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MODELING OF OPTIMAL ELECTRICITY SUPPLY PROBLEM: IMPROVEMENT OF POWER NETWORK USING HIERARCHICAL PLANNING APPROACH

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Abstract

In real power systems, power plants are not in the equal space from the load center, and their fuel cost is different. With common utilization conditions, production capacity is more than total load demand and losses. Therefore, there are different criteria for active and inactive power planning in each power plant. The best selection is to choose a framework in which the utility cost is minimized. On the other hand, planning in power systems has different time horizons, thus, for effective planning in power systems, it is very important to find a suitable mathematical relationship between them. In this paper we propose a modeling by selecting a Fuzzy Hierarchical Production Planning (FHPP) technique with zone covering in the mid-term and long-term time horizons electricity supply modeling in the Iran global compact network with spotting 15 zone electricity study, in terms of inclusive capacity, Max development, Max energy product of each production unit, reliability and autonomy constraints. Other objective functions include parameters that minimize production, development and security costs of the system, capital recovery factor, and interest rate that maximize the total preference weights of power plants.

Keywords and phrases: power plants generation planning (PPGP), hierarchical planning; long-term electricity planning, mid-term electricity planning.

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1. Introduction

Electricity production planning which is called *generation planning* in power systems is divided into: long-term, mid-term and short-term planning [1]. Planning and operating modern electric power systems involve several interlinked and complex tasks. Optimizing a production plan, however, is difficult for thermal and hydro power plants, which could be solved with proper computer tools.

Long-term energy generation planning is of key importance to the operation of electricity generation. It is employed for strategic planning, budgeting, and fuel acquisitions and to provide a framework for short-term energy generation planning.

A long-term planning period (one year) is usually subdivided into shorter intervals of weeks or months, for which parameters like load–duration curve should be predicted, and variables like expected energy generations for each plant unit must be optimized. The Load-Duration Curves (LDC's) predicted for each interval are used as input data, which are equivalent to load-survival functions. This is appropriate since load uncertainty can be suitably described using the LDC. It is assumed that the probability of failure for each thermal unit is known.

In power system management, the problem of planning production for the next 10–30 days is known as the mid-term planning problem. Production planning problems with up to one week time horizon is known as short-term planning.

The short-term and mid-term planning problems could be principally considered alike, except in some specific conditions, when the problems are more or less relevant to the variety of time horizons. Since uncertainty exists in prediction of electricity demand as well as electricity price, the prediction of the mid-term problem can become difficult. On the other hand, the short-term model can be detailed due to the relatively good predictions that can be derived for the next few days. This high level of detail implies that in practice a short-term model, can only implement one district heating system at a time. Another purpose of the mid-term model is the model restrictions that connect the different systems. For example in principal planning procedure, the outputs of solved mid-term problems are used as the inputs to the short-term problems.

Mathematical algorithms have been used over the years for optimizing many power systems planning, operation, and control problems. Mathematical formulations of real-world problems are derived under certain assumptions and even with these assumptions; solution of large-scale power systems is not simple. Furthermore, there are many uncertainties in power system problems because power systems are large, complex, and geographically widely distributed.

An optimization problem is a mathematical model where the main objective is to minimize undesirable components like cost, energy loss, errors, etc., or to maximize desirable outputs like profit, quality, efficiency, etc., that are subject to some constraints. The main advantages of algorithmic methods include:

1. Optimality is mathematically rigorous in some algorithms.

2. Problems can be formulated to take advantage of the existing sparsity techniques applicable to large-scale power systems.

3. There are a wide range of mature mathematical programming technologies, such as Linear Programming (LP), Quadratic Programming (QP), Nonlinear Programming (NLP), integer and mixed integer programming, and Dynamic Programming (DP), etc [2].

In spite of this advantage, in most of the mathematical optimization production planning, the decisions are made from various models with different time horizons and without feedback and hierarchical structure. It is important that these decisions are made at an upper level of planning (long-term) which can be imposed on the lower level (short-term) as constraints. Also in real problems, exact and sufficient detailed data don't exist for short-term planning. However, providing exact data for long-term planning is possible. Then upper level of planning is solved and its outputs are used as validation criteria for lower level outputs.

Production planning in the electricity industry and PPGP problems are very complex with extensive features. Also, due to the specific condition of respective product, electricity generation planning is mainly different from the other production planning problems that have specific characteristics. Some of these characteristics are:

1. Not being able to suppose the backorder state.

2. Generating electricity in a specific time period for use in future time periods is not directly possible.

3. Flexible and specific electricity generation planning generate more electricity than predicted output to satisfy the expected demand.

An appropriate approach to alleviate this deficiency is to use FHPP by introducing imprecise/fuzzy data along with soft constraints, allowing some minor deviations from the outputs of the upper level while making a decision in the lower level.

A rigorous mathematical analysis of Hierarchical Production Planning (HPP) is found in the pioneering work of Hax and Meal [3]. Theoretical work on the topic has followed [4], [5] and [6]. Nowadays, HPP method is used as a structured method in various fields. In general the essential advantages of using an HPP approach are as follows [3]:

1. Planning process complexity reduction are due to the main problem of decomposition into a series of continuous sub problems. Solving these sub problems is very simple and economical. In a common manufacturing system, Mixed Integer Linear Programming (MILP) problem usually includes many variables and constraints, which make it very complex and greatly reduce the scope of useful optimization problems. In a hierarchical approach, an integrated problem is decomposed into sub problems that need very low calculation and computer memory. Therefore, production planning problem solving is possible in an acceptable time.

2. Best contrast with changes and random events occurred inside or outside the organization: In an integrated approach, whenever model parameters change due to the disorders caused by internal or external manufacturing system, the planning problem should be solved once again. Whereas in HPP approach, these random events are gradually absorbed and considered without any requirement to solve all sub problems. It must be mentioned that high level decisions are aggregate and general, and do not need highly detailed information. It needs a few calculations to create total control stability. Therefore, using hierarchical formulation enables to consider the random events. Then it can be modified appropriately to influence the mentioned events on planning problems.

3. There are few requirements for detailed information in long term planning: high levels in hierarchical structure use broad and aggregate demand information. This aggregate forecasting is more accurate and simple to calculate than detailed forecasts which are used in an integrated model. Therefore, long term plans in hierarchical approaches are more accurate than long term plans in integrated models.

4. There is a possibility of using proper decision making criteria in different levels of hierarchical structure. In industrial applications, different criteria are used

in different management levels. For example, worker hire / layoff costs are usually considered in long term planning level and start up costs are known for scheduling level.

5. Hierarchy planning is parallel to management hierarchy. In the lowest level of hierarchical approach, decisions for each workshop plant planning (operational planning) are usually performed by workers. Higher levels of decision making for a factory or a department are made in the proper levels of management hierarchical structure. This relationship between management hierarchical structure and planning will lead to management and organizational improvement.

6. It is possible to use a rolling horizon approach in different levels of planning to update production plans by the latest relevant information in production system. One of the advantages characteristic of HPP compared with integrated approach is to have a feedback from lower levels to the higher levels to update input parameters. In rolling horizon approach, higher level models are again solved by using new information that is generated from lower levels. Then outcomes of higher levels are applied as constraints for lower levels. In this way production systems will have the required ability and flexibility to repel internal and external changes.

In previous studies of power systems, there is very little attention to the hierarchical structure aspects of power system production planning. Also in previous studies there is a lack of a proper updating feedback system to increase reliability and developing performance of the power system production planning on different horizons of planning. A feedback system allows decision makers not only to have very flexible production plans but also to revise the model easily into different levels of long-term, mid-term and short-term levels of electricity planning with the inputs like ‘any unexpected events’, ‘upper manager decision makers’ and ‘actual data which is gained with time lapse’. Moreover in the previous studies, objective functions used in power system production planning models were based on cost, and other criteria of power production such as environmental pollution, proportion in total capacity and so on, were not considered together with economic criterion.

The main purpose of this paper is to improve the performance of the power system generation planning structure practically on different horizons of planning (long-term, mid-term and short-term electricity planning). A feedback system of FHPP is applied with multi objective functions for power production planning. The imprecise input parameters along with some soft constraints are introduced in the model formulation instead of using the crisp data and imposing hard constraints to provide required consistency between decisions of different levels. In practice the result of production plans through FHPP would be more feasible and compatible.

The rest of the paper is organized as follows: The relevant literature is presented in Section 2. The overall structure of the proposed FHPP model along the corresponding fuzzy mathematical models is illustrated in Section 3. In Section 4, the proposed fuzzy HPP structure is elaborated applying appropriate strategies and the associated fuzzy linear programming models are converted into the equivalent auxiliary crisp models. The proposed FHPP structure is implemented for a real power system in Iran. The case study and the obtained results as well as some managerial implications are provided in Section 5. There it is indicated that applying FHPP as a new approach for PPGP, will be conducted toward effective structured and efficient power system as concluding remarks in Section 6.

2. Background

Based on the main characteristics of the research problem, as explained in more detail in the next section, the most relevant and recent literature in three different but somewhat close streams of: 1. Production planning in power systems, 2. Application of mathematical optimization (algorithmic) methods in power systems production planning problems and 3. Applications of fuzzy modeling in production planning are studied.

2.1. Production planning in power systems

The long-term problem is a well-known stochastic optimization problem, as several of its parameters are only known as probability distributions, such as load, the availability of thermal units, hydrogenation and energy generations from renewable sources in general.

Bloom and Gallant [7] proposed a linear model with an exponential number of inequality constraints and used an active set methodology [8] to find the optimal way of matching the LDC of a single interval using thermal unit in the presence of load-matching and other operational non-load-matching constraints.

The Bloom and Gallant model has been successfully extended to multi-interval long-term planning problems, using the active-set method [9], the Dantzig–Wolfe column generation method [10, 11] or the Ford–Fulkerson column-generation method [12, 13]. A quadratic model to formulate the long-term profit maximization of generation companies in a liberalized market has been proposed [14] and column generation procedures have been employed to solve it [15, 16].

Mid-term planning does not frequently appear in the literature. However, the closely related short-term planning, which considers similar questions over a time horizon of up to one week, is well known. The most common version of the short-term planning problem, also known as the unit commitment problem, considers planning of power producing units in a power grid.

Rong et al. [17] introduced in their research the DRDP-RSC algorithm, which is a dynamic regrouping based dynamic programming algorithm based on linear relaxation of the ON/OFF states of the units, sequential commitment of units in small groups. This research addresses the Unit Commitment (UC) in multi-period Combined Heat and Power (CHP) production planning under the deregulated power market.

Currently, the solution approaches to UC of CHP systems are limited to some general-purpose methods. The research follows two lines. The first line applies decomposition techniques such as Lagrangian relaxation (LR) [18,19] and DP based algorithms [20-22]. The second line treats the overall problem as an entity and resorts to a general solver possibly with some modifications such as the Branch and Bound algorithm [23] to solve a MILP formulation of the problem. The application of simulation approaches [24, 25] and artificial intelligence techniques such as genetic algorithms [26] should be placed under this category. It is undoubted that the Interior Point Method (IPM) [27] and the improvement of the formulation for the UC problem [28] can also be applied to CHP systems. Youakim [29] presented necessary and sufficient conditions for the feasibility of unit combinations that can be checked off-line that is, before the start of the unit commitment algorithm, and thus before any economic dispatches are performed, thereby rendering a very efficient unit scheduling algorithm in terms of computer memory and execution time. Patra and Goswami [30] proposed a dynamic programming technique with a fuzzy and simulated annealing based unit selection procedure for the solution of the UC problem.

Jalilzadeh et al. [31] presented a new method with integration of generation and transmission network reliability for the solution of UC problem. In fact, in order to have a more accurate assessment of system reserve requirement, in addition to unavailability of generation units, unavailability of transmission lines are also taken into account. Gomes and Saraiva [32] described the formulations and the solution algorithms developed to include uncertainties in the generation cost function and in the demand on DC OPF studies. The uncertainties are modeled by trapezoidal fuzzy numbers and the solution algorithms are based on multi parametric linear programming techniques. Goransson and Johnsson [33] used a Mixed Integer Programming (MIP) approach to determine the power plant dispatch strategy which yields the lowest systems costs. In the model, each large thermal plant is described separately, including properties such as start-up time, start-up cost and minimum load level. Kumar and Naresh [34] proposed an efficient optimization procedure based on Real Coded Genetic Algorithm (RCGA) for the solution of Economic Load Dispatch (ELD) problem with continuous and non-smooth/non-convex cost function considering various constraints. The effect of the proposed algorithm has been demonstrated on different systems considering the transmission losses and valve point loading effect in thermal units.

For the solution of corresponding optimization problems, several methods have been suggested and implemented, including algorithms based on branch-and-bound [35], dynamic programming [36,37], Lagrangian relaxation [38-40] and genetic algorithms [41,42]. Surveys are given in [43, 44].

2.2. Application of mathematical optimization in power system production planning problems

When the objective function and constraints are linear, this gives the LP [45-47]. LP methods basically fall into two categories: simplex and Integer Programming (IP) methods [48-55]. A variety of IP algorithms have been applied to a number of power system problems, e.g., economic dispatch, reactive power optimization, power system optimization, and etc. Both the simplex and IP methods can be extended to a linear and quadratic objective function when constraints are linear. These methods are called *QP* [56, 57]. LP has been used in various power system applications, including power system optimal power flow [46], load flow [47], reactive power planning [58], and active and reactive power dispatch [59, 60].

When the objective function or the constraints are nonlinear, it forms NLP. IP methods originally developed for LP can be applicable to QP and NLP problems. NLP has been applied to various areas of power systems [61], e.g., optimal power flow [62], hydrothermal scheduling [63], etc.

For many optimization problems (e.g., ON status = 1, and OFF status = 0), some of the independent variables can take only integer values; problem like this is called *integer programming*. When some of the variables are continuous, the problem is called *mixed integer programming*. Mainly two approaches, i.e., 'branch and bound', and 'cutting plane methods', have been used to solve integer problems using mathematical programming techniques [64]. Integer/mixed integer programming has been applied to various areas of power systems, e.g., optimal reactive power planning [65], power systems planning [66, 67], unit commitment [68], generation scheduling [69], etc. DP based on the principle of optimality states that a sub-policy of an optimal policy must in itself be an optimal sub-policy. DP has been applied to various areas of power systems, e.g., reactive power control [70], transmission planning [71], unit commitment [72], and etc. The literature review regarding the application of mathematical optimization in power system production planning problems reveals the lack of using hierarchical and feedback structure in modeling power system production planning. Therefore, in this paper we develop a novel fuzzy HPP model which to the best of our knowledge, has not been addressed in the literature so far.

2.3. Applications of fuzzy modeling in production planning

The fuzzy set theory has been used considerably for modeling and solving the different variants of production planning and scheduling problems in uncertain environments.

Hsu and Wang [73] developed a Possibilistic Linear Programming (PLP) model based on Lai and Hwang's [74] approach to determine appropriate strategies regarding the safety stock levels for assembly materials, regulating dealers' forecast demands and numbers of key machines in an assemble-to order environment. Fung et al. [75] presented a Fuzzy Multi-Product Aggregate Production Planning (FMAPP) model to cater different scenarios under various decision-making preferences by applying integrated parametric programming and interactive methods. Wang and Liang [76] developed a fuzzy multi-objective linear programming model with piecewise linear membership function to solve multi-product Aggregate Production Planning (APP) problems in a fuzzy environment. In another research work, they [77] presented an interactive possibilistic linear programming model using Lai and Hwang's [74] approach to solve the multi-product aggregate production planning problem with imprecise forecast demand, related operating costs and capacity. Moreover, in mid-term supply chain planning domain, Torabi and Hassini [78] presented a novel multi-objective possibilistic mixed integer linear programming model for a Supply Chain Master Planning (SCMP) problem consisting of multiple suppliers, one manufacturer and multiple distribution centers which integrates the procurement, production and distribution aggregate plans considering various conflicting objectives simultaneously as well as the imprecise nature of some critical parameters such as market demands, cost/time coefficients and capacity levels. In another research work [79], the authors extended the above model to multi-site production environments and proposed an interactive fuzzy goal programming solution approach for the problem. Other relevant literature may include [80-83]. It is noteworthy that there are some other research works applying stochastic model to solve the production planning problems in uncertain environments. For a recent review of different approaches for dealing with uncertainty in production planning problems especially HPP approach, an interested reader is referred to Mula et al. [84].The literature review regarding the application of fuzzy approach in production planning and scheduling problems reveals the lack of using fuzzy sets theory in modeling HPP structures.

The proposed fuzzy HPP which has been stimulated by a real industrial case of an Iranian power network consists of three decision-making levels. Monthly consumption of 20 future years is forecasted in the first level. In second level, forecasted demand is allocated to different methods of electricity generation for an aggregate period. Structure of the proposed Fuzzy Aggregate Production Planning (FAPP) model could be considered as a fuzzy linear programming model which generates an optimal production plan to satisfy the aggregate forecasted demands of electricity. Two objective functions are: 1. minimizing the cost of electricity generation by different methods of generation and 2. maximizing the total preference weights of projects that are calculated by Analytical Hierarchy Process (AHP). In the third (disaggregation) level, similar model is applied to determine the production plan at the monthly periods. The next section provides the detailed fuzzy models in the proposed FHPP framework.

3. The Proposed Fuzzy Hierarchical Production Planning Model

Because of insufficient or inaccessible data and also the information acquiring high costs, the modeling parameters for PPGP are usually imprecise. In other words, competitive market persuades managers to implement precise and reliable production plans which could not be achieved with inaccurate and fuzzy market data. Also implementation of production plans with imprecise crisp data and crisp models is very difficult. One of the main motivations of this study is fuzziness. Which made the extracted results from the proposed FHPP to be more accurate, reliable and increase the efficiency of production planning, therefore it will be convenient to obtain production planning model that could handle fuzzy and uncertain data from the market. Fuzzy constraints should be used to increase the efficiency and compatibility among different levels of planning. Hence more optimal and feasible results could be obtained. The integrated problem of PPGP is divided into three levels of Demand level, Aggregate level and Disaggregate or allocation level presented in Figure 1.

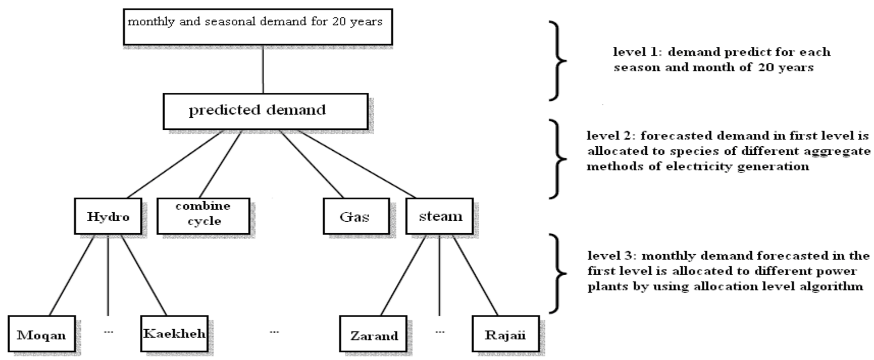

Figure 1. Hierarchical structure of problem solving.

3.1. First level (demand forecast)

The demand forecasting method presented by Sadeghi et al. [85] is applied in this study. The amount of monthly demand for 20 future years is forecasted for planning, and then an optimal planning model is developed to satisfy the demand.

3.2. Second level (**demand allocation to the generation of different methods**)

The forecasted demand in the first level is allocated to different aggregate methods of electricity generation for seasonal aggregate periods in 20 future years. Different methods of electricity generation can be divided into different features. For example we can divide them according to the technology applied such as Fossil, Nuclear, Combine cycle, Small hydro, big hydro, Micro hydro, Wind turbine, PV, Mono crystalline, Multi crystalline and Geothermal. Some of these technologies are not employed in Iran. The most common technologies of Gas, Steam, Combine cycle and Hydro are considered for electricity generation.

3.2.1. Mixed method of AHP with FAPP

AHP is applied to obtain total preference weights for each method of electricity generation in Iran using Expert Choice software. Then FAPP is applied to maximize total preference weights, to determine the best combination of generation methods and to satisfy power plant production demand in Iran using Lingo software (Fig. 2).

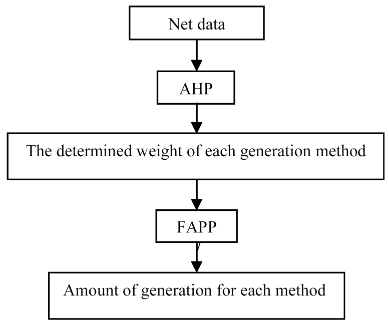

Figure 2. Mixed method of analytical hierarchy process with fuzzy aggregate production planning.

3.2.2. Assigning score to each generation method

For each method of electricity generation in Iran (Gas, Steam, Combine Cycle and Hydro) a score is given based on the following criteria.

1. Amount of environmental pollution in production procedure including SO_2, NO_x and CO_2.

2. The share of each method capacity compared with the total capacity.

Forecasted aggregate seasonal demand for 20 future years is assigned to the different methods of generation by applying heuristic mathematical model. The above criteria are used to rank different production methods, in a hierarchical structure (Figure 3).

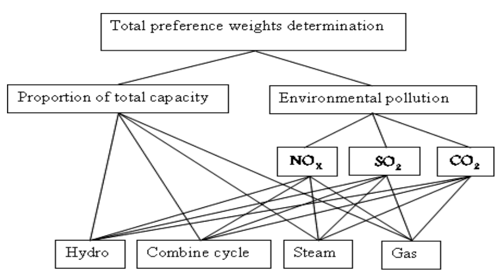

Figure 3. Hierarchical structure to rank production methods.

3.2.3. FAPP of proposed structure to electricity generation planning

The proposed FAPP model is used to provide an optimal aggregate production plan. Then it can satisfy the dynamic demands of electricity over a given long-term planning horizon involving the above mentioned outputs. The main characteristics and assumptions considered in the FAPP formulation are as follows:

- A four-power plant situation is considered.
- There is a seasonal period on 20 future years planning horizon.
- Forecasted demand in seasonal period t_s of year t_y in zone z and peak demand of year t_y in zone z are assumed fuzzy.
- Reliability, autonomy and balance constraints are assumed fuzzy.

The indices, parameters and variables used to formulate the FAPP model are:

Indices:

i	Index of aggregate power plant families $(i = 1, ..., 4)$
t_s	Index of aggregate time periods (seasonal, t_s, 1, ..., 4)
t_y	Index of time horizon planning (annual, $t_y = 1, ..., 20$)
z, z_p	Index of Electricity zones of Iran $(z, z_p = 1, ..., 15)$

Parameters:

$fp(z, i)$	Fuel cost of old power plant i in zone z in base year (Rial/MW)
$HR(z, i)$	Heat rate of old power plant i in zone z in base year (constant)
$fpesc(z, i)$	Regulation rate of old power plant i in zone z in each year toward before year (%)
$OM(z, i)$	Variable operation and maintenance costs of old power plant i in zone z (Rial/MW)
$fpN(z, i)$	Fuel cost of new power plant i in zone z in base year (Rial/MW)
$HRN(z, i)$	Heat rate of new power plant i in zone z in base year (constant)

$fpescN(z, i)$	Regulation rate of new power plant i in zone z in each year toward before year (%)
$OMN(z, i)$	Variable operation and maintenance costs of new power plant i in zone z (Rial/MW)
$UE\cos t(z)$	Unsaved energy cost per blackout (MW) in zone z (Rial/MW)
$UM\cos t$	Unmet reserve requirements cost per MW (Rial/MW)
W_{zi}	Total preference weights of projects that are calculated by AHP (constant)
$\widetilde{D}(t_y, t_s, z)$	Amount of electricity demand in zone z in season t_s of year t_y (MW)
$\widetilde{D}_{peak}(z, t_y)$	Amount of peak electricity demand in zone z of year t_y (MW)
$PGLoss(z, i)$	Inner consumption factor of old power plant i in zone z (%)
$PGNLoss(z, i)$	Inner consumption factor of new power plant i in zone z (%)
$PG\exp step(z, i)$	Capacity mounted in each developing step in old power plant i in zone z (MW)
$PGN\exp step(z, i)$	Capacity mounted in each developing step in new power plant i in zone z (MW)
$PG\max(z, i)$	Upper bound of total development in all years in old power plant i in zone z
Decay	Reduction capacity in each year toward before year (%)
$PG_{init}(z, i)$	Amount of total nominal power of power plants i in zone z (MW)
$PFLoss(z, z_p)$	Loss percentage between two zones (%)
$PF_{init}(z, z_p)$	Amount of initial capacity for lines between two zones (MW)
$AF(z, t_y)$	Autonomy factor for zone z and in year t_y
$Cos\,\theta$	Coefficient of line power that is allocated to active flow (%)
$RESPP(z, i)$	Bound of reserve for power plant i in zone z (%)
$crf(z)$	Capital recovery factor for old power plant in zone z (constant)

$PG \exp \cos t(z, i)$	Fixed cost for developing in old power plant i in zone z (Rial/MW)
$PGN \exp \cos t(z, i)$	Fixed cost for making new power plant i in zone z (Rial/MW)
$PFN \cos t(z, z_p)$	Fixed cost for making new transmission lines between zones z and z_p (Rial/MW)
$crfPNF(z, z_p)$	Capital recovery factor for new transmission lines between zones z and z_p (constant)
disc	Interest rate in any year of planning horizon for all zones (constant)

Variables:

$PG(t_y, t_s, z, i)$	Production amount of old power plant i in zone z and in season t_s of year t_y (MW)
$PGN(t_y, t_s, z, i)$	Production amount of new power plant i in zone z and in season t_s of year t_y (MW)
$PG \exp(t_y, z, i)$	Number of units that add in old power plant i in zone z and in year t_y
$PGN \exp(t_y, z, i)$	Number of units that add in new power plant i in zone z and in year t_y
$PF(t_y, t_s, z, z_p)$	Amount of old transitive power between two zones and in season t_s of year t_y (MW)
$PFN(t_y, t_s, z, z_p)$	Amount of new transitive power between two zones and in season t_s of year t_y (MW)
$PFN \exp(t_y, z, z_p)$	Amount of capacity made of new lines between zones z and z_p in year t_y (MW)
$F_{Max}(t_y, z, z_p)$	Amount of capacity that a zone has reserved for other zone in year t_y (MW)
$UM(z, t_y)$	Amount of unmet reserve requirements in zone z in year t_y (MW)
$UE(t_y, t_s, z)$	Amount of unsaved energy in zone z and in season t_s of year t_y (MW)

Based on the above notations, the FAPP model is formulated as follows:

3.2.3.1 Objective functions

Minimizing the cost of electricity generation using different ranges of technologies is considered as the first objective (Z_1), and the second objective function (Z_2) is to maximize the total preference weights of projects which are calculated by AHP. Knowing the model is long-term projection; total costs in planning horizons of all years. Thus we change the value of the total costs of each year to money value in base year.

$$Min \quad Z_1 = \sum_{t_{yb}=1}^{20} \frac{C_{PG}(t_{yb}) + C_{PGN}(t_{yb}) + C_U(t_{yb})}{(1 + disc)^{(t_{yb}-1)}}$$

$$+ \sum_{t_{yb}=1}^{20} \sum_{tc_y=t_{yb}}^{20} \frac{Ccap_{PG}(t_{yb}) + Ccap_{PGN}(t_{yb}) + Ccap_{PFN}(t_{yb})}{(1 + disc)^{(tc_y-1)}}$$

$$Max \quad Z_2 = \sum_{t_y=1}^{20} \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} W_{zi} \times PG(t_y, t_s, z, i).$$

Fuel and operational cost for old power plants

Production cost of total old power plants with each technology in all zones in t_y year is equal to:

$$C_{PG}(t_y) = \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} PG(t_y,\ t_s,\ z,\ i)$$

$$\times [OM(z,\ i) + HR(z,\ i) \times fp(z,\ i) \times fpesc(z,\ i)^{t_y}]. \tag{2}$$

Fuel and operational cost for new power plants

Production cost of total new power plants with each technology in all zones in t_y year is equal to:

$$C_{PGN}(t_y) = \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} PNG(t_y,\ t_s,\ z,\ i)$$

$$\times [OMN(z, i) + HRN(z, i) \times fpN(z, i) \times fpescN(z, i)^{t_y}]. \tag{3}$$

Blackout costs (*Unsaved Energy and Unmet Reserve*)

Blackout costs in total zones in t_y year are equal to:

$$C_U(t_y) = \sum_{t_s}^{4} \sum_{z=1}^{15} [UE(t_y, t_s, z) \times UE\cos t(z)] + \sum_{z=1}^{15} UM(z, t_y) \times UM\cos t. \quad (4)$$

Capital costs as for developing old power plants

The cost of all total development for all total old power plants in all total zones in t_y year and considering capital recovery factor is equal to:

$$Ccap_{PG}(t_y) = \sum_{i=1}^{4} \sum_{z=1}^{15} PG\exp step(z, i)$$
$$\times PG\exp(t_y, z, t) \times PG\exp\cos t(z, i) crf(z). \quad (5)$$

Capital costs as for developing new power plants

The cost of making power plants for all total new power plants in all total zones in t_y year and considering capital recovery factor is equal to:

$$Ccap_{PGN}(t_y) = \sum_{i=1}^{4} \sum_{z=1}^{15} PGN\exp step(z, i)$$
$$\times PGN\exp(t_y, z, t) \times PGN\exp\cos t(z, i) crf(z). \quad (6)$$

Capital costs that are related to developing new transfer lines

The cost of all total developments for all total new transfer lines between all of the zones of the country in t_y year and considering capital recovery factor is equal to:

$$Ccap_{PFN}(t_y) = \sum_{z=1}^{15} \sum_{z_p=1}^{15} PFN\exp(t_y, z, z_p)$$
$$\times PFN\cos t(z, z_p) \times (0.5) \times crfPNF(z, z_p). \quad (7)$$

3.2.3.2. Constraints

For each period, the following constraints are considered:

(A) *The capacity constraints of old power plants generation*

The amount of electricity product of old power plant i in season t_s of year t_y

in zone z cannot be more than the amount of the total primary mounted capacity plus added capacity until t_y year (by reduction alignment due to decay).

$$PG(t_y, t_s, z, i)$$
$$\leq \left\{PG_{init}(z, i) \times (1 - Decay)^{ty}\right.$$
$$\left. + \sum_{tyb=1}^{ty} PG \exp step(z, i) \times PG \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty-tyb)}\right\}$$
$$\times (1 - PGLoss(z, i)) \quad \forall t_y, t_s, z, i. \tag{8}$$

(B) *Max development of old power plants constraint*

Each productive power plant that is made at first has known measurements and physical close. So the number of all units that can mounted in one power plant, in time horizon is limited. It means that the power plant i in zone z and in all total duration, have known amount of Max output for development. Thus maximum development of old power plants constraint in 20 years time horizon is equal to:

$$\sum_{ty=1}^{20} PG \exp step(z, i) \times PG \exp(t_y, z, i) \leq PG \max(z, i) \quad \forall z, i. \tag{9}$$

(C) *The capacity constraints of new power plants generation*

According to zones geography condition and their abilities to work professionally, construction of different power plants in several zones with their own characteristic presented in the offering projects list is given to the model. By solving the model, the order is determined according to zones demand and costs and other technological and economical problems. The amount of electricity generation in new power plant i in season t_s of year t_y in zone z cannot be more than the amount of total new mounted capacity by reduction alignment due to decay.

$$PGN(t_y, t_s, z, i)$$
$$\leq \left\{\sum_{tyb=1}^{ty} PGN \exp step(z, i) \times PGN \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty-tyb)}\right\}$$
$$\times (1 - PGNLoss(z, i)) \quad \forall t_y, t_s, z, i. \tag{10}$$

(D) *The capacity constraints of old transmission lines*

Development of the old lines between the two zones of the country can not be possible due to the elimination of the filling capacity of power station at the two ends of the lines. Thus the new lines should be developed by Max capacity assuming their development in future is not commercially feasible. The exchange amount between z and z_p zones in season t_s of year t_y related to old lines can be less than or equal to the mounted capacity. So this capacity constraint is for old transferring lines:

$$PF(t_y, t_s, z, z_p)$$

$$\leq \{PF_{init}(z, z_p) \times (1 - Decay)^{ty}\} \times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p. \tag{11}$$

(E) *The capacity constraints of reserve exchange*

In the peak hours, each zone can make part of its capacity as reserve for the other zone named reserve capacity. The reserved capacity should be smaller than initial capacity of transmission lines between two zones, which is necessary to be guaranteed with a constraint.

$$F_{Max}(t_y, z, z_p)$$

$$\leq \left\{PF_{init}(z, z_p) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PFN\exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)}\right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, z, z_p. \tag{12}$$

(F) *The capacity constraints of new transmission lines*

In the transferring lines, that capacity development is possible with each amount of the capacity, when provided continuously. By spotting $\mathrm{PFN}\exp(t_y, z, z_p)$ variable, which determine the amount MW development between two z_p and z zones in season t_s of year t_y, and with spotting the capacity reduction of exhaustion that the Decay parameter show the amount of that, capacity constraint for the new lines is equal to:

$$PFN(t_y, t_s, z, z_p) \leq \left\{\sum_{tyb=1}^{ty} PFN\exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)}\right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p. \tag{13}$$

(G) *The reliability constraints*

Each zone should have a defined reserve bound for main supply source (power plants). The reserve bound with RESPP parameter can be model for thermal power plants for all zones which is typically 0.1 and 0.16 for hydro power plant for all zones. Model for raising reliability of system always has more capacity for network than demand. Certainly if this additional capacity do not supply, amount of network reliability will be less. $UM(z, t_y)$ variable show the amount of network unmet reserve MW in zone z and in t_y year, so for reliability constraint we have:

$$\sum_{i=1}^{4} \frac{\left\{ PG_{init}(z,i)\times(1-Decay)^{ty} + \sum_{tyb=1}^{ty} PG\exp step(z,i) \times PG\exp(t_{yb}, z, i)\times(1-Decay)^{(ty-tyb)} \right\} \times (1-PGLoss(z,i))}{1+RESPP(z,i)}$$

$$+ \sum_{i=1}^{4} \frac{\left\{ \sum_{tyb=1}^{ty} PGN\exp step(z,i)\times PGN\exp(t_{yb}, z, i) \times(1-Decay)^{(ty-tyb)} \right\} \times (1-PGNLoss(z,i))}{1+RESPP(z,i)}$$

$$+ \sum_{z_p=1}^{15} \{F_{Max}(t_y, z_p, z)\times(1-PFLoss(z_p, z))\} + UM\{z, t_y\}$$

$$\tilde{\geq} \sum_{z_p=1}^{15} F_{Max}(t_y, z, z_p) + \tilde{D}_{peak}(z, t_y) \quad \forall t_y, z. \tag{14}$$

(H) *The balance constraints*

Each zone if cannot perform the demand by choosing the blackouts that enter to the supply of the demand/supply balance equation, then the unbalance problem of demand/supply should be solved. $UE(t_y, t_s, z)$ variable represent the amount of MW blackout in zone z and each time.

$$\sum_{i=1}^{4} PG(t_y, t_s, z, i) + PGN(t_y, t_s, z, i) + UE(t_y, t_s, z)$$

$$+ \sum_{z_P=1}^{15} PF(t_y, t_s, z_p, z) \times (1 - PFLoss(z_p, z))$$

$$+ \sum_{z_P=1}^{15} PFN(t_y, t_s, z_p, z) \times (1 - PFNLoss(z_p, z))$$

$$\cong \widetilde{D}(t_y, t_s, z) + \sum_{z_p=1}^{15} PF(t_y, t_s, z, z_p)$$

$$+ \sum_{z_p=1}^{15} PFN(t_y, t_s, z, z_p) \quad \forall t_y, t_s, z. \tag{15}$$

(I) *Autonomy constraints*

In addition by spotting system reliability, because of non-technological, political, and economical reasons, it is better to preserve the internal capacity in the determinate fraction of internal peak demand, uninterested to economical profits of “cheap power importation” or “reserve capacity importation”.

Autonomy factor $AF(z, t_y) >= 0,$ returns the percent of autonomy of each zone. Thus if it want to be completely autonomy should $AF \geq 1$ and if it want to be completely attached to the fixed importation is the peak time, to be economically safe it is $AF = 0$ and for the remains between $0 < AF < 1.$ So the production autonomy constraint is like this:

$$\Bigg\{PG_{init}(z, i) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PG \exp step(z, i)$$

$$\times PG \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty-tyb)}\Bigg\} (PGLoss(z, i))$$

$$+ \Bigg\{\sum_{tyb=1}^{ty} PGN \exp step(z, i) \times PGN \exp(t_{yb}, z, i)$$

$$\times (1 - Decay)^{(ty-tyb)}\Bigg\} \times (PGNLoss(z, i))$$

$$\tilde{\geq}\ AF(z,\ t_y) \times \widetilde{D}_{peak}(z,\ t_y) \quad \forall t_y,\ z,\ i. \tag{16}$$

(J) *Non-negativity constraint*

Equation (17) is a non-negativity constraint.

$$\begin{aligned} & PG(t_y,\ t_s,\ z,\ i),\ PGN(t_y,\ t_s,\ z,\ i),\ UE(t_y,\ t_s,\ z),\ F_{Max}(z,\ z_p), \\ & PF(t_y,\ t_s,\ z,\ z_p),\ PFN(t_y,\ t_s,\ z,\ z_p),\ PG\exp(t_y,\ z,\ i), \\ & PGN\exp(t_y,\ z,\ i), PFN\exp(t_y,\ z,\ z_p), UM(z,\ t_y) \\ & \geq 0 \quad \forall t_s,\ z,\ z_p,\ i. \end{aligned} \tag{17}$$

3.3. Third level (different methods of production allocation to dependent power plants)

In the third level of hierarchical structure, monthly demand forecasted in the first level is allocated to different power plants using allocation level algorithm as follows.

3.3.1. Mixed method of AHP with fuzzy disaggregate production planning

AHP is applied to obtain total preference weights for each power plant by using Expert Choice software. Then Fuzzy Disaggregate Production Planning (FDPP) is applied to maximize the total preference weights, to determine the best combination of power plants and to satisfy production demand in Iran using Lingo software (Figure 4).

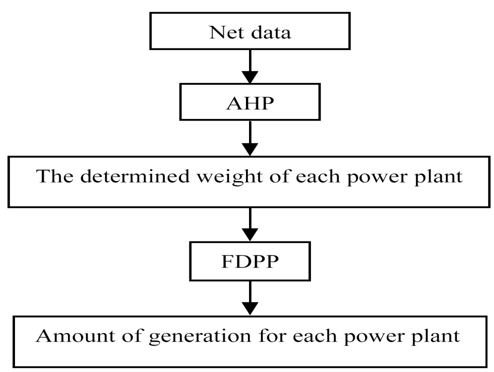

Figure 4. Mixed method of analytical hierarchy process with fuzzy disaggregate production planning.

3.3.2. Ranking power plants that produce electricity by the same method

To rank power plants, AHP method is used as illustrated in the Figure 5. Two criteria are used to score each power plant:

- Efficiency of different power plants
- Power plant activity in a year

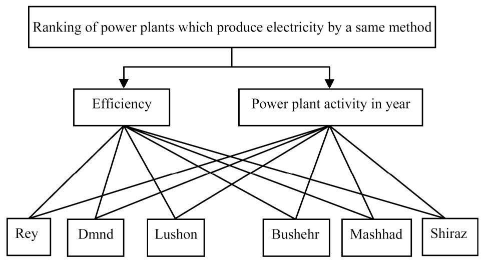

Figure 5. Hierarchical structure to rank power plants which produce electricity by the same method.

After ranking of different power plants using the above method, the forecasted monthly demand for first season of the first year should be satisfied with allocation of the demand to the power plants, using FDPP. FDPP model is as follows:

3.3.3. FDPP of proposed structure for electricity generation planning

The aggregate production plan generated by FAPP model cannot be implemented in practice because of its aggregate nature regarding both the power plants and time periods. Therefore, in order to develop a detailed production plan, disaggregated model is required to provide a master production plan (MPS). Thus, another fuzzy linear programming model (FDPP) is proposed in which its main assumptions and structure are similar to those of FAPP model. FDPP model must be solved separately for each period of FAPP model. It means that we should solve one FDPP model for each season of 20 future years. For example, we solved the FDPP model for autumn $(t_s = 3)$ of first year. After solving the aggregate model, and specified the amount of generation of Steam, Gas, Combine Cycle and Hydro power plants in each season of 20 years planning horizon, then the disaggregate model for the first season of first year should solve the disaggregate model for the first season of first year to determine the monthly production rate of each final power plants. Then do the same calculation with the result for other seasons of first year by using the obtained results from the previous stages which should do the necessary reforms for future stages to get the monthly production planning of power plants for first year. This calculation should work for the next 20 years horizon by spotting the pervious years results and planning the necessary reforms. This work in hierarch planning is named rolling horizon approach that the regulation and correction is always based on pervious results. Here we perform the sample of autumn season of first year of disaggregate model.

The main characteristics and assumptions considered in the FDPP formulation are as follows:

- There is a three-period planning horizon that each period is a month.
- Forecasted demand in period tm of zone z and peak demand in zone z are assumed fuzzy.
- Reliability and balance constraints and forced constraints (24-26) of aggregate planning level are assumed fuzzy.

The indices, parameters and variables used to formulate the FDPP model are as follows:

Indices:

I	Index of aggregate power plant families $(i = 1, ..., 4)$
t_s	Index of aggregate time periods $(t_s = 1, ..., 4)$
z, z_p	Index of Electricity zones of Iran $(z, z_p = 1, ..., 15)$
k	Index of Disaggregate power plant $(k = 1, ..., n_i)$
t_m	Index of Disaggregate period $(t_m = 1, ..., 3)$

Parameters:

$FC(z, i, k)$	Fuel cost of power plant k of i family in zone z (Rial/MW)
$OM(z, i, k)$	Variable operation and maintenance costs of power plant k of i family in zone z (Rial/MW)
$UE\cos t(z)$	Unsaved energy cost per outage (MW) in zone z (Rial/MW)

$UM\cos t$	Unmet reserve requirements cost per MW (Rial/MW)
W_{zik}	Total preference weights of power plant *k* of *i* family that are calculated by AHP (Constant)
$\widetilde{D}(t_m, z)$	Amount of electricity demand per zone for per season of next year (MW)
$\widetilde{D}_{peak}(z)$	Amount of peak electricity demand per zone of next year (MW)
$PGLoss(z, i, k)$	Inner consumption factor of power plant k of *i* family in zone *z* (%)
$PG_{init}(z, i, k)$	Amount of total nominal power of power plants *k* of *i* family in zone *z* (MW)
$PFLoss(z, z_p)$	Loss percentage between two zones (%)
$PF_{init}(z, z_p)$	Amount of initial capacity for lines between two zones (MW)
$Cos\theta$	Coefficient of line power that is allocated to active flow (%)
$RESTHM(z, i, k)$	Bound of reserve for power plant *k* of *i* family in zone *z* (%)

Variables:

$PG(t_m, z, i, k)$	Production amount of power plant *k* of *i* family in zone *z* and in period t_m (MW)
$PF(t_m, z, z_p)$	Amount of transitive power between two zones and in period t_m (MW)
$F_{Max}(z, z_p)$	Amount of capacity that a zone has reserved for other zone (MW)
$UM(z)$	Amount of unmet reserve requirements in zone *z* (MW)
$UE(t_m, z)$	Amount of unsaved energy in zone *z* and in period t_m (MW)

Based on the above notations, the FDPP model is formulated as follows:

$$Min \quad Z_1 = \sum_{t_m=1}^{3}\sum_{z=1}^{15}\sum_{i=1}^{4}\sum_{k=1}^{n_i} PG(t_m, z, i, k) \times [OM(z, i, k) + FC(z, i, k)]$$

$$+ \sum_{t_m=1}^{3}\sum_{z=1}^{15} [UE(t_m, z) \times UE \cos t(z)] + \sum_{z=1}^{15} UM(z) \times UM \cos t$$

$$Max \quad Z_2 = \sum_{t_m=1}^{3}\sum_{z=1}^{15}\sum_{i=1}^{4}\sum_{k=1}^{n_i} W_{zik} \times PG(t_m, z, i, k) \tag{18}$$

s.t.

$$PG(t_m, z, i, k) \le PG_{init}(z, i, k) \times (1 - PGLoss(z, i, k)) \quad \forall t_m, z, i, k \tag{19}$$

$$PF(t_m, z, z_p) \le PF_{init}(z, z_p) \times (1 - PFLoss(z, z_p)) \times C0s\theta \quad \forall t_m, z, z_p \tag{20}$$

$$F_{Max}(z, z_p) \le PF_{init}(z, z_p) \times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall z, z_p \tag{21}$$

$$\sum_{i=1}^{4}\sum_{k=1}^{n_i} \frac{PG_{init}(z, i, k) \times (1 - PGLoss(z, i, k))}{1 + RESPP(z, i, k)}$$

$$+ \sum_{z_p=1}^{15} \{F_{Max}(z_p, z) \times (1 - PFLoss(z_p, z))\} + UM(z)$$

$$\tilde{\ge} \sum_{z_p=1}^{15} F_{Max}(z, z_p) + \tilde{D}_{peak}(z) \quad \forall z \tag{22}$$

$$\sum_{i=1}^{4}\sum_{k=1}^{n_i} PG(t_m, z, i, k) + UE(t_m, z) + \sum_{z_P=1}^{15} PF(t_m, z_p, z) \times (1 - PFLoss(z_p, z))$$

$$\cong \tilde{D}(t_m, z) + \sum_{z_p=1}^{15} PF(t_m, z, z_p) \quad \forall t_m, z. \tag{23}$$

$$\sum_{t_m=1}^{3}\sum_{k=1}^{n_i} PG(t_m, z, i, k) \tilde{\ge} PG(t_s, z, z_p) \quad \forall z, i;\ t_s = 3. \tag{24}$$

$$\sum_{t_m=1}^{3} PF\left(t_m, z_p, z\right) \tilde{\ge} PF\left(t_s, z_p, z\right) \quad \forall z, z_p;\ t_s = 3. \tag{25}$$

$$\sum_{t_m=1}^{3} UE(t_m, z) \tilde{\geq} UE(t_s, z) \quad \forall z;\, t_s = 3 \tag{26}$$

$$PG(t_m, z, i, k),\, UE(t_m, z),\, F_{Max}(z, z_p),\, PF(t_m, z, z_p),\, UM(z)$$

$$\geq 0 \quad \forall t_m, z, z_p, i, k \tag{27}$$

3.3.3.1 *Objective functions* (18)

Minimizing the cost of electricity generation by different power plants is considered as the first objective, and the second objective function is to maximize the total preference weights of power plants which are calculated by AHP.

3.3.3.2. *Constraints*

For each period, the following constraints are considered:

(A) *The capacity constraints of power plant generation* (19):

Production amount of power plant k of i family in zone z and in period t_m should not be greater than the amount of total nominal power of power plants k of i family in zone z.

(B) *The capacity constraints of transmission lines* (20)

The amount of exchange that can be transited between two zones is smaller or equal to installed lines capacity between two zones.

(C) *The capacity constraints of reserve exchange* (21)

In the peak hour, each zone can make part of its capacity as reserve for the other zone named reserve capacity, which is smaller than the initial capacity of transmission lines between two zones. It is necessary to be guaranteed with a constraint.

(D) *The reliability constraints* (22)

Reliability constraints guarantee the existence of a suitable reserve bound between installed capacity and peak period demand.

(E) *The balance constraints* (23)

Load balance forces the supply and demand to be equal in each period.

(F) *Forced constraints of aggregate planning level* (24-26)

The solution of a higher level subsystem represents a constraint to be imposed on the next level subsystem and thus, decisions at each level constitute a chain. Moreover, in the HPP problem, solutions of higher level subsystem are considered as inputs of the next level subsystem. Hence it is important to create suitable compatibility among the levels of subsystem. Crisp constraints reduce flexibility of HPP problems and the probability of having a feasible solution in any level whereas by using fuzzy constraints, flexibility of HPP problems increases, put a suitable compatibility between each level, and increases the probability of having a feasible solution for the problem [86].

4. Solution Procedure

In order to reach a preferred solution of the proposed FHPP structure, the associated mathematical programming models should be converted into the equivalent crisp ones. In this regard, three main stages are considered as the solution procedure for the proposed FHPP as follows:

1. Converting the FAPP model into its equivalent auxiliary crisp model.

2. Converting the FDPP into its equivalent auxiliary crisp model.

3. Applying an interactive fuzzy programming solution algorithm to obtain the final preferred solution.

4.1. Formulating the FAPP as an auxiliary crisp model

In order to solve the FAPP model, it should be transformed to an auxiliary crisp model. Here we present efficient strategies to convert the fuzzy objective function and soft constraints into equivalent crisp equations.

4.1.1. Treating the objective functions of FAPP

Since all the coefficients in the objective functions are crisp, it is sufficient that the multi objective FAPP model be converted into an equivalent single-objective FAPP model.

In linear programming, in order to convert the Multi Objective Linear Programming (MOLP) model into an equivalent single-objective LP model, it requires an aspiration level for each objective function and defines a new objective function based on the minimizing or maximizing objective functions. Then primary objective functions along with free variables and aspiration levels should be defined as additional constraints in model [87].

First objective (Z_1) is minimizing the cost of electricity generation by different power plants. Aspiration level for Z_1 is stated as follows:

$$A = \text{total load on 20 future years} \times \text{minimum production cost.}$$

In other words, all production methods are assumed with minimum production cost. Then a non-negative variable (d_1) are considered in the first objective function as the following:

$$Z_1 - d_1 = A. \tag{28}$$

Second objective (Z_2) is maximizing the total preference weights of power plants that are calculated by AHP. Aspiration level for Z_1 is considered as follows:

$$B = \text{total load on 20 future years maximum preference weight.}$$

In other words, it is assumed that total load has been produced with a method which has the highest preference. Then a non-negative variable (d_2) are considered in the second objective function as the following:

$$Z_2 + d_2 = B. \tag{29}$$

According to the relation of (28), to minimize Z_1, d_1 should be minimized. Also according to the relation of (29), to maximize Z_2, d_2 should be minimized. Therefore we should define $Z = d_1 + d_2$ as objective function of FAPP problem and also consider Eqs. (28) And (29) as constraint in FAPP problem. The new single-objective function defined for FAPP problem is as the following:

$$\text{Min } Z = d_1 + d_2. \tag{30}$$

Therefore problem is transformed into a FAPP problem with the single-objective function.

4.1.2. Treating the soft constraints

Due to incompleteness and/or unavailability of required data over the long-term decision horizon, the environmental data and operational parameters are typically uncertain and imprecise (fuzzy) in nature. Therefore, Forecasted demand in period t_m of zone z and peak demand in zone z are assumed to be fuzzy numbers characterized by triangular possibility distribution. These triangular possibility distributions which are determined by using both objective and subjective data are the most common tool for modeling the ambiguous parameters due to their computational efficiency and simplicity in data acquisition [77, 78, 79]. Generally, a possibility distribution can be stated as the degree of occurrence of an event with imprecise data. Figure 6 represents the triangular possibility distribution of imprecise parameter which can be symbolized as

$$\widetilde{D}(t_y, t_s, z) = (D^p_{(t_y, t_s, z)}, D^m_{(t_y, t_s, z)}), D^o_{(t_y, t_s, z)}),$$

where $D^p_{(t_y, t_s, z)}$, $D^m_{(t_y, t_s, z)}$ and $D^o_{(t_y, t_s, z)}$ are the most pessimistic value, the most possible value and the most optimistic value of $\widetilde{D}_{(t_y, t_s, z)}$ which are estimated by the decision maker. The other fuzzy data can be modeled in the same manner in which:

$$\widetilde{D}_{peak(z, t_y)} = (D^p_{peak(z, t_y)}, D^m_{peak(z, t_y)}, D^o_{peak(z, t_y)})$$

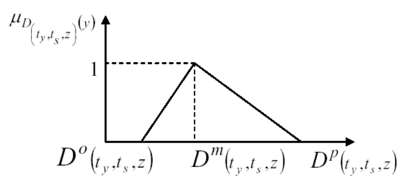

Figure 6. The triangular possibility distribution of fuzzy parameter $\widetilde{D}_{(t_y, t_s, z)}$.

To resolve the vagueness of Constraints (14-16) which permit these constraints to be satisfied as much as possible, they can be modeled by the preference-based membership functions. For example, a typical membership function of soft equation $a \cong b$ with tolerance p has been depicted in Figure 7.

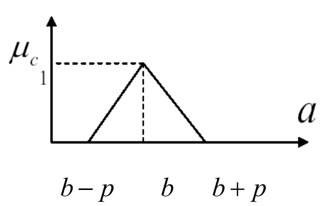

Figure 7. A preference-based membership function of soft equation $a \cong b$.

$$\sum_{i=1}^{4} PG(t_y, t_s, z, i) + PGN(t_y, t_s, z, i) + UE(t_y, t_s, z)$$

$$+ \sum_{z_P=1}^{15} PF(t_y, t_s, z_p, z) \times (1 - PFLoss(z_p, z))$$

$$+ \sum_{z_P=1}^{15} PFN(t_y, t_s, z_p, z) \times (1 - PFNLoss(z_p, z))$$

$$- \sum_{z_p=1}^{15} PF(t_y, t_s, z, z_p) - \sum_{z_p=1}^{15} PFN(t_y, t_s, z, z_p)$$

$$= A_{(ty, ts, z)}(x) \le \widetilde{D}(t_y, t_s, z) + p^{1}_{(ty, ts, z)} \quad \forall t_y, t_s, z. \tag{31}$$

And

$$\sum_{i=1}^{4} PG(t_y, t_s, z, i) + PGN(t_y, t_s, z, i) + UE(t_y, t_s, z)$$

$$+ \sum_{z_P=1}^{15} PF(t_y, t_s, z_p, z) \times (1 - PFLoss(z_p, z))$$

$$+ \sum_{z_P=1}^{15} PFN(t_y, t_s, z_p, z) \times (1 - PFNLoss(z_p, z))$$

$$- \sum_{z_p=1}^{15} PF(t_y, t_s, z, z_p) - \sum_{z_p=1}^{15} PFN(t_y, t_s, z, z_p)$$

$$= A_{(ty, ts, z)}(x) \ge \widetilde{D}(t_y, t_s, z) + p^{1}_{(ty, ts, z)} \quad \forall t_y, t_s, z. \tag{32}$$

The inequality relation of Eq. (14,16) can be constructed in the same way:

$$\sum_{i=1}^{4} \frac{\left\{ \begin{array}{l} PG_{init}(z, i) \times (1 - Decay)^{ty} + \displaystyle\sum_{tyb=1}^{ty} PG \exp step(z, i) \\ \times PG \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty - tyb)} \end{array} \right\} \times (1 - PGLoss(z, i))}{1 + RESPP(z, i)}$$

$$+\sum_{i=1}^{4}\frac{\left\{\sum_{tyb=1}^{ty} PGN\exp step(z,i)\times PGN\exp(t_{yb},z,i)\times(1-Decay)^{(ty-tyb)}\right\}\times(1-PGNLoss(z,i))}{1+RESPP(z,i)}$$

$$+\sum_{z_p=1}^{15}\{F_{Max}(t_y,\ z_p,\ z)\times(1-PFLoss(z_p,\ z))\}$$

$$+UM(z,\ t_y)-\sum_{z_p=1}^{15}F_{Max}(t_y,\ z,\ z_p)$$

$$=B_{(ty,z)}(x)\geq\widetilde{D}_{peak}(z,\ t_y)-p^2_{(ty,z)}\quad\forall t_y,\ z, \tag{33}$$

$$\left\{PG_{init}(z,\ i)\times(1-Decay)^{ty}+\sum_{tyb=1}^{ty}PG\exp step(z,\ i)\right.$$

$$\left.\times\ PG\exp(t_{yb},\ z,\ i)\times(1-Decay)^{(ty-tyb)}\right\}$$

$$\times\ (PGLoss(z,\ i))+\left\{\sum_{tyb=1}^{ty}PGN\exp step(z,\ i)\times PGN\exp(t_{yb},\ z,\ i)\right.$$

$$\left.\times\ (1-Decay)^{(ty-tyb)}\right\}\times(PGNLoss(z,\ i))$$

$$=C_{(ty,z,i)}\geq AF(z,\ t_y)\times\widetilde{D}_{peak}(z,\ t_y)-p^3_{(ty,z,i)}\quad\forall t_y,\ z,\ i, \tag{34}$$

where the $p^1_{(t_y,t_s,z)}$, $p^2_{(t_y,z)}$ and $p^3_{(ty,z,i)}$ denote the associated allowable tolerances.

Regarding the constraints (31-34) we should now compare the fuzzy right-hand sides with the crisp left-hand sides. An efficient approach to deal with such fuzzy constraints is to convert them into their equivalent crisp ones by obtaining crisp representative numbers for the corresponding fuzzy right-hand sides. To do so, we apply the well-known weighted average method [77, 88, 79]. This approach seems to be the simplest and the most reliable defuzzification method in converting the fuzzy constraints into their crisp ones. In this regard, we also need to determine a minimal acceptable possibility level, β which denotes the minimum acceptable possibility level of occurrence for the corresponding imprecise/fuzzy data. Then the equivalent auxiliary crisp constraints can be represented as follows:

$$A_{(t_y, t_s, z)}(x) \leq w_1 D^p_{(t_y, t_s, z), \beta} + w_2 D^m_{(t_y, t_s, z), \beta} + w_3 D^o_{(t_y, t_s, z), \beta} + p^1_{(t_y, t_s, z)}; \quad \forall t_y,\, t_s,\, z \tag{35}$$

$$A_{(t_y, t_s, z)}(x) \geq w_1 D^p_{(t_y, t_s, z), \beta} + w_2 D^m_{(t_y, t_s, z), \beta} + w_3 D^o_{(t_y, t_s, z), \beta} - p^1_{(t_y, t_s, z)}; \quad \forall t_y,\, t_s,\, z \tag{36}$$

$$B_{(t_y, z)}(x) \geq w_1 D^p_{peak, \beta}(z,\, t_y) + w_2 D^m_{peak, \beta}(z,\, t_y) + w_3 D^o_{peak, \beta}(z,\, t_y) - p^2_{(t_y, z)}; \quad \forall t_y,\, Z \tag{37}$$

$$C_{(t_y, z, i)}(x) \geq AF(z,\, t_y) \times \{w_1 D^p_{peak, \beta}(z,\, t_y) + w_2 D^m_{peak, \beta}(z,\, t_y) + w_3 D^o_{peak, \beta}(z,\, t_y)\} - p^3_{(ty, z, i)}; \quad \forall t_y, Z \tag{38}$$

where $w_1 + w_2 + w_3 = 1$ and $w_1,\, w_2$ and w_3 represent the weights of the most pessimistic, the most possible and optimistic value of the related fuzzy demands, respectively. In practice, the suitable values for these weights as well as β are usually determined subjectively by the experience and knowledge of the decision maker. Based on the concept of the most likely values proposed by Lai and Hwang [74] and considering several relevant works [77, 88, 79], we set these parameters $(w_2 = 4/6\,,\ w_1 = w_3 = 1/6 \text{ and } \beta = 0.5)$ in our numerical experiments.

4.2. Formulating the FDPP as an auxiliary crisp model

Recalling the FDPP model, regarding the objective functions (18) along with the constraints (22) up to (26), we can apply the same approaches as used in thes FAPP model.

4.3. Applying an interactive solution algorithm

In the previous section, we described how the original FAPP and FDPP models could be replaced with an equivalent crisp single objective LP model, respectively. Generally, to solve the LP models, there are different techniques in the literature among them; the fuzzy programming approaches are being increasingly applied due to their ability in determining the satisfaction degree of each objective function explicitly. Thus, the decision makers can take their final decision by choosing a preferred efficient solution according to the satisfaction degree and preference (relative importance) value of each objective function. Here, we propose an interactive solution algorithm for implementation of the proposed FHPP as follows:

Step 1. Determining appropriate triangular possibility distributions for the imprecise parameters and formulating the FAPP and FDPP models.

Step 2. Transforming the FAPP model into its equivalent single objective LP crisp model.

Step 3. Transforming the FDPP model into its equivalent single objective LP crisp model.

Step 4. Solving the above-mentioned crisp models.

To solve the single objective APP model, the Werner fuzzy programming method is used as follows:

I. Suppose that Z_U is the low bound of objective function which has been gained of the below model (APP) solving:

$$Min\ Z_U = d_1 + d_2. \tag{39}$$

S.t.

$$\sum_{t_{yb}=1}^{20} \frac{C_{PG}(t_{yb}) + C_{PGN}(t_{yb}) + C_U(t_{yb})}{(1+disc)^{(t_{yb}-1)}}$$

$$+ \sum_{t_{yb}=1}^{20} \sum_{tc_y=t_{yb}}^{20} \frac{Ccap_{PG}(t_{yb}) + Ccap_{PGN}(t_{yb}) + Ccap_{PFN}(t_{yb})}{(1+disc)^{(tc_y-1)}} - d_1 = A \tag{40}$$

$$\sum_{t_y=1}^{20} \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} W_{zi} \times PG(t_y, t_s, z, i) + d_2 = B \tag{41}$$

$$PG(t_y, t_s, z, i)$$

$$\leq \left\{ PG_{init}(z, i) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PG \exp step(z, i) \right.$$

$$\left. \times PG \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty - tyb)} \right\} (1 - PGLoss(z, i)) \quad \forall t_y, t_s, z, i \quad (42)$$

$$\sum_{ty=1}^{20} PG \exp step(z, i) \times PG \exp(t_y, z, i) \leq PG \max(z, i) \quad \forall z, i \quad (43)$$

$$PGN(t_y, t_s, z, i)$$

$$\leq \left\{ \sum_{tyb=1}^{ty} PGN \exp step(z, i) \times PGN \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PGNLoss(z, i)) \quad \forall t_y, t_s, z, i \quad (44)$$

$$PF(t_y, t_s, z, z_p) \leq \{PF_{init}(z, z_p) \times (1 - Decay)^{ty}\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p \quad (45)$$

$$F_{Max}(t_y, z, z_p)$$

$$\leq \left\{ PF_{init}(z, z_p) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PFN \exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, z, z_p \quad (46)$$

$$PFN(t_y, t_s, z, z_p)$$

$$\leq \left\{ \sum_{tyb=1}^{ty} PFN \exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p \quad (47)$$

$$A_{(t_y,t_s,z)}(x) \le w_1 D^p_{(t_y,t_s,z),\beta} + w_2 D^m_{(t_y,t_s,z),\beta} + w_3 D^o_{(t_y,t_s,z),\beta}; \quad \forall t_y, t_s, z \tag{48}$$

$$A_{(t_y,t_s,z)}(x) \ge w_1 D^p_{(t_y,t_s,z),\beta} + w_2 D^m_{(t_y,t_s,z),\beta} + w_3 D^o_{(t_y,t_s,z),\beta}; \quad \forall t_y, t_s, z \tag{49}$$

$$B_{(t_y,\, z)}(x) \ge w_1 D^p_{peak,\beta}(z,\, t_y) + w_2 D^m_{peak,\beta}(z,\, t_y)$$

$$+ w_3 D^o_{peak,\beta}(z,\, t_y); \quad \forall t_y,\, Z \tag{50}$$

$$C_{(t_y,z,i)}(x) \ge AF(z,\, t_y) \times \{w_1 D^p_{peak,\beta}(z,\, t_y) + w_2 D^m_{peak,\beta}(z,\, t_y)$$

$$+ w_3 D^o_{peak,\beta}(z,\, t_y)\}; \quad \forall t_y,\, Z \tag{51}$$

$$PG(t_y,\, t_s,\, z,\, i),\ PGN(t_y,\, t_s,\, z,\, i),\ UE(t_y,\, t_s,\, z),\ F_{Max}(z,\, z_p),$$

$$PG(t_y,\, t_s,\, z,\, i),\ PGN(t_y,\, t_s,\, z,\, i),\ UE(t_y,\, t_s,\, z),\ F_{Max}(z,\, z_p),$$

$$PF(t_y,\, t_s,\, z,\, z_p),\ PFN(t_y,\, t_s,\, z,\, z_p),\ PG\exp(t_y,\, z,\, i),$$

$$PGN\exp(t_y,\, z,\, i),\ PFN\exp(t_y,\, z,\, z_p),\ UM(z,\, t_y)$$

$$\ge 0 \quad \forall t_s,\, z,\, z_p,\, i \tag{52}$$

II. Suppose that Z_L is the low bound of objective function which has been gained of the below model (APP) solving:

$$Min\ Z_L = d_1 + d_2. \tag{53}$$

$$\sum_{t_{yb}=1}^{20} \frac{C_{PG}(t_{yb}) + C_{PGN}(t_{yb}) + C_U(t_{yb})}{(1+disc)^{(t_{yb}-1)}}$$

$$+ \sum_{t_{yb}=1}^{20} \sum_{tc_y=t_{yb}}^{20} \frac{Ccap_{PG}(t_{yb}) + Ccap_{PGN}(t_{yb}) + Ccap_{PFN}(t_{yb})}{(1+disc)^{(tc_y-1)}} - d_1 = A \tag{54}$$

$$\sum_{t_y=1}^{20} \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} W_{zi} \times PG(t_y,\, t_s,\, z,\, i) + d_2 = B \tag{55}$$

$$PG(t_y, t_s, z, i)$$

$$\leq \left\{ PG_{init}(z, i) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PG \exp step(z, i) \right.$$

$$\left. \times PG \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PGLoss(z, i)) \quad \forall t_y, t_s, z, i \tag{56}$$

$$\sum_{ty=1}^{20} PG \exp step(z, i) \times PG \exp(t_y, z, i) \leq PG \max(z, i) \quad \forall z, i \tag{57}$$

$$PGN(t_y, t_s, z, i)$$

$$\leq \left\{ \sum_{tyb=1}^{ty} PGN \exp step(z, i) \times PGN \exp(t_{yb}, z, i) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PGNLoss(z, i)) \quad \forall t_y, t_s, z, i \tag{58}$$

$$PF(t_y, t_s, z, z_p)$$

$$\leq \{PF_{init}(z, z_p) \times (1 - Decay)^{ty}\} \times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p \tag{59}$$

$$F_{Max}(t_y, z, z_p)$$

$$\leq \left\{ PF_{init}(z, z_p) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PFN \exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, z, z_p \tag{60}$$

$$PFN(t_y, t_s, z, z_p) \leq \left\{ \sum_{tyb=1}^{ty} PFN \exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty - tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p \tag{61}$$

$$A_{(t_y, t_s, z)}(x) \leq w_1 D^p_{(t_y, t_s, z), \beta} + w_2 D^m_{(t_y, t_s, z), \beta}$$

$$+ w_3 D^o_{(t_y, t_s, z), \beta} + p^1_{(t_y, t_s, z)}; \quad \forall t_y, t_s, z \tag{62}$$

$$A_{(t_y, t_s, z)}(x) \geq w_1 D^p_{(t_y, t_s, z), \beta} + w_2 D^m_{(t_y, t_s, z), \beta}$$
$$+ w_3 D^o_{(t_y, t_s, z), \beta} - p^1_{(t_y, t_s, z)}; \quad \forall t_y, t_s, z \tag{63}$$

$$B_{(t_y, z)}(x) \geq w_1 D^p_{peak, \beta}(z, t_y) + w_2 D^m_{peak, \beta}(z, t_y)$$
$$+ w_3 D^o_{peak, \beta}(z, t_y) - p^2_{(t_y, z)}; \quad \forall t_y, Z \tag{64}$$

$$C_{(t_y, z, i)}(x) \geq AF(z, t_y) \times \{w_1 D^p_{peak, \beta}(z, t_y) + w_2 D^m_{peak, \beta}(z, t_y)$$
$$+ w_3 D^o_{peak, \beta}(z, t_y)\} - p^3_{(ty, z, i)}; \quad \forall t_y, Z \tag{65}$$

$$PG(t_y, t_s, z, i), PGN(t_y, t_s, z, i), UE(t_y, t_s, z), F_{Max}(z, z_p),$$
$$PF(t_y, t_s, z, z_p), PFN(t_y, t_s, z, z_p), PG \exp(t_y, z, i),$$
$$PGN \exp(t_y, z, i), PFN \exp(t_y, z, z_p), UM(z, t_y)$$
$$\geq 0 \quad \forall t_s, z, z_p, i \tag{66}$$

III. Determine the membership function for the objective function and constraints of APP.

Membership function for the objective function is defined in Figure 8. Membership function to minimize objective function $Z = d_1 + d_2$ is displayed:

$$\mu_Z(d_1 + d_2) = \begin{cases} 1, & \text{if } d_1 + d_2 < Z_L, \\ \dfrac{Z_U - (d_1 + d_2)}{Z_U - Z_L}, & \text{if } Z_L \leq d_1 + d_2 \leq Z_U, \\ 0, & \text{if } d_1 + d_2 > Z_U. \end{cases} \tag{67}$$

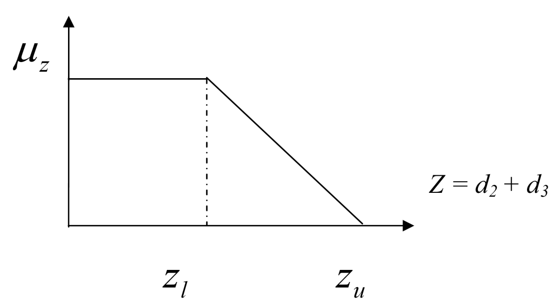

Figure 8. Membership function for minimizing objective function $Z = d_1 + d_2$.

Membership functions for the constraints (35-38) are defined as follows:

If: $w_1 D^p_{(t_y,t_s,z),\beta} + w_2 D^m_{(t_y,t_s,z),\beta} + w_3 D^o_{(t_y,t_s,z),\beta} = M$.

If: $w_1 D^p_{peak,\beta}(z,\, t_y) + w_2 D^m_{peak,\beta}(z,\, t_y) + w_3 D^o_{peak,\beta}(z,\, t_y) = N$

$$\mu^1_{D(t_y,t_s,z)}(A_{(t_y,t_s,z)}(x))$$

$$= \begin{cases} 1 & \text{if } A_{(t_y,t_s,z)}(x) < M \\ \dfrac{M + p^1_{(t_y,t_s,z)} - A_{(t_y,t_s,z)}(x)}{p^1_{(t_y,t_s,z)}} & \text{if } M \le A_{(t_y,t_s,z)}(x) \le M + p^1_{(t_y,t_s,z)} \\ 0 & \text{if } A_{(t_y,t_s,z)}(x) > M + p^1_{(t_y,t_s,z)} \end{cases} \tag{68}$$

$$\mu^2_{D(t_y,t_s,z)}(A_{(t_y,t_s,z)}(x))$$

$$= \begin{cases} 1 & \text{if } A_{(t_y,t_s,z)}(x) > M \\ \dfrac{A_{(t_y,t_s,z)}(x) - M - p^1_{(t_y,t_s,z)}}{p^1_{(t_y,t_s,z)}} & \text{if } M - p^1_{(t_y,t_s,z)} \le A_{(t_y,t_s,z)}(x) \le M \\ 0 & \text{if } A_{(t_y,t_s,z)}(x) < M - p^1_{(t_y,t_s,z)} \end{cases} \tag{69}$$

$$\mu_{Dpeak(z,t_y)}(B_{(t_y,z)}(x))$$

$$= \begin{cases} 1 & \text{if } B_{(t_y,z)}(x) > N \\ \dfrac{B_{(t_y,z)}(x) - N - p^2_{(t_y,z)}}{p^2_{(t_y,z)}} & \text{if } N - p^2_{(t_y,z)} \le B_{(t_y,z)}(x) \le N \\ 0 & \text{if } B_{(t_y,z)}(x) < N - p^2_{(t_y,z)} \end{cases} \tag{70}$$

$$\mu_{Dpeak(z,t_y)}(C_{(t_y,z,i)}(x))$$

$$= \begin{cases} 1 & \text{if } C_{(t_y,z,i)}(x) > N \\ \dfrac{C_{(t_y,z,i)}(x) - N - p^3_{(t_y,z,i)}}{p^3_{(t_y,z,i)}} & \text{if } N - p^3_{(t_y,z,i)} \le C_{(t_y,z,i)}(x) \le N \\ 0 & \text{if } C_{(t_y,z,i)}(x) < N - p^3_{(t_y,z,i)} \end{cases} \tag{71}$$

IV. Having membership functions for fuzzy constraints and objective function, the APP problem can be transformed into a crisp optimization system as follows:

$$\lambda = Min \begin{Bmatrix} \mu_Z(d_1 + d_2),\ \mu^1_{D(t_y, t_s, z)}(A_{(t_y, t_s, z)}(x)),\ \mu^2_{D(t_y, t_s, z)}(A_{(t_y, t_s, z)}(x)), \\ \mu_{Dpeak(z, t_y)}(B_{(t_y, z)}(x)), \mu_{Dpeak(z, t_y)}(C_{(t_y, z, i)}(x)) \end{Bmatrix}$$

$$Max\ \lambda \tag{72}$$

s.t.

$$\lambda \le \mu_Z(d_1 + d_2) \tag{73}$$

$$\lambda \le \mu^1_{D(t_y, t_s, z)}(A_{(t_y, t_s, z)}(x)) \quad \forall (t_y,\ t_s,\ z) \tag{74}$$

$$\lambda \le \mu^2_{D(t_y, t_s, z)}(A_{(t_y, t_s, z)}(x)) \quad \forall t_y,\ t_s,\ z \tag{75}$$

$$\lambda \le \mu_{Dpeak(z, t_y)}(B_{(t_y, z)}(x)) \quad \forall t_y,\ z \tag{76}$$

$$\lambda \le \mu_{Dpeak(z, t_y)}(C_{(t_y, z, i)}(x)) \quad \forall t_y,\ z \tag{77}$$

$$\sum_{t_{yb}=1}^{20} \frac{C_{PG}(t_{yb}) + C_{PGN}(t_{yb}) + C_U(t_{yb})}{(1 + disc)^{(t_{yb}-1)}}$$

$$+ \sum_{t_{yb}=1}^{20} \sum_{tc_y = t_{yb}}^{20} \frac{Ccap_{PG}(t_{yb}) + Ccap_{PGN}(t_{yb}) + Ccap_{PFN}(t_{yb})}{(1 + disc)^{(tc_y - 1)}} - d_1$$

$$= A \tag{78}$$

$$\sum_{t_y=1}^{20} \sum_{t_s=1}^{4} \sum_{z=1}^{15} \sum_{i=1}^{4} W_{zi} \times PG(t_y,\ t_s,\ z,\ i) + d_2 = B \tag{79}$$

$$PG(t_y,\ t_s,\ z,\ i)$$

$$\le \Bigg\{ PG_{init}(z,\ i) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PG \exp step(z,\ i) \times PG \exp(t_{yb} z,\ i)$$

$$\times (1 - Decay)^{(ty - tyb)} \Bigg\} (1 - PGLoss(z,\ i)) \quad \forall t_y,\ t_s,\ z,\ i \tag{80}$$

$$\sum_{ty=1}^{20} PG\exp step(z, i) \times PG\exp(t_y, z, i) \le PG\max(z, i) \quad \forall z, i \tag{81}$$

$$PGN(t_y, t_s, z, i)$$

$$\le \left\{ \sum_{tyb=1}^{ty} PGN\exp step(z, i) \times PGN\exp(t_{yb}, z, i) \times (1 - Decay)^{(ty-tyb)} \right\}$$

$$\times (1 - PGNLoss(z, i)) \quad \forall t_y, t_s, z, i \tag{82}$$

$$PF(t_y, t_s, z, z_p)$$

$$\le \{PF_{init}(z, z_p) \times (1 - Decay)^{ty}\} \times (1 - PFLoss(z, z_p)) \times Cos\theta \quad \forall t_y, t_s, z, z_p \tag{83}$$

$$F_{Max}(t_y, z, z_p)$$

$$\le \left\{ PF_{init}(z, z_p) \times (1 - Decay)^{ty} + \sum_{tyb=1}^{ty} PFN\exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty-tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\,\theta \quad \forall t_y, z, z_p \tag{84}$$

$$PFN(t_y, t_s, z, z_p)$$

$$\le \left\{ \sum_{tyb=1}^{ty} PFN\exp(t_{yb}, z, z_p) \times (1 - Decay)^{(ty-tyb)} \right\}$$

$$\times (1 - PFLoss(z, z_p)) \times Cos\,\theta \quad \forall t_y, t_s, z, z_p \tag{85}$$

$$PG(t_y, t_s, z, i),\ PGN(t_y, t_s, z, i),\ UE(t_y, t_s, z),\ F_{Max}(z, z_p),$$

$$PF(t_y, t_s, z, z_p),\ PFN(t_y, t_s, z, z_p),\ PG\exp(t_y, z, i),$$

$$PGN\exp(t_y, z, i),\ PFN\exp(t_y, z, z_p),\ UM(z, t_y)$$

$$\ge 0 \quad \forall t_s, z, z_p, i \tag{86}$$

$$0 \le \lambda \le 1 \tag{87}$$

To solve the single objective Disaggregate Production Planning (DPP) model the Werner fuzzy programming method is used similar to the above manner.

Step 5. To solve the above-mentioned crisp models for APP and DPP, the required parameters including the minimal acceptable level of satisfaction of soft constraints, α, the minimal acceptable possibility degree of imprecise data, β and also the tolerances of soft constraints, should be given by the decision maker. Moreover, if the decision maker is satisfied with the current efficient compromise solution, we should stop. Otherwise, we provide another efficient solution by changing the value of some controllable parameters say α and β.

5. Implementation of FHPP Model for Electricity Generation Planning in Iran

The proposed model has been implemented in Iran by using Iranian Electricity Industry Statistics, data of energy balance of Iran and Tavanir Co. using Lingo version 8 software. The output of the model is presented in Table 1 and 2. table (1) shows the production amount of aggregate methods of electricity generation i in zone z and in season $t_s = 3$ of year $t_y = 1$ $(PG(t_y, t_s, z, i))$ which is an output of FAPP model for autumn season of first year and data of this table is used as inputs for FDPP model.

Table 1. Amount of electricity generated with each aggregate method of electricity generation for autumn season of first year

Zone	Production amount of aggregate methods of electricity generation i in zone z and in autumn season $t_s = 3$ of first year $t_y = 1 : PG(t_y, t_s, z, i)$ MW			
	Steam $(i = 1)$	Gas $(i = 2)$	Combine Cycle $(i = 3)$	Hydro $(i = 4)$
1	1251.2	222.656	342.314	40.877
2	2240.2	87.0744	0.0	94.5156
3	2116.0	59.64	0.0	9.42165
4	1769.436	3865.765	2681.574	268.4921
5	662.4	1307.607	1359.456	0.0
6	1738.8	490.6384	0.0	5976.018
7	0.0	0.0	0.0	0.0

8	235.52	273.0518	0.0	0.0
9	588.8	632.184	0.0	0.0
10	0.0	1323.213	1014.594	18.69375
11	55.2	1264.368	0.0	0.0
12	220.8	119.28	132.888	87.2375
13	1619.2	0.0	427.28	0.0
14	1177.6	1033.76	0.0	0.0
15	0.0	215.698	399.644	0.0

Table 2 shows the outputs of FDPP model which present the amount of power plants production only for each month in autumn season of first year. The results indicate a very close relation between the real load trend and our model outputs. In the real planning which is made in Iran's Electricity Network, there is no relationship between long-term planning and mid-term planning, no suitable balance in Electricity zones, and high rate of electricity blackout and loss. However, taking this approach, all these shortcomings are removed and electricity production is made with the least rate of electricity blackout and loss. The electricity demand is also satisfied with the best combination of power plants.

Table 2. Amount of power plant production in each month of autumn season of the first year

steam power plants $(i = 1)$ in zone $= 1$	power plants production in each month of autumn $PG(t_m, z, i, k)$ MW		
	October $(t_m = 1)$	November $(t_m = 2)$	December $(t_m = 3)$
k			
1	143.28	516.612	0
2	0	601.3	0
Gas power plants $(i = 2)$ in zone $= 1$			
1	61.24	0	0
2	1.132	55.2	0
3	97.3	0	0

Combine cycle power plants $(i = 3)$ in zone $= 1$			
1	0	314.08	0
2	0	0	0
Hydro power plants $(i = 4)$ in zone $= 1$			
1	23.01	24.28	0
2	3.106	3.106	0
3	11.987	11.987	0
…	…	…	…
steam power plants $(i = 1)$ in zone $= 15$			
1	0	0	0
Gas power plants $(i = 2)$ in zone $= 15$			
1	95.706	0	0
2	0	0	117.15
Combine cycle power plants $(i = 3)$ in zone $= 15$			
1	412.496	412.496	412.496
Hydro power plants $(i = 4)$ in zone $= 5$			
1	0	0	0

The advantages of the addressed new approach (FHPP) are as follows:

1. Attention to the hierarchical structure aspects of power systems production planning.

2. The proposed model decreases complexity of problem using disaggregation of the problem at different levels.

3. The proposed model is a proper updating feedback system to increase reliability and developing performance of the power system production planning.

4. By applying the proposed approach, power system production planning problem will be solved in less time and with less computer memory.

5. FHPP is very consistent with changes, due to its high flexibility.

6. Weakness of using imprecise data is solved appropriately by fuzzy theory and increased reliability of model's solutions.

7. Accurate planning is done and blackout problems will be nearly solved.

8. By using AHP technique, those technologies would be chosen for electricity generation which are unpolluted and efficient, besides their economical views for electricity industry.

6. Conclusion

In this paper FHPP has been applied as a new approach for PPGP. The proposed approach converts complex electricity generation planning problem to small sub-problems which could be easily solved and need less computer memory. This approach is relatively more effective than traditional approaches which are used in Iranian power plant planning system. Besides, the feedback system increases the flexibility of the system and dynamically allows the model to be well-suited with changes. Unexpected consumer behavior makes uncertainty for demand prediction thus the outputs of demand models are not accurate. This is a very important issue for electricity generation planning. Fuzzy theory can solve this weakness appropriately and increase reliability of model’s solutions. Therefore accurate planning could be done and any shortage in electricity demand satisfaction could nearly be solved. In this research also with the combination of the AHP method and linear programming model, environmental pollution, efficiency, proportion in total capacity and power plant activity in year criteria are considered in addition to the previously considered cost based models.

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