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SUM-PRODUCT DECODING OF NON-SYSTEMATIC CONVOLUTIONAL CODES

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Abstract

This paper proposes decoding methods for non-systematic convolutional codes by using sum-product algorithm (SPA). For these codes, the conventional SPA decoding cannot provide good decoding performance. To improve the SPA decoding performance, this paper proposes two-step SPA decoding method (Proposed Method 1: PM1). The PM1 is as follows: (1) Only parity bits are decoded by SPA. (2) With decoded parity bits, information bits are regenerated. Furthermore, this paper proposes the combined use of PM1 and method with higher degree parity check equation (Proposed Method 2: PM2). The PM2 provides good performance near BCJR performance. At $\mathrm{BER} = 10^{-5}$, PM2 performance is only 0.7 [dB] inferior to BCJR performance. Complexity for PM2 is 5.2×10^{-2} times of that for BCJR algorithm.

1. Introduction

Non-systematic convolutional codes (NSCC) have been used in some communication standards. An example is the wireless local area network (wireless-

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LAN) standard [4]. Sum-product algorithm (SPA) [7] used as decoding algorithm for low-density parity-check (LDPC) codes [3], is a suitable algorithm for high speed decoders, since its operations are simple and it is feasible by parallel processing. Therefore, it is expected to implement high speed NSCC decoder by using SPA. In addition, it is expected to apply for concatenated codes [1], turbo equalization [2], [6] constructed by NSCC since SPA gives soft-input soft-output (SISO) decoding. However, SPA decoding of NSCC can not give good performance. This paper proposes performance improvement methods. This paper takes up the NSCC used in the wireless LAN standard as an example and considers on the code.

A sum-product decoding method for convolutional codes has been introduced in [5]. The SPA uses a Wiberg-type graph that represents a code trellis with hidden variables as code states and visible variables as code bits. In this case, the Wiberg-type graph is equivalent to the code trellis and SPA becomes precisely identical to BCJR algorithm [5]. This method only gives interpretation of BCJR algorithm as SPA. To avoid confusion, the method of [5] is referred to as BCJR. In this paper, SPA uses a Tanner graph that represents a parity check matrix of the code. This SPA is the same as that for low-density parity-check (LDPC) code.

This paper is constructed as follows: In Section 2, the NSCC used in the wireless LAN standard and the reason for poor SPA decoding performance of NSCC are shown. In Section 3, a performance improvement method (Proposed Method 1) is proposed based on the consideration in Section 2. In Section 4, a further improvement method (Proposed Method 2) is proposed. In Section 5, decoding complexity is discussed.

2. Conventional Sum-product Decoding

2.1. Convolutional code for wireless LAN

The convolutional code for the wireless LAN is a non-systematic code with rate $1/2$, [4]. Let a sequence of information bits be denoted by $x_0, x_1, \ldots, x_{N-1}$, a sequence of parity bits 1 be denoted by $p_{1,0}, p_{1,1}, \ldots, p_{1,N-1}$, and a sequence of parity bits 2 be denoted by $p_{2,0}, p_{2,1}, \ldots, p_{2,N-1}$. Then polynomial representation for each sequence is as follows:

$$X(D) = x_0 + x_1 D + x_2 D^2 + \cdots + x_{N-1} D^{N-1}, \tag{1}$$

$$P_1(D) = p_{1,0} + p_{1,1} D + p_{1,2} D^2 + \cdots + p_{1,N-1} D^{N-1}, \tag{2}$$

$$P_2(D) = p_{2,0} + p_{2,1} D + p_{2,2} D^2 + \cdots + p_{2,N-1} D^{N-1}. \tag{3}$$

Parity bit polynomials are given by

$$P_1(D) = G_1(D) X(D), \tag{4}$$

$$P_2(D) = G_2(D) X(D). \tag{5}$$

For the wireless LAN standard, $G_1(D)$ and $G_2(D)$ are given by

$$G_1(D) = 1 + D^2 + D^3 + D^5 + D^6, \tag{6}$$

$$G_2(D) = 1 + D + D^2 + D^3 + D^6. \tag{7}$$

In the sequel, polynomials $X(D)$, $P_1(D)$ and $P_2(D)$ are also represented by X, P_1 and P_2.

2.2. Parity check equation of convolutional code for wireless LAN

From (4)-(5), we can obtain following equations:

$$G_1(D) X + P_1 = 0, \tag{8}$$

$$G_2(D) X + P_2 = 0. \tag{9}$$

Let left parts of (8) and (9) be defined as parity check equation (PCE). Then

$$H_{org,1}(X, P_1) = G_1(D) X + P_1, \tag{10}$$

$$H_{org,2}(X, P_2) = G_2(D) X + P_2. \tag{11}$$

A tuple of polynomials (X, P_1, P_2) is a code word if following equations are satisfied:

$$H_{org,1}(X, P_1) = 0, \tag{12}$$

$$H_{org,2}(X, P_2) = 0. \tag{13}$$

The degree of a PCE is denoted by ν, that is, the maximum degree of multipliers for X, P_1 and P_2 in PCE. For example, since multipliers for X, P_1 in $H_{org,1}(X, P_1)$ are $\{G_1(D), 1\}$, the maximum degree is $\nu = 6$, that is, the maximum degree of $G_1(D)$.

2.3. Consideration

Sum-product decoding can be performed by using (10) and (11) as parity check equations. However, the decoding gives bad performance, specifically, bit error rate becomes around 0.5 at any E_b/N_0. We consider that bad performance is caused by stopping set [8] composed by information bits. The reason is as follows. It can be seen from (10), (11) that each check node has more than one information bit connections. In addition, channel values of information bits are zero, since the code under consideration is a non-systematic code. Therefore, information bits composes stopping set. Consequently, conventional SPA cannot realize good performance.

3. Proposed Method 1 (PM1)

3.1. Decoding for parity bits

This paper proposes to use parity check equation that does not include information bits. By using the parity check equation for SPA decoding, stopping set composed by information bits can be avoided.

From equations (12) and (13), we have

$$G_2(D)P_1(D) + G_1(D)P_2(D) = 0. \tag{14}$$

The left part of the equation is defined as parity check equation of PM1,

$$H'(P_1, P_2) = G_2(D)P_1 + G_1(D)P_2. \tag{15}$$

Parity bits P_1 and P_2 can be decoded by SPA based on parity check equation (15). By using the decoded parity bits, information bits can be regenerated.

3.2. Decoding for information bits

Following equation is used to decode information bit X,

$$X = G_{x,1}(D)P_1 + G_{x,2}(D)P_2. \tag{16}$$

By substituting (4) and (5) into (16), we obtain the following equation:

$$X = \{G_{x,1}(D)G_1(D) + G_{x,2}(D)G_2(D)\}X. \tag{17}$$

From (17), we obtain following equation:

$$1 = G_{x,1}(D)G_1(D) + G_{x,2}(D)G_2(D). \tag{18}$$

It is necessary to solve $G_{x,1}(D)$ and $G_{x,2}(D)$, since $G_1(D)$ and $G_2(D)$ under consideration are prime each other, a solution of $G_{x,1}(D)$ and $G_{x,2}(D)$ exists. It can be obtained by using the extended Euclidean algorithm. The solution is as follows:

$$G_{x,1}(D) = D^4 + D^2, \tag{19}$$

$$G_{x,2}(D) = D^4 + D^3 + D^2 + D + 1. \tag{20}$$

Decoded information bit $\hat{X}$ can be obtained by (16) with decoded parity bits $\hat{P}_1$ and $\hat{P}_2$. From (16), (19) and (20), it can be seen that information bit can be regenerated by using a non-recursive convolutional encoder with input $\hat{P}_1$, $\hat{P}_2$ and output $\hat{X}$ as shown in Figure 1:

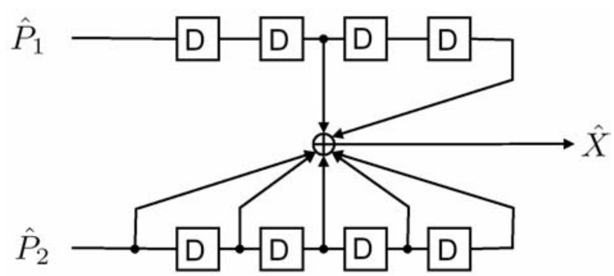

Figure 1. Information bits regenerator.

3.3. Simulation results

The simulation condition is shown in Table 1. Figure 2 shows simulation results. The figure shows that the conventional SPA provides $\text{BER} = 4 \times 10^{-1}$, and PM1 provides $\text{BER} = 6 \times 10^{-4}$ at $E_b/N_0 = 5.0$ [dB]. It can be seen that PM1 provides much better performance in comparison with the conventional SPA. The figure also shows that decoding performance for information bit is 0.2 [dB] inferior to that for parity bits at $BER = 10^{-5}$. The reason is that one information bit is regenerated from several decoded parity bits. The figure also shows that decoding performance for information bit of PM1 is 2.2 [dB] inferior to that of BCJR at $BER = 10^{-5}$. The difference shows that further performance improvement is necessary for PM1.

Table 1. Simulation condition

Number of info bits per block	1024[bit]
Termination	Zero-termination
Channel	Additive white Gaussian noise
Maximum iterations	200

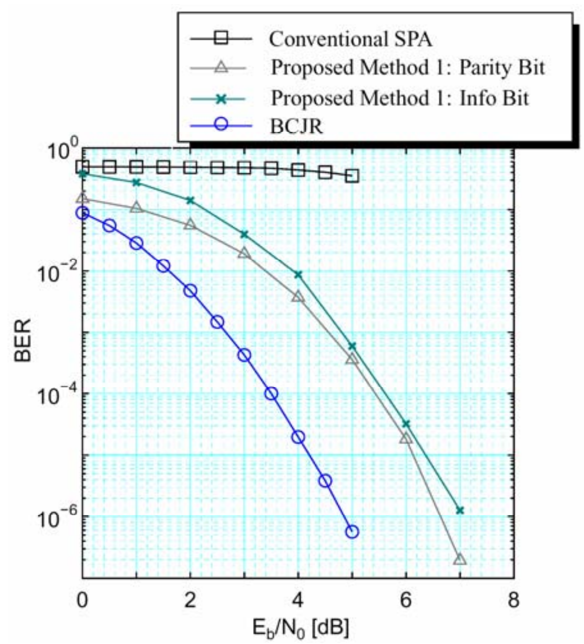

Figure 2. BER performance of Proposed Method 1.

4. Proposed Method 2 (PM2)

4.1. Decoding with higher degree parity check equation

For systematic convolutional code, a performance improvement method for SPA decoding has been proposed in [9]. The method is a SPA decoding with higher degree parity check equation (HDPCE) than that of the original parity check equation. In this subsection, the method is applied to improve the SPA decoding performance for parity bits. The HDPCE is denoted by $H_1''(P_1, P_2)$, that is, given by

$$H_1''(P_1, P_2) = M(D)H_1'(P_1, P_2) \tag{21}$$

$$= M(D)G_2(D)P_1 + M(D)G_1(D)P_2 \tag{22}$$

$$= G_{H,2}(D)P_1 + G_{H,1}(D)P_2, \tag{23}$$

where $M(D)$ is a non-zero polynomial. Among possible HDPCE's, we aim to select the optimum HDPCE by experiments and to use it for SPA decoding. However, the number of prospective objects becomes too much when we include all possible HDPCE's in the experimental objects. Therefore, we limit the range of degree of HDPCE. For those HDPCE's, we further limit the prospective objects by using n_{f_c}, that is the number of four-cycles (FC's) per one check node [9]. For every degree of HDPCE, we select the HDPCE that has the minimum n_{f_c} among HDPCE's of object degree and include it in the experimental objects. In this way, selected HDPCE's are shown in Table. 2. By using those HDPCE's, SPA decoding simulations were performed. The results are shown in Figure 3.

Table 2. HDPCE

ν	n_{FC}	$G_{H,2}(oct)$	$G_{H,1}(oct)$
6	29	117	155
7	24	321	267
8	52	563	731
9	17	1067	1405
10	36	3131	2417
11	11	4015	6243
12	28	13103	16111
13	13	21003	30611
14	22	45203	65011
15	17	100001	145207
16	25	221001	322207

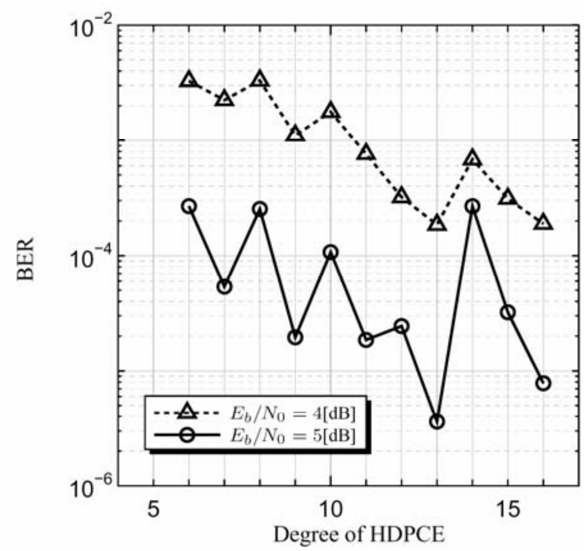

Figure 3. BER performance relating to degree of HDPCE.

From Figure 3, it can be seen that HDPCE of degree $\nu = 13$ gives the best performance. Therefore, following HDPCE is used for SPA decoding in this paper

$$H_1''(P_1, P_2) = G_{H,2}(D)P_1 + G_{H,1}(D)P_2, \tag{24}$$

$$G_{H,1}(D) = 1 + D^3 + D^7 + D^8 + D^{12} + D^{13}, \tag{25}$$

$$G_{H,2}(D) = 1 + D + D^9 + D^{13}. \tag{26}$$

By using SPA, parity bits are decoded. With the decoded parity bits, information bits are regenerated by the same method as PM1.

4.2. Simulation results

Simulation condition is shown in Table 1. Figure 4 shows simulation results. Figure 4 shows that the performance for parity bits of PM2 is 1.2 [dB] superior to that of PM1 at $\text{BER} = 10^{-5}$. The figure also shows that the performance for information bits of PM2 is 1.5 [dB] superior to that of PM1 at $\text{BER} = 10^{-5}$. This improvement is due to using the HDPCE. The figure also shows that the performance for information bits of PM2 is only 0.7 [dB] inferior to that of BCJR at $\text{BER} = 10^{-5}$.

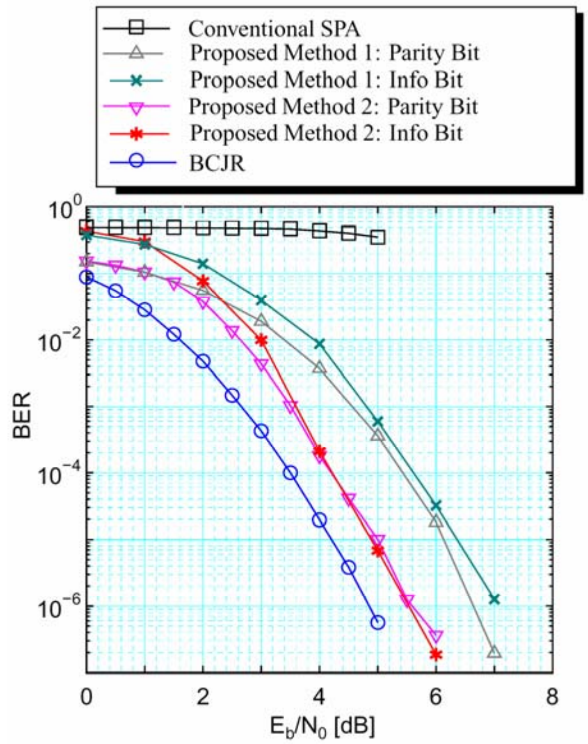

Figure 4. BER performance of Proposed Method 2.

5. Complexity

Tables 3 and 4 show the number of operations per code block. In this case, the code block is a coded sequence for information sequence with length $N = 1030$ that includes termination bits. Table 3 shows the number of operations for PM2. "Special" in the table denotes the number of operations for $\tanh(\cdot)$ and $\tanh^{-1}(\cdot)$. The number of additions for information bits shows the number of XOR's. Table 4 shows the number of operations for BCJR. "Special" in the table denotes the number of operations for $\exp(\cdot)$ and $\log(\cdot)$. Comparison between Tables 3 and 4 shows that the number of operations for PM2 is 4.0×10^{-2} times of that for BCJR.

Next, in consideration of the number of iterations for PM2, the complexity is evaluated. The average number of iterations for PM2 was 1.3 at $E_b/N_0 = 6.0$ [dB].

In consideration of that, the number of operations for PM2 is 5.2×10^{-2} times of that for BCJR, where it is necessary to notice that the SPA iteration is required for parity bits decoding only.

Table 3. Complexity of Proposed Method 2

	Additions	Multiplications	Special	Total
Parity Bit	8,240	19,570	12,360	40,170
Info Bit	1,030(XOR)	0	0	1,030
	9,270	19,570	12,360	41,200

Table 4. Complexity of BCJR

Additions	Multiplications	Special	Total
336,810	684,950	9,270	1,031,030

6. Conclusion

Sum-product algorithm (SPA) is expected to bring a high speed decoder, since decoding rule of SPA is simple and SPA is feasible for parallel processing. This paper proposes SPA decoding methods for non-systematic convolutional codes. In this paper, the non-systematic convolutional code used in the wireless LAN standard is taken up as one of examples. For the code, the conventional SPA cannot provide good decoding performance. To improve the SPA decoding performance, this paper proposes two-step SPA decoding method (Proposed Method 1: PM1). The PM1 is as follows: (1) Only parity bits are decoded by SPA. (2) With decoded parity bits, information bits are regenerated. Proposed Method 1 provides much better performance in comparison to the conventional SPA. However, the performance of PM1 is 2.2 [dB] inferior to that of BCJR. Therefore, further performance improvement method is proposed (Proposed Method 2: PM2). In Proposed Method 2, the higher degree parity check equation is used for parity bit decoding. The performance of PM2 is 1.5 [dB] superior to that of PM1 at $\text{BER} = 10^{-5}$. The performance of PM2 is only 0.7 [dB] inferior to that of BCJR at $\text{BER} = 10^{-5}$. Complexity for PM2 is 5.2×10^{-2} times of that for BCJR algorithm.

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