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USE OF ENSEMBLE KALMAN FILTER FOR BOUNDED RESERVOIR

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Abstract

The **En**semble **K**alman **F**ilter (EnKF) has been used for history matching of the reservoir models. The methodology can be used to estimate the correlation between reservoir properties (permeability, porosity) and reservoir response (pressure, flow rate). Recent studies show that the methodology is promising, however the methodology is novel to reservoir simulation, and further research of the filter is needed for practicing community. The main scope of this paper is to explore the applicability of the EnKF to estimate the reservoir properties in constant rate inner boundary no flow outer boundary reservoir models. The radial flow of the reservoir is modeled using diffusivity flow equation. The analytical solution was derived using Laplace transform, and its inverse was derived using complex residual theorem. The analytical solution is used to develop a nonlinear state space model where the analytical solution is the forward model. In practice, the analytical solution is replaced by the reservoir simulator (numerical solution of flow equation). An EnKF sequential updating is proposed to estimate the reservoir properties based on pressure observations of a well testing experiment. The case study supports the expectation that the methodology is applicable for reservoir simulation environment.

1. Introduction

Reservoir simulation aims to produce a model that represents the true reservoir system. The results can be used to predict reservoir performance. For multiphase flow, numerical methods appear the recourse for well testing. It is important issue to validate the numerical method by comparing with the analytical results. Parameter estimation in reservoir simulation, often named history matching, is to find reservoir properties which better match the production history. Integrated in an expert system, reservoir simulator has been used to determine the production strategy to optimize the oil recovery. Lorentzen et al. [4] provided the applicability of EnKF for estimation of permeability and porosity in reservoir models. The focus was on the sensitivity of the results with respect to the choice of the initial ensemble. EnKF is a promising method for solving the history matching problem (Aanonsen et al. [1]). The methodology EnKF was applied on 2D reservoir models. The EnKF is able to track the measurements and estimate the permeability, and the forecasts are improved as more observations are assimilated (Naevdal et al. [6]). EnKF was introduced as a sequential updating for near well reservoir model. A two phase flow with isotropic permeability was used. The forecasts are improved as more observations available (Naevdal et al. [7]). A finite difference reservoir simulator is used for forward the state. EnKF provides satisfactory results while requiring less computation than traditional methods (Gu and Oliver [5]). Hugen et al. [3] used an eclipse simulation model to present a successful study of the EnKF. Although the results are promising, the methodology is new for reservoir simulation, and further validations are needed for reservoir application. The main scope of this paper is to explore the applicability of the EnKF to estimate the reservoir properties in constant rate inner boundary no flow outer boundary reservoir models. The radial flow of the reservoir is modeled using diffusivity flow equation. The analytical solution was derived using Laplace transform, and its inverse was derived using complex residual theorem. The analytical solution is used to develop a nonlinear state space model. An EnKF sequential updating is proposed to estimate the reservoir properties based on pressure observations of a well testing experiment. The case study supports the expectation that the methodology is applicable in reservoir simulation environment.

2. Methodology

The diffusivity equation is considered as one most important expression in flow modeling. The model is derived under assumption that the permeability and viscosity are constant over pressure, time and distance, and the fluid is assumed slightly compressible. The parameters are: P (pressure in psi), r (radius in ft), t (time in hours), k (permeability in mD), μ (viscosity in cp), ϕ (porosity in %), c (total compressibility in psi^{-1}). The equation is defined in a cylindrical reservoir with a hole as the well. Figure 1 shows a slice of the reservoir with geometry parameters: r_w (well radius), r_e (external radius), P (bottom hole pressure), and h (thickness of the reservoir).

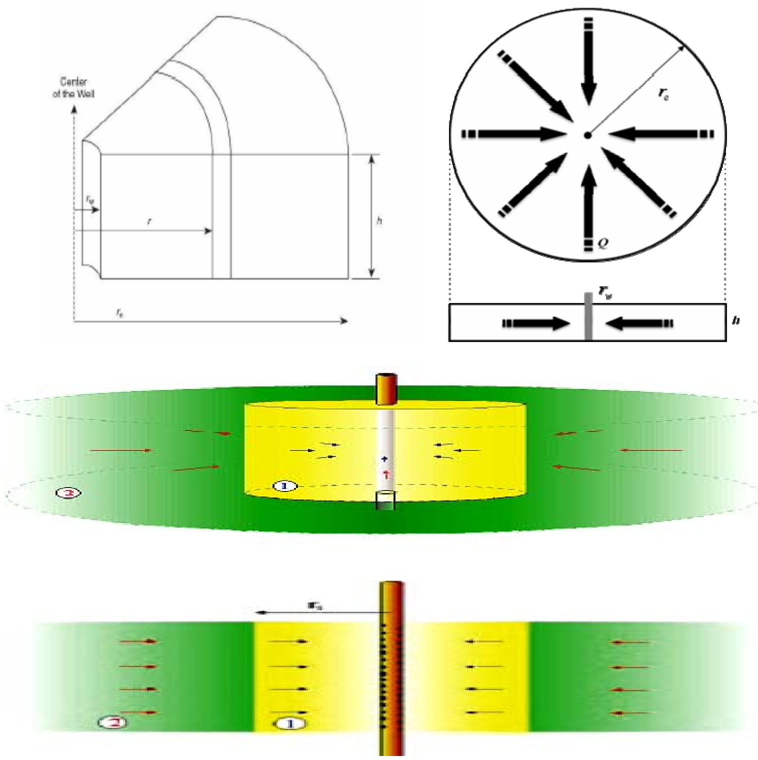


Figure 1. A slice of geometry of a bounded reservoir (Almendral-Vazquez and Syversveen [2]), r_w is the well radius, r_e is the reservoir radius, and h is the reservoir thickness. The reservoir is modeled as a cylinder with a hole at the center.

Consider a radial flow into a well bore at the center. The flow into a well will follow radial flow lines. In petroleum modeling, oil is flowing through a porous medium. The equation of it is written in terms of pressure. The pressure depends only on radius r and time t; $P(r, t)$. The equation of the flow under constant rate inner boundary and infinite outer boundary is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = \frac{\phi\mu c}{.0002637k}\frac{\partial P}{\partial t}, \quad P(r, 0) = P_i,$$

$$P(\infty, t) = P_i, \quad .001127\frac{2\pi rhk}{\mu}\frac{\partial P}{\partial r}\Big|_{r=r_w} = q.$$

The fluid moves from high pressure regions into low pressure. Using Hankel transform, the solution is given by

$$P(r,\, t) = P_i - \frac{70.61Q\mu B}{kh} Ei\left(\frac{948\phi\mu c r^2}{kt}\right),$$

$$Ei(x) = \int_x^{\infty} \frac{e^{-t}}{t}\, dt.$$

B is the oil formation volume factor (bbl/STB, well field barrels/stock tank barrels), Q is the oil flow in STB/day, $q = BQ$.

Consider a bounded reservoir model with inner and outer boundaries: constant rate inner boundary and no flow outer boundary, in dimensionless notation

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D \frac{\partial P_D}{\partial r_D}\right) = \frac{\partial P_D}{\partial t_D}, \quad P(r_D,\, 0) = 0, \quad r_D \frac{\partial P_D}{\partial r_D}\Big|_{r_D=1} = -1, \quad \frac{\partial P_D}{\partial r_D}\Big|_{r_D=r_{eD}} = 0.$$

Using Laplace transform, the solution is given by

$$P(r_w,\, t) = P_i - \frac{Q\mu}{2\pi kh}\left[2\frac{t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4} + 2\sum_{n=1}^{\infty}\frac{1}{\alpha_n^2}\left[\frac{J_1^2(\alpha_n r_{eD})}{J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)}\right]e^{-\alpha_n^2 t_D}\right]$$

with α_n is the root of

$$J_1(\alpha_n r_{eD})Y_1(\alpha_n) - J_1(\alpha n)Y_1(\alpha_n r_{eD}) = 0,$$

the dimensionless parameters are

$$r_D = r/r_w, \quad P_D = \frac{P_i - P}{P_{ch}}, \quad P_{ch} = \frac{141.2qB\mu}{kh}, \quad t_D = t/t_{ch}, \quad t_{ch} = \frac{\phi\mu c r_w^2}{.0002637k}.$$

The EnKF was introduced to handle nonlinear model such as diffusivity flow model. Given the nonlinearity of the model, the moments of the state are difficult to drive analytically. A possible solution is the Monte Carlo simulation. The EnKF is a Monte Carlo Kalman filter. The interest is in estimating the permeability based on bottom hole pressure observations. The bottom hole pressure is obtained from evaluating $P(r,\, t)$ at the well radius $r = r_w$. The state consists of two parameters: static (permeability) and dynamic (bottom hole pressure): $X = (kP(r_w,\, t))^t$, $Y = P_{obs}(r_w,\, t)$. The static (permeability) is modeled as random walk $k_0 \sim N(E(k_0),\, Var(k_0))$, $k_k = k_{k-1}$, $t_k = k\Delta t$. The analytical solution provides a model for bottom hole pressure $P_k = P(r_w,\, t_k,\, k_k) + \varepsilon_k^{obs}$. In practice, the analytical solution is replaced by a finite difference solver. Consider a nonlinear model f which propagates the state according to $X_t = f(X_{t-1}) + \varepsilon_t^m$, $\varepsilon_t^m \sim N(0, Q)$. At time t, an observation Y_t is available and is related to state X_t as $Y_t = HX_t + \varepsilon_t^{obs}$, $\varepsilon_t^{obs} \sim N(0, R)$. In the prior (prediction) step, the model is evaluated for each sample $X_{k,i}^f = f(X_{k-1,i}^a) + \varepsilon_{k,i}^m$. The prior step is repeated using time step Δt until reaching an observation. The equation for posterior (update) step $K_t = P_t^f H^t (HP_t^f H^t + R)^{-1}, Y_{t,i} = Y_t + \varepsilon_{t,i}^{obs}$, $X_{k,i}^a = X_{k,i}^f + K_t(Y_{k,i} - HX_{t,i}^f)$. The iterative process is stopped after some observation period T.

3. Results and Discussion

The focus of the results and discussion is on the applicability of EnKF methodology for the constant rate bounded no flow reservoir. Figure 2 shows the results of well testing experiment. The observation is the well pressure after the well was closed at $t_p = 13630$ hours, as expected the pressure increases as function of time. Figure 3 shows the plot of well pressure P_{WS} on $\log((t_p + t)/t)$. The regression equation is $P_{WS} = 4882 - 276 \log((13630 + t)/t)$, $R^2 = 87\%$. Using Middle Time Region (MTR) data, the regression is $P_{WS} = 4575 - 69.86 \log((13630 + t)/t)$, $R^2 = 99\%$, $k = (162.62Q\mu B)/\hat{\beta}_1 h = 7.664mD$. The value $k = 7.664mD$ is used as a true value, $k_{true} = 7.664mD$ in validating the EnKF methodology. The objective is to show that the EnKF will converge to the true permeability after a number of iterations. The experiments aims to show the EnKF may be used to recover the permeability. The experiment time is set up for $T = 72$ hours, $r_{eD} = 7520$, $\mu = 0.8$, $P_i = 5000$, $h = 69$, $k_{true} = 7.664$, $\Delta t = 0.05$, ensemble size $m = 100$, number of observations = 20. Figure 4 shows the results for the case where the ensemble mean is larger than the true permeability value. The initial ensemble is sampled from normal distribution N(11.3; 1). As expected, observations update the permeability and the simulated pressure gets closer to the observations. Figure 5 shows the results for the case where the mean ensemble is smaller the true permeability value. The evolution of permeability shows a convergence toward the true value, however the simulated pressures are not converged to the observations.

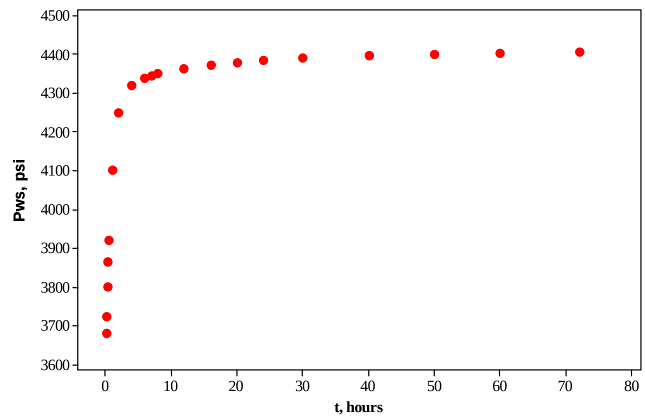


Figure 2. Pressure observations in build up test; 72 hours after $t_s = 13630$ hours production. The pressures are increases as function of time *t*. There are three time regions: Early Time Region (ETR), Middle TR (MTR), and Late TR (LTR).

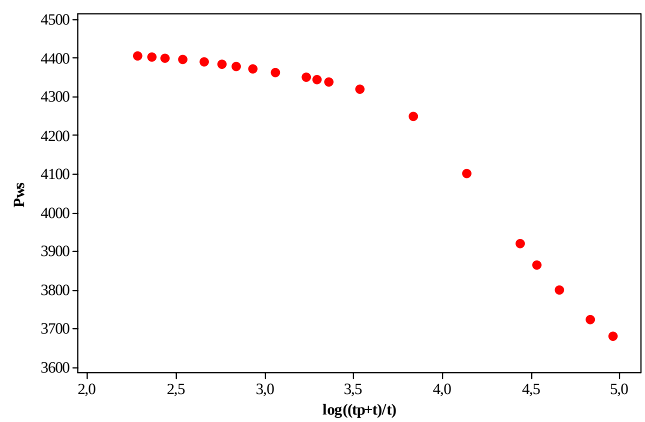
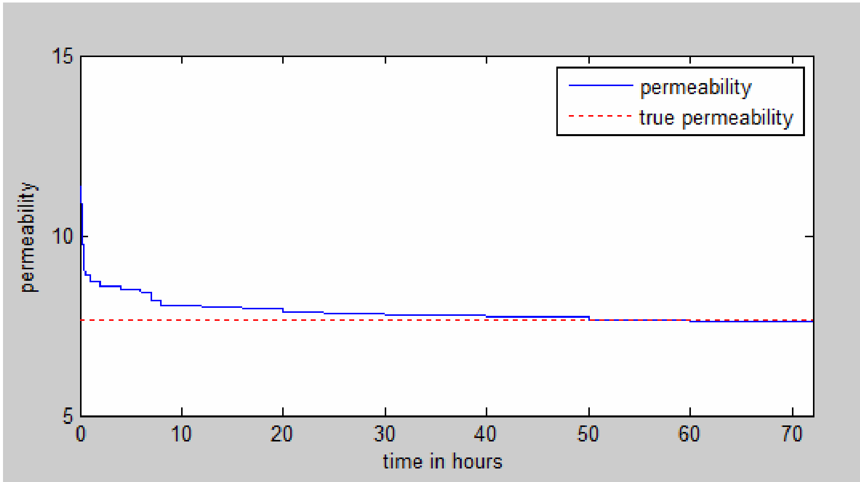
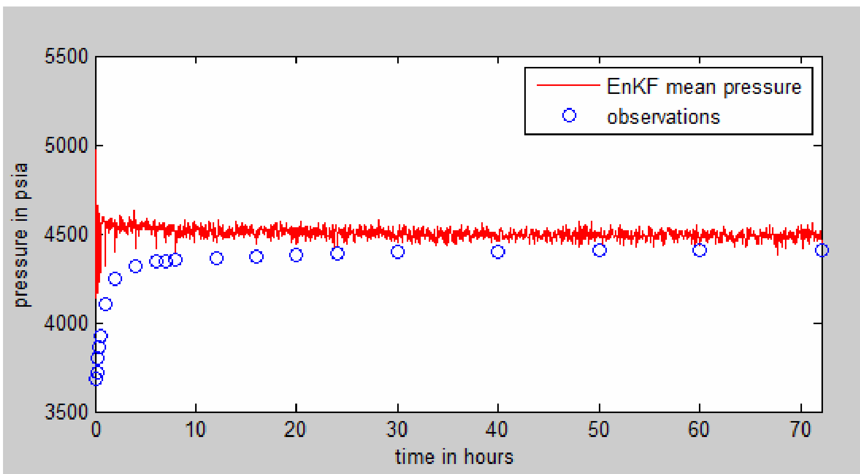


Figure 3. The scatter plot of P_{WS} on $\log((t_p + t)/t)$, $t_p = 13630$ hours. The regression equation is $P_{WS} = 4882 - 276\log((13630 + t)/t)$, $R^2 = 87\%$. Using Middle Time Region (MTR) data, the regression is, $P_{WS} = 4575 - 69.86\log((13630 + t)/t)$, $R^2 = 99\%$, $k = (162.62Q\mu B)/\hat{\beta}_1$, $h = 7.664mD$. The value $k = 7.664mD$ is used as a true value, $k_{true} = 7.664mD$ in validating the EnKF methodology.

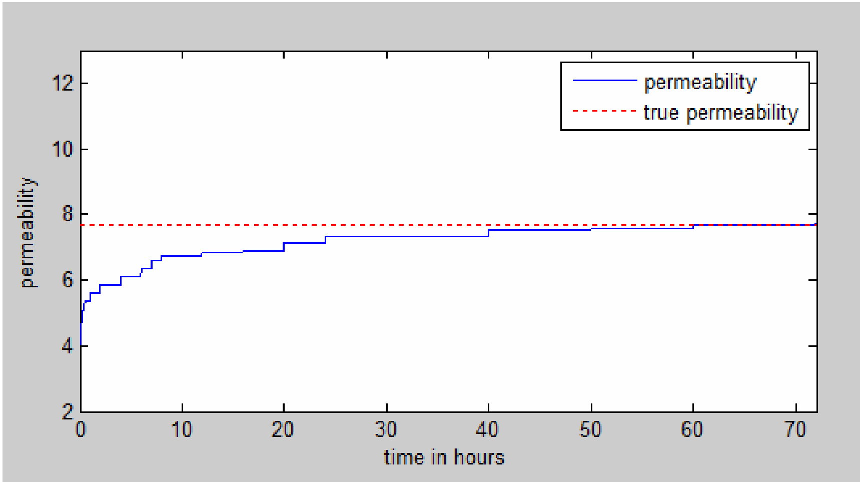


(a) The evolution of sample average of reservoir parameter permeability

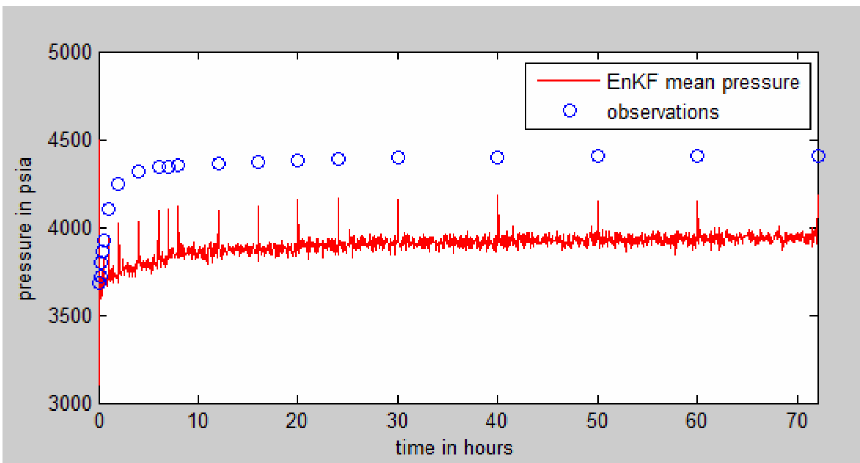


(b) The simulated and observations of P_{WS}

Figure 4. Results from experiment 1; the mean of initial permeability distribution is larger than the true permeability. The initial permeabilities was generated from N(11, 3; 1), the ensemble size is $m = 100.$ The update of permeabilities converge to $k_{true} = 7.664mD.$ The simulated P_{WS} was obtained from analytical solution with parameters $P_i = 5000$ psi, time step 0.05 hours, experiment time 72 hours, number of observations 20. The simulated pressure converge to the observed pressure.



(a) The evolution of sample average of reservoir parameter permeability



(b) The simulated and observations of P_{WS}

Figure 5. Results from experiment 2; the mean of initial permeability distribution is smaller than the true permeability. The initial permeabilities was generated from N(4; 1), the ensemble size is $m = 100$. The update of permeabilities converge to $k_{true} = 7.664mD$. The simulated P_{WS} was obtained from analytical solution with parameters $P_i = 4500$ psi, time step 0.05 hours, experiment time 72 hours, number of observations 20. The updated pressure does not converge the pressure observations.

4. Summary and Conclusions

In the present paper, the applicability of EnKF for constant rate inner boundary and no flow outer boundary reservoir model has been considered. The radial flow from reservoir to the well is modeled using diffusivity equation. The equation was solved using Laplace transform Bessel function, and Cauchy residual theorem. The methodology was successfully implemented for the case constant rate infinite reservoir. The case study shows that the methodology can be applied for the constant rate no flow bounded reservoir. The EnKF is a promising method for estimating the reservoir properties such as permeability. Using EnKF, the reservoir properties are always keep up to date.

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```
% PARAMETERS
clear all
reD = 7520.20;
mu = 0.8;
q = 250;
pi = 5000;
h = 69;
ktrue = 7.664;
a = [0.00051; 0.000935; 0.00135; 0.00177; 0.0022; 0.0026; 0.003; 0.00345;
0.00385; 0.0043; 0.0047];
time = [0.15; 0.2; 0.3; 0.4; 0.5; 1; 2; 4; 6; 7; 8; 12; 16; 20; 24; 30; 40; 50; 60; 72];
pres = [3680; 3723; 3800; 3866; 3920; 4103; 4250; 4320; 4340; 4344; 4350; 4364;
4373; 4379; 4384; 4393; 4398; 4402; 4405; 4407];
N = 100;        % Ensemble size
m = 20;         % Number of observations
ts = 0.05;      % Time steps
TT = 72;        % Experiment time
H = [0 1];
SIGMAk = 1.5;   % Standard deviation of kappa
SIGMApb = 250;% Standard deviation of pb
k = 1;
t = 0; % Initial time
F(1) = 0;
FC = 0;
% Step 1: Initialize
kappa (1, :) = normrnd(11.3, SIGMAk, N, 1); % Initial ensemble for permeability
pb (1, :) = normrnd(pi, SIGMApb, N, 1); % Initial ensemble for pressure
for i = 1:m
   FC = FC + F(i);
    F(i+1) = (time(i)*100)/(ts*100)-FC;
```

```
    G = F(i+1);
    for j = 1:G
        % Step 21: Forecast (ensemble forecast)
        t = t+ts;
        k = k+1;
        kappa(k, :) = kappa(k-1, :);
         P1 = (q*mu)./(2*pi().*kappa(k, :)*h);
        P2 = 2*t/(reD^2)+log(reD)-3/4;
        SS1 = exp(-t.*a.^2).*((besselj(1, reD.*a)).^2);
        SS2 = (a.^2).*(((besselj(1, reD.*a)).^2)-
((besselj(1, a)).^2));
        SS = SS1./SS2;
        P3 = 2*sum(SS);
        W = normrnd(0, SIGMApb, 1, N);
        pb(k, :) = pi-P1.*(P2+P3)+W;
    end
    % Step 22: Forecast (mean & covariance of the forecast ensemble)
    Xf = [kappa(k, :); pb(k, :)];
    Pef = cov(Xf');
    Ref = var(W);
    % Step 3: Analysis
    Ze = normrnd(pres(i), SIGMApb, 1, N);
    Xa = Xf+Pef*H'*(inv(H*Pef*H'+Ref))*(Ze-H*Xf);
    kappa(k, :) = Xa(1, :);
    pb(k, :) = Xa(2, :);
end
T = 0:0.05:72;
Tobs = time;
pbebar = mean(pb');
pbeSD = std(pb');
```

```
kappaeSD = std(kappa');
kappaebar = mean(kappa');
% Figure 1: EnKF mean and observations for pressure
subplot (2, 2, 1)
plot(T, pbebar,'-r',Tobs,pres,'bo')
axis([0 TT 3500 5500])
xlabel('time in hours')
ylabel('pressure in psia')
legend('EnKF mean pressure', 'observations')
% Figure 2: EnKF standard deviation for pressure
subplot(2, 2, 2)
plot(T, pbeSD, '-r')
axis([0 TT 0 600])
xlabel('time in hours')
ylabel('pressure standard deviation')
legend('EnKF std pressure')
% Figure 3: Permeability progress vs true permeability
subplot(2, 2, 3)
plot(T, kappaebar,'-b', [0 TT], [ktrue ktrue], ':r')
legend('permeability', 'true permeability')
axis([0 TT 5 15])
xlabel('time in hours')
ylabel('permeability')
% Figure 4: EnKF standard deviation for permeability
subplot(2, 2, 4)
plot(T, kappaeSD,'-r')
axis([0 TT 0 2])
xlabel('time in hours')
ylabel('permeability standard deviation')
legend('EnKF std permeability')
```