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AN ASSESSMENT OF THE DFI ON DWD-HRM

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Abstract

An optimal Digital Filter Initialization (DFI) technique is used in the HRM (High Resolution Model) in order to suppress spurious high frequency oscillations caused by an imbalance between initial mass and wind fields. The filter is calibrated for a large Brazilian domain and its efficiency on removing high frequency oscillations is analyzed. The DFI is compared to the nonlinear Normal-Mode Initialization (NMI) also

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implemented on HRM. Both methods yield to satisfactory results in suppressing noise in the forecast, but the DFI appears to be more efficient.

1. Introduction

Primitive equations models produce spurious high frequency oscillations, caused by gravity waves components of large amplitude, noticeable mainly during the first hours of a numerical integration. This large amplitude wave results from imbalance between the mass and wind fields in the initial condition of numerical models without previous adjustment. This imbalance is due to minor errors on observation, and imperfections of the model, such as difficulty for cut-off errors in spectral models. Schemes devised to avoid imbalances are termed initialization. A detailed review of such concepts is described by Huang and Lynch [1].

Some initialization techniques have been applied to numerical models in order to remove this noise of the forecast, such as Dynamic Initialization [2], and Laplace Transform Initialization [3]. Indeed, the most successful technique for initialize global models is based on normal modes of model. Independently, Machenhauer [4] developed the Nonlinear Normal Mode Initialization (NNMI). In this method, the slow rotational components or Rossby modes are separated of the high frequency gravity modes by process of spectral analysis. In order to avoid this high frequency grow during the integrations period, the gravity wave initial rate-of-change is set zero. However, the application of NMI for limited area models is not straightforward due to the mathematical complexities to obtain the normal modes in the artificial boundaries, although the Deutscher Wetterdienst, commonly abbreviated as DWD (translated from German as German Meteorological Service), has done this task in a very adequate interface to the global model [5].

An attractive approach to filter spurious high frequency gravity waves, termed Digital Filter Initialization (DFI), was introduced by Lynch and Huang [6]. The mathematical formulation of the method consists on obtaining a filtered value from a linear combination of a given sequence. Thus, the output value substitutes the central value in the sequence. In Huang and Lynch [1], the filter is applied to time series of variables generated by short range adiabatic forward and adiabatic backward integrations from the initial time. Both integrations are combined as for generating a balanced initial condition. In Lynch [3], the filter is applied to time series generated by adiabatic backward and diabatic forward integrations of the model. Lynch [7] described an optimal filter based on Dolph-Chebyshev window.

This study exploits a DFI with Dolph-Chebyshev window, implemented by DWD in the HRM (High Resolution Model) for the South America domain. The most important goals in this research are: (i) show that the initialization procedure is able to avoid unbalance in initial data; (ii) choose the best setting for applying the digital filter (span, cut-off frequency and strategy of integration of the filter, diabatic or adiabatic).

Section 2 presents the theoretical basis and formulation of the digital filter with Dolph-Chebyshev window following the derivation of Lynch and Huang [6] and Lynch [7]. Section 3 presents an overview about the HRM. Section 4 examines the optimization of the filter and its effects on HRM. A summary and additional discussion are presented in Section 5.

2. Theoretical Framework

The method applied in this research arises from digital signal processing theory, so that a deep discussion about digital filter could be found in Oppenheim and Schafer [8]. An overview of the theory to initialization problem, as applied in this work, is as follows.

a. The digital filter

The formulation of the DFI (Digital Filter Initialization) can be found in several researches (e.g., Huang and Lynch [1]; Fillion et al. [9]; Lynch [7]). Here the DFI is introduced such as in Lynch and Huang [6] and Lynch [7].

(1) The low-pass filter

Consider a sequence $\{f_n\}$ and its Fourier transform

$$f_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega)e^{i\omega n}d\omega, \tag{2.1-a}$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} f_n e^{-i\omega n}. \tag{2.1-b}$$

The relation (2.1-a) follows from multiplying (2.1-b) by $e^{i\omega m}$, integrating from $\omega = -\pi$ to $\omega = \pi$, and using the orthogonality relation

$$\int_{-\pi}^{\pi} e^{i(m-n)\omega}d\omega = \begin{cases} 2\pi, & \text{if } m = n, \\ 0, & \text{otherwise.} \end{cases} \tag{2.2}$$

Defining a function of the frequency ω by

$$H(\omega) = \begin{cases} 1, & \text{if } |\,\omega\,| \le |\,\omega_c\,|, \\ 0, & \text{otherwise,} \end{cases} \tag{2.3}$$

where ω_c is the cut-off frequency, the product

$$F(\omega)H(\omega) \tag{2.4}$$

retains only frequencies greater than ω_c. The filtered sequence $\{f_n^*\}$ is constituted by elements in the form:

$$f_n^* = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega)H(\omega)e^{\text{int}}d\omega. \tag{2.5}$$

(2) The convolution of the filtered sequence

The use of the convolution theorem allows to obtain the filtered sequence without the necessity of computing the integral (2.5). Given two sequences $\{f_n\}$ and $\{h_n\}$, the convolution is defined by

$$(h * f)(n) = \sum_{k=-\infty}^{\infty} h_k f_{(n-k)}$$

and the theorem of convolution establishes that

$$H(\omega)F(\omega) = \sum_{k=-\infty}^{\infty} (h * f)(k)e^{-i\omega k}.$$

Then, the filtered sequence given by (2.5) has the form

$$f_n^* = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sum_{k=-\infty}^{\infty} (h * f)(k)e^{-i\omega k}\right]e^{i\omega n}d\omega$$

and with the orthogonality relation (2.2), one obtains

$$f_n^* = (h * f)(n). \tag{2.6}$$

(3) The coefficients of the low-pass filter

The sequence $\{h_n\}$ with its Fourier transform given by (2.3) is

$$h_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{i\omega n}d\omega$$

or

$$h_n = \frac{1}{2\pi}\int_{-\omega_c}^{+\omega_c} e^{i\omega n}d\omega = \frac{1}{2\pi in}[e^{i\omega_c n} - e^{-i\omega_c n}]$$

$$= \frac{sen(n\omega_c)}{n\pi}.$$

Therefore, according to (2.6), one element of the filtered sequence $\{f_n^*\}$ is

$$f_n^* = \sum_{k=-\infty}^{\infty} \frac{sen(k\omega_c)}{k\pi} f_{n-k}.$$

In practical problems, the digital filter is applied to a finite sequence. This introduces the Gibbs oscillations near the cut-off frequency ω_c. Several windows have been proposed to solve this problem. The new filtered sequence with $2N + 1$ elements is given by

$$f_n^* = \sum_{k=-N}^{k=+N} \frac{sen(k\omega_c)}{k\pi} f_{(n-k)}\sigma(k, N), \tag{2.7}$$

where $\sigma(N, k)$ is the controlling factor of the Gibbs phenomenon. Some common windows are the Hamming [10]. Alternatively, the window may be used directly as a low-pass filter. The code implemented in the HRM by DWD is based on Lynch [7], where the filter is a direct convolution of model variables and Dolphy-Chebyshev window, introduced below.

(4) The Chebyshev polynomials

Defining a function of the frequency ω by

$$H(\omega) = \frac{T_{2M}(x_0\cos(\omega/2))}{T_{2M}(x_0)},$$

where $x_0 > 1$. The stop-band or cut-off frequency is ω_s such that $x_0\cos(\omega_s/2) = 1$. As ω varies from 0 to ω_s, $H(\omega)$ falls from 1 to $r = 1/T_{2M}(x_0)$. For $\omega_s \le \omega \le \pi$,

$H(\omega) \in [-r, +r]$. Under this formulation $H(\omega)$ is a low-pass filter, with a cut-off at $\omega = \omega_s$. T_{2M} comes from Chebyshev polynomials, defined by the equations

$$T_n(x) = \begin{cases} \cos(n \cos^{-1} x), & \text{if } |x| \le 1, \\ \cosh(n \cosh^{-1} x), & \text{if } |x| > 1. \end{cases}$$

From trigonometric relation $T_0(x) = 1$, $T_1(x) = x$, and $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$, if $n \ge 2$. By definition of Chebyshev polynomials, the low-pass filter $H(\omega)$ can be written as a finite expansion

$$H(\omega) = \sum_{n=-M}^{M} h_n e^{(-in\omega)}$$

and the filter coefficients h_n are obtained from the inverse Fourier transform

$$h_n = \frac{1}{N}\left[1 + 2r\sum_{m=1}^{M} T_{2M}\left(x_0 \cos\frac{\omega_m}{2}\right)\cos m\omega_n\right],$$

where $|n| \le M$, the filter order is $N = 2M + 1$ and $\omega_m = 2\pi m/N$. Since $H(\omega)$ is real and even, h_n are also real so that $h_{-n} = h_n$. In fact, this property reduces the computation cost of the filter. The parameters x_0 and r are given by $\frac{1}{x_0} = \cos\left(\frac{\omega_s}{2}\right)$ and $r = 1/T_{2M}(x_0)$, since $T_K = \frac{1}{2}[(x + \sqrt{x^2 - 1})^K + (x - \sqrt{x^2 - 1})^K]$.

3. Forecast Model

In this section, an overview of the numerical model used for performing the tests with DFI scheme is shown. The DWD High Resolution Model (Heise and Schrodin [11]), hereafter denoted HRM, is a hydrostatic atmospheric model of primitive equations solved in an Arakawa C grid formulation. The numerical model is integrated using 40 layers vertically in sigma-pressure coordinate, horizontal domain of 301 by 301 grid points in the east and north directions, in Mercator projection. The grid distance on the map is 25 km true at 30° of latitude. The boundary-layer processes are evaluated using the level 2 closure according to Mellor and Yamada [12] and the mass flux convection scheme, as formulated by Tiedtke [13], for deep, shallow and mid-level convection types. The soil model was formulated as seven-stratified layers, including snow and interception storage [11]. The precipitation process considers the grid scale scheme including parameterized cloud microphysics (Doms and Schättler [5]). The sea surface temperature, fixes during the integration, is obtained from monthly climatic data.

4. Results

The DFI can be applied to time series of the model variables generated by short-range just forward integration (Adiabatic Digital Filter Initialization - ADFI) from the initial time or by backward and forward integration (Diabatic Digital Filter Initialization - DDFI) from the initial time. Huang and Lynch [1] have shown that the last one is more efficient for removing high frequency oscillation, otherwise more expensive. The efficiency of the filter depends on adjustments in cut-off frequency set on the code. Thereby some experiments are performed in order to check the most appropriated strategy to apply the filter as well as the best cut-off frequency.

A common way to demonstrate the performance of an initialization scheme is to show the time evolution of the surface pressure at a model grid point, because it is sensitive to noise in a vertically integrated sense (Williamson and Temperton [14]).

Figure 1 brings the time evolution of the Surface Pressure Tendency (SPT) at Santiago (capital of Chile), localized in Andes mountain, source of fast gravity waves, due to the topography in the region. The NOI graph shows a severe initial shock resulting from the unbalance in the initial data. There is a damping due to the diffusivity of the numerical method in few time steps, but it oscillates noisily during the approximately first 2 hours of forecast reaching an asymptotic value around 1 hPa/s. Both NMI, ADFI and DDFI remove spurious high frequency of forecast. Furthermore, the NMI is more efficient than NOI but less than digital filters for reducing the initial value of SPT. Comparing ADFI to DDFI, the results suggest that including diabatic process in the initialization (DDFI) produces an initial condition field better balanced than corresponding adiabatic initialization.

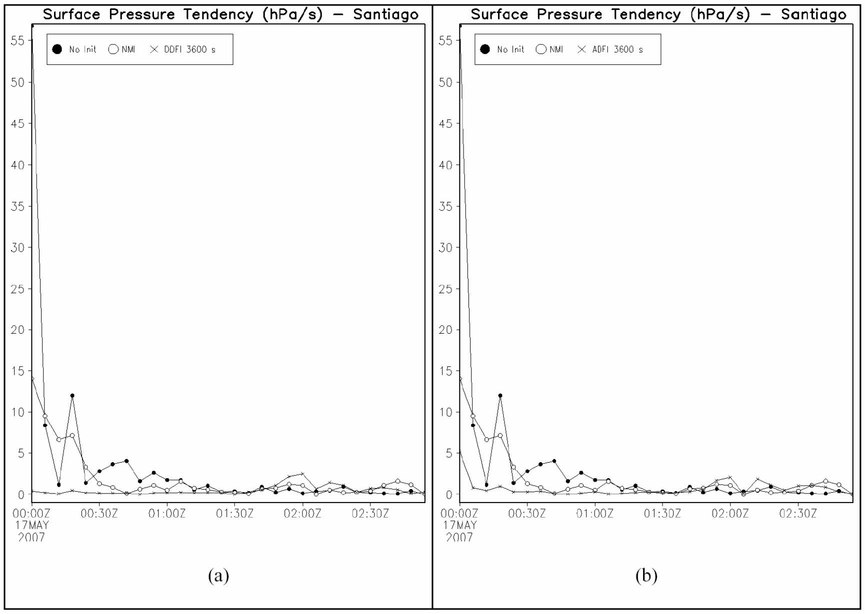


Figure 1. Time evolution of the Surface Pressure Tendency at Santiago: (a) no-initialized, Normal Mode Initialization and Diabatic Digital Filter Initialization; (b) no-initialized, Normal Mode Initialization and Adiabatic Digital Filter Initialization.

Figure 2 shows the DDFI, hereafter termed, DFI to different cut-off frequencies. The goal is to find the filter that most reduces the spurious high frequency and avoid large changes in the initial field (see Figure 3). Until $\tau_S = 3600\text{s},$ just after the reduction of the cut-off frequency (or when the cut-off period is enlarged) more noise is eliminated of the forecast. Cut-off period bigger than 3600s does not seem to make effect in the noise.

The above mentioned tests carry out that the digital filter including the diabatic process, span and cut-off period of 3600s are the best settings for the filter. By this reason, the experiments hereafter are carried on considering such results.

According to Lynch and Huang [6], any satisfactory initialization scheme has 3 essential characteristics: high frequency noise is effectively removed from the forecast (it was shown in the experiment above); changes made to the analyzed fields are acceptably small and the forecast is not degraded by application of the initialization. In order to carry on the tests for verifying those characteristics, some experiments are performed in the next sections with cut-off period and window span of 3600s for DFI and NMI.

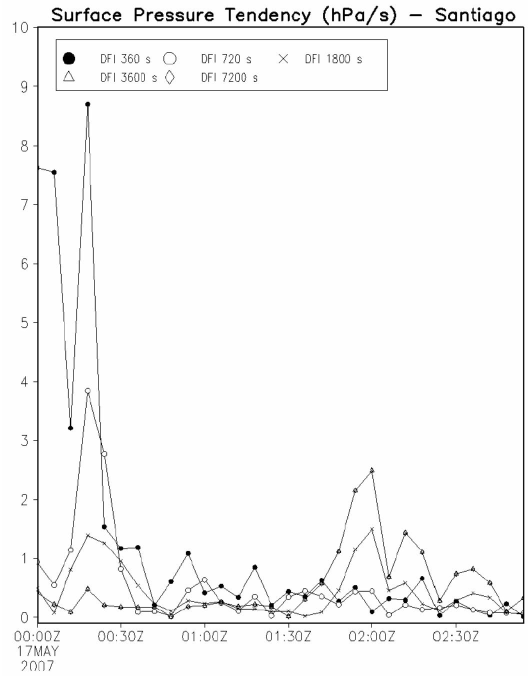


Figure 2. Digital Filter Initialization to different cut-off periods.

4.1. Changes in the analysis

Initialization scheme should imply small changes in the analysis fields. In general, these changes should be similar in size or smaller than the expected analysis error to ensure that the analysis is not degraded by the initialization process (Lynch and Huang [6]). To verify the changes in the initial fields, the analyses created by NOI, NMI and DFI are depicted in Figure 3. In Figure 4, the differences: NMI-NOI and DFI-NOI are shown.

In Figure 3, it is shown that both initialization procedures practically do not change the analysis, as required by a satisfactory initialization method according to Lynch and Huang [6].

The small changes in the analysis are better shown in Figure 4, where the differences between the NOI analysis and analysis produced by NMI and DFI are depicted. The largest differences are produced in area with the highest mountain, such as Andes. This is an expected result, agreement with the fact that mountains are source of fast gravity waves.

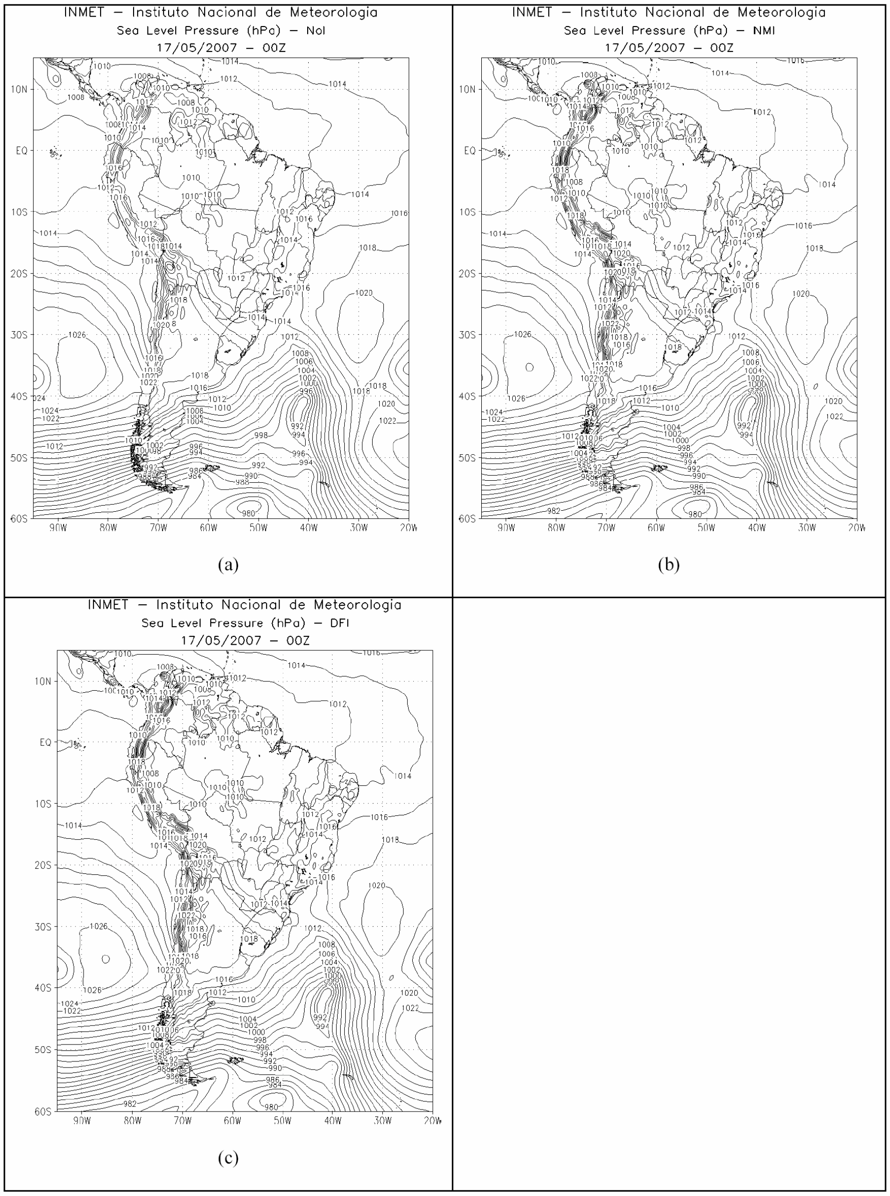


Figure 3. Analysis for (a) NOI, (b) NMI and (c) DFI.

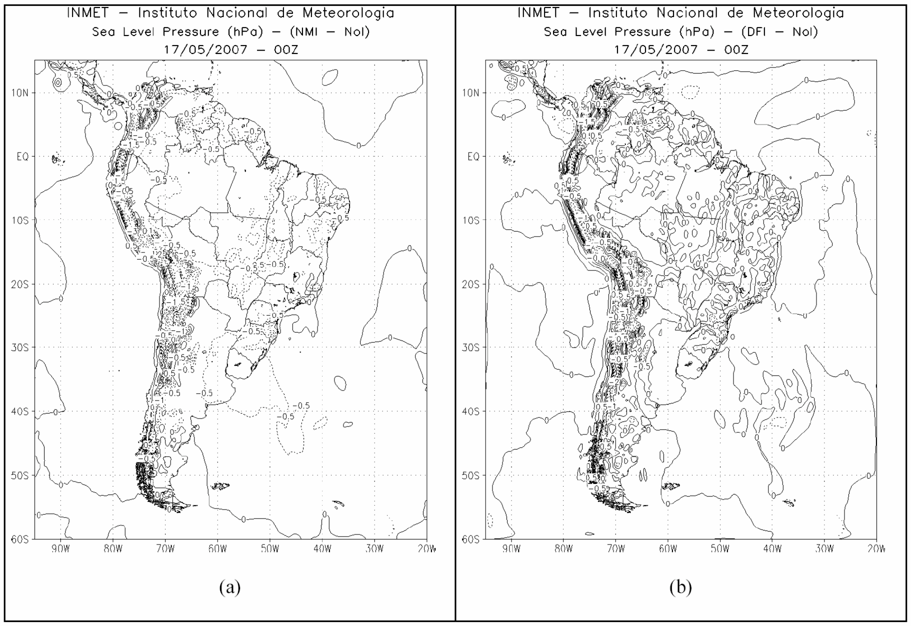


Figure 4. Differences in the analysis due to (a) NMI and (b) DFI.

4.2. Changes in the forecasts

Due to small changes in the initial fields, it is required that a forecast from initialized data should resemble the no initialized forecast. This characteristic is depicted in Figure 5, for 24 hours forecasting for NOI, NMI and DFI.

After approximately 3 hours of forecasting, the numerical method used in the integration of the Navier-Stokes equations smooth out the noise generated by fast waves by diffusivity process, so that the differences in the 24 hours of forecasting must be very small or it does not exist comparing no initialized to initialized fields. This characteristic is double checked in Figures 6(a) and (b).

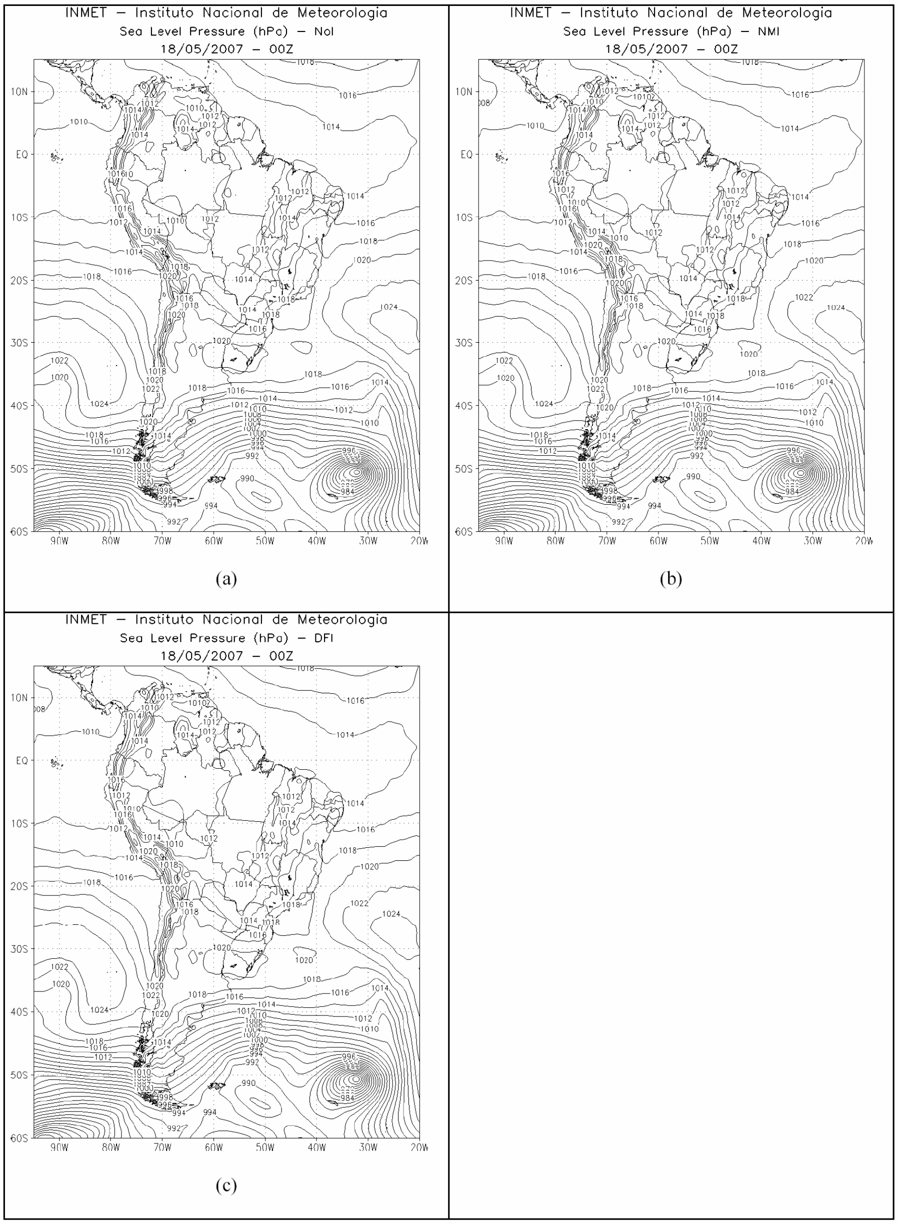
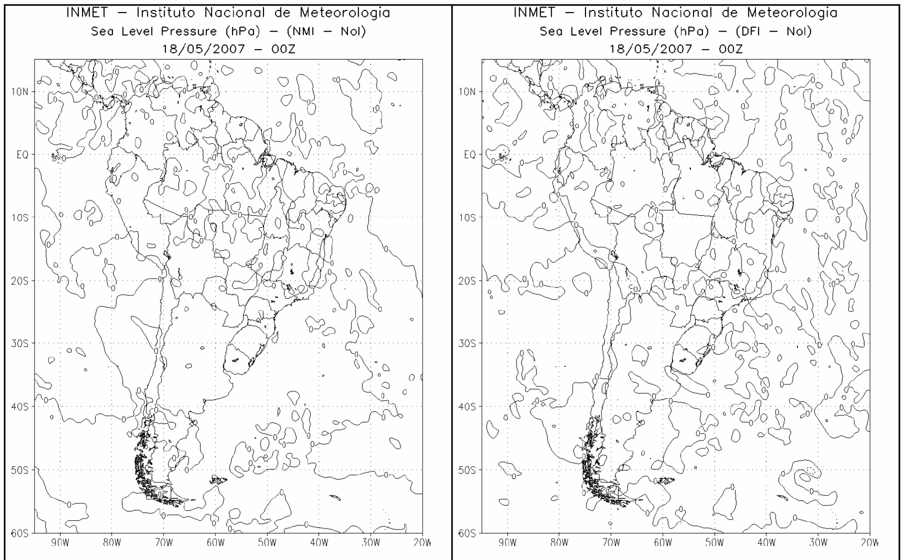


Figure 5. One day forecasting for (a) NOI, (b) NMI and (c) DFI.



(a) (b)

Figure 6. Differences in the 24 hours forecast due to (a) NMI and (b) DFI.

5. Summary and Conclusions

In this work, a low-pass non-recursive filter is applied to initialize the HRM model. According to Lynch and Huang [6], this technique has some advantages when compared to available alternatives, such as: no need to compute or store normal modes; no need to separate vertical modes; complete compatibility with model discretisation; exotic coordinates and distorted grids cause no problems; no iterative numerical procedure which may diverge; ease of implementation and maintenance, due to the simplicity of scheme; applicable to all prognostic model variables; applicable to non-hydrostatic models.

The experiments performed in this paper shown that the DFI scheme applied to HRM model is a satisfactory initialization scheme, as it agrees with the three characteristics pointed out by Lynch and Huang [6] essential for good initialization scheme: (1) high frequency noise is effectively removed from the forecast; (2) changes made to the analyzed fields are acceptably small and; (3) the forecast is not degraded by application of the initialization.

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