order kernel density estimator given by (5) does not necessarily satisfy condition (6). To make it satisfy condition (6), we replace each value x_i with x_i and its reflection $-x_i$, and then applying equation (5) on the extended sample

$x_1, -x_1, x_2, -x_2, \ldots, x_n, -x_n$. If a fourth-order kernel estimator $\hat{f}_1(x)$ is constructed from this data set of size $2n$, then an estimator based on the original data set can be given by putting

$$\hat{f}(x) = \begin{cases} 2\hat{f}_1(x), & x \geq 0, \\ 0, & x < 0 \end{cases}$$

for the extended sample $x_1, -x_1, x_2, -x_2, \ldots, x_n, -x_n$ of size $2n$. Thus, if $x_1, x_2, \ldots, x_n$ is a random sample of perpendicular distances, then the fourth-order kernel estimator of $f(x)$ is given by

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}\left[K_{(4)}\left(\frac{x - x_i}{h}\right) + K_{(4)}\left(\frac{x + x_i}{h}\right)\right], \quad x \geq 0$$

and $\hat{f}(x) = 0$ for $x < 0$. Since $K(u)$ is a symmetric kernel about zero, $K_{(4)}(u)$ is symmetric about zero, and so

$$\hat{f}(0) = \frac{2}{nh}\sum_{i=1}^{n} K_{(4)}\left(\frac{x_i}{h}\right). \tag{7}$$

3. Asymptotic Properties

In this Section, we derive the asymptotic bias and variance of the fourth-order kernel estimator given by (7). Assume that the underlying probability density function $f(x)$ of a random sample of perpendicular distances $x_1, x_2, \ldots, x_n$ has a continuous fourth-order derivative over the positive half line. Assume also that $K(u)$ is a symmetric function satisfying condition (2). Then the expected value of $\hat{f}(0)$ for given the sample size n is

$$E(\hat{f}(0)) = \frac{2}{h}\int_0^{\infty} K_4\left(\frac{x_1}{h}\right) f(x_1)\, dx_1$$

$$= 2\int_0^{\infty} K_4(t) f(ht)\, dt,$$

where $t = x_1/h$. Expand $f(ht)$ around zero by using Taylor series, then if $h \to 0$

as $n \to \infty$,

$$E(\hat{f}(0)) = f(0) + 2hf'(0)\int_0^\infty tK_{(4)}(t)\,dt + \frac{2h^3}{3!} f'''(0)\int_0^\infty t^3 K_{(4)}(t)\,dt$$

$$+ \frac{2h^4}{4!} f^{(4)}(0)\int_0^\infty t^4 K_{(4)}(t)\,dt + O(h^5).$$

Thus, the asymptotic bias of $\hat{f}(0)$ is

$$Bias(\hat{f}(0)) = 2hf'(0)\int_0^\infty tK_{(4)}(t)\,dt + \frac{2h^3}{3!} f'''(0)\int_0^\infty t^3 K_{(4)}(t)\,dt$$

$$+ \frac{2h^4}{4!} f^{(4)}(0)\int_0^\infty t^4 K_{(4)}(t)\,dt. \tag{8}$$

If the shoulder condition is true, then a $O(h^3)$ bias is achieved. While a $O(h^2)$ bias is achieved for the traditional kernel estimator under the shoulder condition assumption (Chen [7]), to achieve a $O(h^4)$ bias, the following condition should be valid:

$$f'(0) = 0 \text{ and } f'''(0) = 0. \tag{9}$$

If condition (9) is not true and we aim to achieve a $O(h^4)$ bias, then it becomes necessary to use a boundary kernel to force $\int_0^\infty tK_{(4)}(t)\,dt = 0$ and $\int_0^\infty t^3 K_{(4)}(t)\,dt = 0,$ or to assume that the first two terms in the right hand side of (8) are equal with different signs. But the task is still not easy to determine a kernel function with such of the above properties because we need to integrate over $[0, \infty)$, not on $(-\infty, \infty)$.

In nonparametric density estimation using line transect sampling, a natural choice for the family of key probability density functions is constituted by the half-normal model (see Chen [7]; Mack and Quang [13]), that is,

$$f(x) = \frac{2}{\sigma}\phi(x/\sigma)I_{[0,\infty)}(x), \tag{10}$$

where $\phi(x)$ represents the standard normal probability density function and $I_A(x)$ is the indicator function of the set A. If we assume that the underlying probability density function is as given by (10), then condition (9) will be valid and then a $O(h^4)$ bias is achieved. We now turn to the variance. If condition (9) is true, then

$$\operatorname{var}(\hat{f}(0)) = \frac{4}{nh^2}\operatorname{var}\left(K_{(4)}\left(\frac{X_1}{h}\right)\right)$$

$$= \frac{4}{nh}\int_0^\infty K_{(4)}^2(t)f(ht)\,dt - \frac{1}{n}[f(0) + bias(\hat{f}(0))]^2.$$

If h is related to n in such a way that $h \to 0$ and $nh \to \infty$ as $n \to \infty$, then the variance of $\hat{f}(0)$ is

$$\operatorname{var}(\hat{f}(0)) = \frac{4}{nh}f(0)\int_0^\infty K_{(4)}^2(t)\,dt + O(n)^{-1}. \tag{11}$$

It is obvious that the variance is of order $O(nh)^{-1}$, which is the same order for the variance of the traditional kernel estimator (Chen [7]). Accordingly, the asymptotic mean square error (AMSE) of the proposed estimator $\hat{f}(0)$ is given by

$$\operatorname{AMSE}(\hat{f}(0)) = \frac{4}{nh}f(0)\int_0^\infty K_{(4)}^2(t)\,dt + \frac{4h^8}{576}\left[f^{(4)}(0)\int_0^\infty t^4 K_{(4)}(t)\,dt\right]^2, \tag{12}$$

where the first term in the right hand side of (12) is the variance and the second term is the squared bias.

4. Bandwidth Selection

To implement the new estimator in practice, we need to choose the value of the bandwidth h, which is the crucial problem in nonparametric density estimation. The bandwidth controls the smoothness of the fitted density curve. A larger h gives smoother estimate with smaller variance and larger bias. A smaller h produces a rougher estimate with larger variance and smaller bias (see equation (12)). One of the most common methods in nonparametric estimation is to find h that minimizes the AMSE and compromises between the variance and bias of the estimate. Consider the AMSE as a function of h (say $d(h)$), differentiate $d(h)$ with respect to h and equate to zero, so as to get

$$h = \left\{288 f(0) \int_0^\infty K_{(4)}^2(t)\,dt\right\}^{1/9} \left\{2 f^{(4)}(0) \int_0^\infty t^4 K_{(4)}(t)\,dt\right\}^{-2/9} n^{-1/9}. \tag{13}$$

Let $K(t)$ be the standard normal kernel. Then $s_4 = 3$ and $s_2 = 1$. Thus, $K_{(4)}(t) = \dfrac{(3 - t^2) K(t)}{2}$, where $K(t) = (1/\sqrt{2\pi})\exp(-t^2/2)$. Substituting these values back in (13), we have

$$h = 1.253064\{f(0)\}^{1/9}\{f^{(4)}(0)\}^{-2/9} n^{-1/9}. \tag{14}$$

The value of h given by (14) can be substituted back into (12) to give us the minimum achievable AMSE for $\hat{f}(0)$, which indicates that the convergence rate for the MSE of $\hat{f}(0)$ is of order $n^{-8/9}$, while it is of order $n^{-4/5}$ for the traditional kernel estimator (Chen [7]). In other words, as $n \to \infty$, the MSE of $\hat{f}(0)$ approaches to zero in a faster way compared to the MSE of the traditional kernel estimator.

The formula (14) is somewhat disappointing since it shows that h itself depends on unknown parameter $f(0)$ being estimated. A natural method for choosing h in line transect sampling is to assume half-normal as the underlying model. Assume that the underlying model to be half-normal, which satisfies condition (9). Then plugging (10) into (14), we have

$$h = 1.00657\,\sigma\, n^{-1/9}, \tag{15}$$

where σ can be estimated by using the maximum likelihood estimator $\hat{\sigma}$ given by $\hat{\sigma} = \sqrt{\sum_{i=1}^{n} x_i^2 / n}$. The simulation results given in Section 6 are obtained based on formula (15).

5. A Modified Estimator

Equation (8) implies that the fourth-order kernel achieves a $O(h^4)$ bias if condition (9) is true. If $f'(0) \neq 0$, then $\hat{f}(0)$ achieves only a $O(h)$ bias, which is significantly greater than $O(h^4)$. As stated earlier, despite of the logical shoulder condition assumption in line transect sampling, in practice in some cases, the validity of this condition is doubtful. Accordingly, assume that we are not sure whether the shoulder condition is valid or not. On this basis, a modified estimator for $f(0)$ is introduced in this section. The proposed estimator $(\text{say } \hat{f}^*(0))$ is considered as a semiparametric estimator, which combines the negative exponential model which does not satisfy the shoulder condition at the origin and the fourth-order kernel model. While the fourth-order kernel estimator performed generally well in line transect sampling as the simulation results indicated, some improvements can be obtained by using the semiparametric estimator, especially when the shoulder condition fails to remain valid. Gates et al. [11] suggested the negative exponential model to fit the perpendicular distances in line transect sampling. The model does not have the shoulder condition, which is given by

$$f(x, \lambda) = \frac{1}{\lambda} e^{-x/\lambda}, \quad x \geq 0.$$

In this setting, we estimate the parameter $f(0, \lambda) = 1/\lambda$ by using the maximum likelihood method which gives $f(0, \hat{\lambda}) = 1/\bar{x}$, where $\bar{x}$ represents the mean of the observed perpendicular distances $x_1, x_2, \ldots, x_n$. Thus, the semiparametric estimator in this case is of the form

$$\hat{f}^*(0) = (1 - m) f(0, \hat{\lambda}) + m\hat{f}(0).$$

The parameter m $(0 \leq m \leq 1)$ is estimated from the data and its estimate $\hat{m}$ is then used in $\hat{f}^*(0)$ as the proposed estimate for $f(0)$. What is less clear in the above semiparametric estimator is how m should be chosen in the estimator $\hat{f}^*(0)$. The main idea here is that we need to force $\hat{m}$ to be close to unity when the shoulder condition for the underlying model of the data at hand holds and to be far from unity towards zero when the shoulder condition fails to hold. In other words, a good $\hat{f}^*(0)$ is expected to give high weight for the fourth-order kernel estimator when the shoulder condition holds and less weight when it does not.

Zhang [17] proposed a procedure for testing the shoulder condition of a model based on line transect sampling. Assume that a random sample $x_1, x_2, ..., x_n$ of perpendicular distances is drawn from a distribution with probability density function $f(x)$. Consider the test $H_0 : f'(0) = 0$ vs. $H_1 : f'(0) \neq 0$, according to Zhang [17], we reject H_0 for large value of $Z = \frac{\sqrt{\sum_{i=1}^{n} x_i^2}}{\sum_{i=1}^{n} x_i}$. Zhang constructed a table of critical values of the sampling distribution for Z with respect to different sample sizes by Monte Carlo simulation. The idea to choose the weighted parameter m is based on the test statistics Z. The value of Z (which always lies between zero and one) indicates how strongly H_0 is supported by the data. A large value of $1 - Z$ leads us to accept the shoulder condition which indicates in some sense that $f(0)$ is close to the fourth-order kernel estimator $\hat{f}(0)$. Thus, we can use this $1 - Z$ value to estimate the parameter m. Thus, the proposed semiparametric estimator is

$$\hat{f}^*(0) = zf(0, \hat{\lambda}) + (1 - z)\hat{f}(0). \tag{16}$$

6. Simulation Study

Because the exact behavior of the proposed semiparametric estimator $\hat{f}^*(0)$ is complex, we chose to study the sample properties of $\hat{f}^*(0)$ in addition to the first estimator $\hat{f}(0)$ through simulation techniques. The proposed estimators were compared with the nonparametric traditional kernel estimator $\hat{f}_k(0)$; the smoothing parameter h is computed by using the formula $h = 1.06\hat{\sigma}n^{-1/5}$ (Chen [7]). The Buckland [3] semiparametric estimator $\hat{f}_H(0)$ based on a key half-normal model with Hermite polynomial correction and the semiparametric estimator $\hat{f}_L(0)$ proposed by Barabesi [1] based on the local likelihood approach are also considered. Our simulation design is similar to that of Barabesi [1], in which three families of models which are commonly used as references in line transect studies were considered in the simulation. The exponential power (EP) family (Pollock [15])

$$f(x) = \frac{1}{\Gamma(1 + 1/\beta)} e^{-x^{\beta}}, \quad x \geq 0, \quad \beta \geq 1.$$

The hazard-rate (HR) family (Hayes and Buckland [12])

$$f(x) = \frac{1}{\Gamma(1 - 1/\beta)}\left(1 - e^{-x^{-\beta}}\right), \quad x \geq 0, \quad \beta > 1$$

and the beta (BE) model (Eberhardt [8])

$$f(x) = (1+\beta)(1-x)^{\beta}, \quad x \geq 0, \quad \beta \geq 0.$$

In our simulation design, these three families were truncated at some distance w which required in computing of $\hat{f}_H(0)$. Four models were selected from the EP family with parameter values $\beta = 1.0, 1.5, 2.0, 2.5$ and corresponding truncation points given by $w = 5.0, 3.0, 2.5, 2.0$. Four models were selected from the HR family with parameter values $\beta = 1.5, 2.0, 2.5, 3.0$ and corresponding truncation points given by $w = 20, 12, 8, 6$. Moreover, four models were selected from the BE model with parameter values $\beta = 1.5, 2.0, 2.5, 3.0$ and $w = 1$ for all the cases. The considered models cover a wide range of perpendicular distance probability density functions which vary near zero from spike to flat. It should be remarked that the EP model with $\beta = 1$ and the BE model do not satisfy the shoulder condition. This choice was made in order to assess the robustness of the considered estimators with respect to the shoulder condition.

For each model and for sample sizes $n = 50, 100, 200$, one thousand runs are iterated. For each model and for each sample size, Table 1 reports the simulated value of the relative bias (RB)

$$\text{RB} = \frac{E(\hat{f}(0)) - f(0)}{f(0)},$$

and the relative mean error (RME)

$$\text{RME} = \frac{\sqrt{\text{MSE}(\hat{f}(0))}}{f(0)},$$

for each considered estimator.

7. Results and Conclusion

Depending on the simulation results given in Table 1, we conclude in summary the following:

(1) The local likelihood estimator $\hat{f}_L(0)$ of Barabesi [1] has very small RBs for each sample size and model, even if it is not very accurate, generally showing large RMEs. Indeed, it turns out to be the best estimator only for the EP model with

$\beta = 1.0$ and the HR model with $\beta = 1.5$, that is, for the most spiked model (the EP model with $\beta = 1.0$) and for the model that markedly falls as distance increases (the HR with $\beta = 1.5$), in which the two cases are barely suitable for line transect data.

(2) The Hermite polynomial correction estimator $\hat{f}_H(0)$ of Buckland [3] generally produces rather small RBs but it is not the best among the other estimators for any model with exception case when the model is the BE with $\beta = 1.5$ and $n = 200$.

(3) The traditional kernel estimator $\hat{f}_H(0)$ of Chen [7] which can be viewed as a second-order kernel estimator is with large $|\text{RB}|$s for the EP model with $\beta =$ $1.0, 1.5$; for the HR model with $\beta = 1.5, 2.0$ and for the BE model with different values of β. However, its performance is quite well for the EP and HR models when the shape parameter increases, which increases the smoothness of the underlying model near $x = 0$. The estimator turns out to be the best estimator for the HR model with $\beta = 3$ and $n = 100, 200$ and for the NE model with $\beta = 2.5$. In the last case, its performance is similar to that of the fourth-order kernel estimator $\hat{f}(0)$.

(4) The proposed estimator $\hat{f}(0)$ generally produces rather small RBs. The values of RBs are generally reduced when the parametric negative exponential estimator is introduced to form the semiparametric estimator $\hat{f}^*(0)$, especially when the model of the simulated data does not satisfy the shoulder condition (i.e., the NE model with $\beta = 1.0$ and the BE model). On the other hand, comparing the $|\text{RB}|$s of $\hat{f}_k(0)$ with that of $\hat{f}(0)$, the simulation results demonstrate clearly that the $|\text{RB}|$s of $\hat{f}(0)$ is smaller than the $|\text{RB}|$s of $\hat{f}_k(0)$ which coincides with our discussion in Section 3.

(5) Regarding to RME, the estimator $\hat{f}(0)$ turns out to be the best estimator for the NE model with $\beta = 2.0, 2.5$ and for the HR model with $\beta = 2.5, 3.0$ in which the shoulder condition is valid for these models. On the other hand, some improvements are obtained when we use the semiparametric $\hat{f}^*(0)$ instead of $\hat{f}(0)$ especially for the BE model (in which the shoulder condition is not true) and for the NE model when the shape parameter decreases, which decreases the smoothness of

the model near $x = 0$. Moreover, the performance of $\hat{f}^*(0)$ is better than $\hat{f}(0)$ for the HR model in the case that the model sharply falls as distance far away from $x = 0$ (i.e., when $\beta = 1.5$, 2.0).

Finally, we have seen that fourth-order kernel approach is a useful nonparametric tool for analyzing line transect data. The method generally generates small RBs and RMEs. Some improvements over the fourth-order kernel estimator can be obtained by applying the semiparametric estimator in the absence of any information about the validity of the shoulder condition. Comparing the traditional kernel method with the fourth-order kernel method, the latest performed better than the traditional kernel method theoretically and numerically. Accordingly, the fourth-order kernel method is recommend in line transect sampling.

8. Numerical Example

We apply the proposed estimator to the classical wooden stakes data set, given in Burnham et al. [6, p. 61]. The data are collected from line transect survey to estimate the density of stakes in a given area. The stakes data are the perpendicular distances (in meters) of detected a stake to the transect line, in which 150 stakes were placed at random in an area of 1000 meters long. Out of 150 stakes, 68 stakes are detected using line transect technique. The true form of $f(x)$ is unknown, but the true value of $f(0)$ is known which equals $f(0) = 0.110294$, thus the actual density of stakes was 37.5 stakes/ha. Calculation shows that $Z = 0.1624$, the empirical critical value for $\alpha = 0.05$ and $n = 68$ is 0.1605 (Zhang [17]), so the shoulder condition is rejected. In this case, the estimator which given by (16) should be used. Calculation shows that $\hat{\lambda} = 6.115$, $\hat{\sigma} = 8.190$, $f(0, \hat{\lambda}) = 0.163$, $h = 5.158$ and $\hat{f}(0) = 0.10639$. Accordingly, the semiparametric estimator of $f(0)$ is $\hat{f}^*(0) = 0.11567$ and the corresponding estimator for the density of stakes is $\hat{D}^* = 39.38$ stakes/ha. Burnham et al. [6] analyzed the same data set by using a cosine series estimator, and they obtain an estimate for $f(0)$ given by 0.1148 with corresponding density estimate $\hat{D} = 39.00$ stakes/ha. It should be remarked that the cosine series estimator employs an exact value for the maximum perpendicular distance (take to be 20 meters for this example), that is, more information is used in this case.

Table 1. RME and RB (in parentheses) for the different five estimators of $f(0)$

Exponential Power Model	n	$\hat{f}_L(0)$	$\hat{f}_H(0)$	$\hat{f}_k(0)$	$\hat{f}(0)$	$\hat{f}^*(0)$
β = 1, w = 5	50	0.24 (–015)	0.31 (–0.28)	0.37 (–0.35)	0.31 (–0.29)	0.26 (–0.23)
	100	0.19 (–0.14)	0.29 (–0.27)	0.33 (–0.32)	0.29 (–0.28)	0.25 (–0.23)
	200	0.17 (–0.13)	0.26 (–0.24)	0.30 (–0.29)	0.27 (–0.26)	0.24 (–0.23)
β = 1.5, w = 3	50	0.19 (–0.04)	0.16 (–0.08)	0.21 (–0.17)	0.16 (–0.11)	0.13 (–0.03)
	100	0.15 (–0.03)	0.13 (–0.08)	0.17 (–0.15)	0.13 (–0.10)	0.10 (–0.04)
	200	0.11 (–0.03)	0.11 (–0.08)	0.14 (–0.13)	0.11 (–0.09)	0.08 (–0.05)
β = 2, w = 2.5	50	0.20 (0.01)	0.19 (0.01)	0.16 (–0.09)	0.14 (–0.03)	0.16 (–0.07)
	100	0.15 (0.01)	0.15 (0.02)	0.12 (–0.07)	0.11 (–0.03)	0.12 (0.05)
	200	0.11 (0.00)	0.11 (0.02)	0.10 (–0.06)	0.08 (–0.02)	0.08 (0.03)
β = 2.5, w = 2	50	0.22 (0.03)	0.24 (0.06)	0.15 (–0.05)	0.15 (0.01)	0.19 (0.12)
	100	0.17 (0.02)	0.19 (0.06)	0.11 (–0.04)	0.11 (0.00)	0.14 (0.09)
	200	0.12 (0.01)	0.17 (0.06)	0.08 (–0.03)	0.08 (0.00)	0.10 (0.06)
Hazard Rate Model						
β = 1.5, w = 20	50	0.22 (–0.05)	0.39 (–0.37)	0.43 (–0.42)	0.37 (–0.35)	0.33 (–0.30)
	100	0.16 (–0.05)	0.38 (–0.36)	0.39 (–0.38)	0.35 (–0.33)	0.31 (–0.30)
	200	0.13 (–0.03)	0.38 (–0.36)	0.33 (–0.33)	0.32 (–0.31)	0.30 (–0.29)
β = 2, w = 12	50	0.20 (0.02)	0.21 (-0.10)	0.30 (–0.27)	0.23 (–0.19)	0.19 (–0.12)
	100	0.15 (0.03)	0.15 (–0.08)	0.24 (–0.22)	0.19 (–0.17)	0.16 (–0.12)
	200	0.11 (0.03)	0.14 (–0.08)	0.19 (–0.18)	0.16 (–0.14)	0.13 (–0.11)
β = 2.5, w = 8	50	0.20 (0.05)	0.19 (0.08)	0.18 (–0.14)	0.14 (–0.05)	0.15 (0.04)
	100	0.16 (0.04)	0.15 (0.07)	0.13 (–0.10)	0.10 (–0.02)	0.11 (0.04)
	200	0.11 (0.03)	0.11 (0.07)	0.09 (–0.07)	0.07 (–0.01)	0.08 (0.03)
β = 3, w = 6	50	0.23 (0.05)	0.20 (0.10)	0.14 (–0.06)	0.13 (0.02)	0.18 (0.11)
	100	0.17 (0.03)	0.17 (0.11)	0.10 (–0.04)	0.10 (0.03)	0.14 (0.10)
	200	0.13 (0.02)	0.16 (0.11)	0.07 (–0.02)	0.08 (0.04)	0.11 (0.09)
Beta Model						
β = 1.5, w = 1	50	0.21 (–0.03)	0.16 (–0.03)	0.22 (–0.18)	0.18 (–0.13)	0.14 (–0.04)
	100	0.15 (–0.03)	0.12 (–0.04)	0.19 (–0.17)	0.16 (–0.13)	0.11 (–0.06)
	200	0.11 (–0.02)	0.09 (–0.03)	0.16 (–0.15)	0.14 (–0.12)	0.10 (–0.07)
β = 2.0, w = 1	50	0.20 (–0.06)	0.15 (–0.08)	0.24 (–0.22)	0.20 (–0.15)	0.15 (–0.07)
	100	0.15 (–0.06)	0.12 (–0.08)	0.21 (–0.20)	0.18 (–0.15)	0.13 (–0.09)
	200	0.11 (–0.06)	0.11 (–0.08)	0.19 (–0.17)	0.15 (–0.14)	0.12 (–0.09)
β = 2.5, w = 1	50	0.22 (–0.06)	0.18 (–0.13)	0.26 (–0.23)	0.21 (–0.17)	0.16 (–0.09)
	100	0.15 (–0.07)	0.15 (–0.12)	0.23 (–0.21)	0.19 (–0.17)	0.15 (–0.11)
	200	0.12 (–0.07)	0.14 (–0.13)	0.20 (–0.18)	0.17 (–0.15)	0.13 (–0.11)
β = 3.0, w = 1	50	0.21 (–0.08)	0.19 (–0.15)	0.27 (–0.25)	0.22 (–0.18)	0.16 (–0.10)
	100	0.16 (–0.08)	0.18 (–0.16)	0.24 (–0.23)	0.20 (–0.18)	0.15 (–0.12)
	200	0.13 (–0.08)	0.17 (–0.16)	0.21 (–0.20)	0.18 (–0.17)	0.14 (–0.13)

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