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ONE MACHINE SCHEDULING PROBLEM WITH INTERVAL NUMBERS

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Abstract

In this paper, the single machine scheduling problem with parameters given in the form of fuzzy interval numbers is considered. A possibilistic approach to scheduling is proposed. The objective is to minimize the possibility of delays of jobs, the problem is solved by using the generalized Lawer's algorithm, and the special case of this problem is also considered.

1. Introduction

In the literature on a single machine scheduling problem, jobs' processing times and due dates are fixed and certain values. In some actual cases, however, the exact

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values of parameters are not known beforehand and this uncertainty needs to be taken into account. In the recent decade, there have appeared some papers dedicated to the fuzzy scheduling which seem to be an interesting research topic. A wide review of the fuzzy scheduling problems can be found in [1], Han et al. [2], Tanaka and Vlach [3] and Ishii et al. [4] have studied the single machine problem in which the due dates of jobs are fuzzy. The fuzzy due date of a job expresses the degree of satisfaction with completion time of this job. The single machine problems with fuzzy processing times have been studied by Itoh and Ishii [5], Sung and Vlach [6], Chanas and Kaserski [7]. Ishii and Tada [8] have studied single machine scheduling problem with fuzzy precedence relation. In these papers, different criteria have been applied to calculate the optimal solution.

In the field of scheduling problem, $1 | \text{prec} | f_{\max}$ is very important. This problem can be solved in polynomial time by Lawler's algorithm. In this paper, we consider one problem which is a case of the fuzzy problem $1 | \text{prec} | f_{\max}$, where the case that all processing times and due dates are fuzzy interval numbers. In the first part of this paper, we study the problem of ranking interval numbers based on possibility degree, and propose a simple possibility degree formula for comparing two interval numbers. In the second part, we show that if the cost functions f_i are fuzzy monotone with respect to fuzzy completion time, the generalized Lawer's algorithm can be also used for the fuzzy scheduling problem. In the end, we consider the special case of the above fuzzy scheduling problem.

2. Preliminaries

In this section, we summarize some concepts required in the next part of the paper. A general definition of an interval number is as follows:

Definition 1. $\tilde{a}$ is called an *interval number*, if $\tilde{a} = [a^-, a^+] = \{x \mid a^- \leq x \leq a^+\}$, especially when $a^- = a^+$, $\tilde{a}$ degenerates to a real number.

Definition 2. An interval number $\tilde{a}$ is *non-negative*, if and only if $a^- \geq 0$.

There are two interval numbers $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$, the basic operations of interval numbers are as follows:

(i) $\tilde{a} + \tilde{b} = [a^- + b^-, a^+ + b^+]$,

(ii) $\lambda\tilde{a} = [\lambda a^-, \lambda a^+]$, $\lambda \in R^+$.

There are many different approaches for interval numbers comparison in the literature [9]. Here we use the possibility degree method for ranking interval numbers.

Definition 3. Let $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$ be two interval numbers, and l_{ab} be the length of intersection of $\tilde{a}$ and $\tilde{b}$. If $[a^-, a^+] \cap [b^-, b^+] = \phi$, then $l_{ab} = 0$, the degree of possibility that $\tilde{a} > \tilde{b}$ is defined as follows:

$$\text{poss}(\tilde{a} > \tilde{b}) = \frac{1}{2}\left(1 + \frac{(a^+ - b^+) + (a^- - b^-)}{| a^+ - b^+ | + | a^- - b^- | + l_{ab}}\right).$$

In this paper, we consider that the interval numbers are non-negative. Let $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$, $\tilde{c} = [c^-, c^+]$ are three non-negative fuzzy numbers. From the above definition, the degree of possibility satisfies the following properties:

(1) $0 \le \text{poss}(\tilde{a} > \tilde{b}) \le 1$.

(2) $\text{poss}(\tilde{a} > \tilde{b}) + \text{poss}(\tilde{b} > \tilde{a}) = 1$.

(3) $\text{poss}(\tilde{a} > \tilde{b}) \ge \frac{1}{2}$ and $b^- \le a^- \le b^+ \le a^+$, then $\text{poss}(\tilde{a} > \tilde{b}) \ge \text{poss}(\tilde{b} > \tilde{c})$.

Lemma 1. *$poss(\tilde{a} + \tilde{c} > \tilde{b}) \ge poss(\tilde{a} > \tilde{b})$, where $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ are three non-negative fuzzy numbers*.

Proof. From $\text{poss}(\tilde{a} + \tilde{c} > \tilde{a}) \ge \frac{1}{2}$ and by property (3), the proof is straightforward.

3. Main Results

In this section, we present and prove our main results. There are n jobs $J = \{J_1, J_2, \ldots, J_n\}$ to be scheduled for processing on a single machine. It is assumed that preemption of jobs is not allowed and there is a precedence constraint between jobs. If $J_i \to J_j$, $J_i, J_j \in J$, then job J_j cannot start before finishing job J_i. For each job $J_i \in J$, there are given: a processing time P_i and a due date D_i. It is assumed that the processing times and the due dates are non-negative interval numbers. Let us denote by $\pi = (\pi(1), ..., \pi(n))$ a feasible schedule, $\pi(i) \in J$, $i = 1, 2, ..., n$. Let $C_i(\pi)$, $J_i \in J$, denote the completion time of job J_i in schedule π. If $J_i = \pi(k)$, $k = 1, ..., n$, then $C_i(\pi) = \sum_{j=1}^{k} P_{\pi(j)}$.

Let us assume that for each job $J_i \in J$, we have a function $f_i : FN(R) \to R$ determining the completion cost of job J_i, i.e., $f_i(\tilde{C}_i(\pi))$ is a completion cost of J_i under a given schedule π.

Definition 4. A function $f : FN(R) \to R$ is fuzzy monotone if and only if for any $\tilde{a}, \tilde{b} \in FN(R)$, where $\tilde{b}$ is non-negative, the following relation holds: $f(\tilde{a} + \tilde{b}) \geq f(\tilde{a})$.

Now, we formulate the fuzzy scheduling problem, denoted by **FP1:**

FP1. $1 | \text{prec} | \max\{\text{poss}(\tilde{C}_i(\pi) > \tilde{D}_i)\}$.

In this problem, for a given job $J_i \in J$, the value of $\text{poss}(\tilde{C}_i(\pi) > \tilde{D}_i)$ denotes the possibility of a tardy completion, i.e., the possibility of the event that the completion time of J_i will exceed the due date $\tilde{D}_i$.

Lemma 2. *The index* $poss(\tilde{C}_i(\pi) > \tilde{D}_i)$, *treated as real valued function of* $\tilde{C}_i(\pi)$, *is F-monotone.*

Proof. The proof is straightforward. It results from Lemma 1.

We assume that $\tilde{P}_i = (p_i^-, p_i^+)$ and $\tilde{D}_i = (d_i^-, d_i^+)$, $J_i \in J$ are non-negative interval numbers. The algorithm for solving problem **FP1** can be viewed as a generalization of Lawler's algorithm to the fuzzy case.

Algorithm. 1. $S \leftarrow \{J_1, ..., J_n\}$.

2. $\tilde{T} \leftarrow \left(\sum_{J_i \in S} p_i^-, \sum_{J_i \in S} p_i^+ \right)$.

3. for $k \leftarrow n$ down to 1 do.

4. Find job $J_j \in S$ which has no successor in S and has a minimal value $\text{poss}(\tilde{T} > \tilde{D}_j)$.

5. $S \leftarrow S \backslash \{J_j\}$.

6. $\pi(k) \leftarrow J_j$.

7. Output (π).

8. End.

Theorem. *If for each job* $J_i \in J$, $i = 1, ..., n$, *function* $poss(\tilde{C}_i(\pi) > \tilde{D}_i)$ *is fuzzy monotone, then algorithm constructs an optimal sequence, and the time complexity of the algorithm is* $O(n^2)$.

Proof. Let $\pi = (\pi(1), ..., \pi(n))$ be the sequence constructed by the algorithm and $\sigma = (\sigma(1), ..., \sigma(n))$ be an optimal sequence with $\sigma(i) = \pi(i)$ for $i = n, \; n-1, ..., r$ and $\sigma(r-1) \neq \pi(r-1)$, where r is minimal. This means that there is no an optimal sequence with less value of r. Suppose that $\sigma(k) = \pi(r-1)$, where $1 \leq k \leq r-1$. It is possible to schedule $J_{\sigma(k)}$ (i.e., $J_{\pi(r-1)}$) immediately before $J_{\sigma(r)}$ because $J_{\sigma(k)}$ and $J_{\sigma(r)}$ have no successors in the set of jobs $\{J_{\sigma(1)}, ..., J_{\sigma(r-1)}\}$. We can create the feasible sequence σ' by moving $J_{\sigma(k)}$ immediately before $J_{\sigma(r)}$. We obtain the sequence σ' in which

$$\sigma'(i) = \sigma(i), \quad i = 1, ..., k-1,$$

$$\sigma'(i) = \sigma(i+1), \quad i = k, ..., r-2,$$

$$\sigma'(r-1) = \sigma(k), \quad \sigma'(i) = \sigma(i), \quad i = r, ..., n.$$

The relations between the sequences π, σ and σ' are presented as follows:

$$\pi : [\pi(1), ..., \pi(r-1), \pi(r), ..., \pi(n)],$$

$$\sigma : [\sigma(1), ..., \sigma(k-1), \sigma(k), \sigma(k+1), ..., \sigma(r-1), \sigma(r), ..., \sigma(n)],$$

$$\sigma' : [\sigma'(1), ..., \sigma'(k-1), \sigma'(k), \sigma'(k+1), ..., \sigma'(r-1), \sigma'(r), ..., \sigma'(n)]$$

$$= [\sigma(1), ..., \sigma(k-1), \sigma(k+1), ..., \sigma(r-1), \sigma(k), \sigma(r), ..., \sigma(n)].$$

It is obvious that for $i = 1, \ldots, k-1$ and $i = r, \ldots, n,$

$$\mathrm{poss}(\tilde{C}_{\sigma(i)}(\sigma) > \tilde{D}_{\sigma(i)}) = \mathrm{poss}(\tilde{C}_{\sigma'(i)}(\sigma') > \tilde{D}_{\sigma'(i)}). \tag{1}$$

Because $\sigma'(r-1) = \sigma(k) = \pi(r-1)$ and π is constructed by the algorithm, so

$$\mathrm{poss}(\tilde{C}_{\sigma(r-1)}(\sigma) > \tilde{D}_{\sigma(r-1)}) \geq \mathrm{poss}(\tilde{C}_{\sigma'(r-1)}(\sigma') > \tilde{D}_{\sigma'(r-1)}). \tag{2}$$

For each $i = k, \ldots, r-2,$ according to $\mathrm{poss}(\tilde{C}_i(\pi) > \tilde{D}_i)$ is fuzzy monotone, we obtain

$$\begin{aligned}
&\mathrm{poss}(\tilde{C}_{\sigma(i+1)}(\sigma) > \tilde{D}_{\sigma(i+1)}) \\
&= \mathrm{poss}(\tilde{T} + \tilde{P}_{\sigma(k)} + \sum\nolimits_{j=k}^{i} \tilde{P}_{\sigma'(j)} > \tilde{D}_{\sigma(i+1)}) \\
&\geq \mathrm{poss}\left(\tilde{T} + \sum\nolimits_{j=k}^{i} \tilde{P}_{\sigma'(j)} > \tilde{D}_{\sigma(i+1)}\right) \\
&= \mathrm{poss}(\tilde{C}_{\sigma'(i)}(\sigma') > \tilde{D}_{\sigma'(i)}),
\end{aligned} \tag{3}$$

where $\tilde{T} = \sum\nolimits_{i=1}^{k-1} \tilde{P}_{\sigma(i)}.$

From (1)-(3), we can conclude that the sequence σ' is not worse than $\sigma,$ thus it is also optimal. This contradicts the minimality of r.

The next problem, denoted by **FP 2**, is a special case of **FP 1**, where P_i is a real number and D_i is an interval number.

FP 2. $1 | \mathrm{prec} | \max\{\mathrm{poss}(C_i(\pi) > \tilde{D}_i)\}.$

By Definition 3, we get

$$\mathrm{poss}(C_i(\pi) > \tilde{D}_i) = \frac{1}{2}\left(1 + \frac{(C_i - d_i^+) + (C_i - d_i^-)}{|C_i - d_i^+| + |C_i - d_i^-|}\right)$$

$$= \begin{cases} 1, & C_i > d_i^+, \\ \dfrac{C_i - d_i^-}{d_i^+ - d_i^-}, & d_i^- < C_i \leq d_i^+, \\ 0, & C_i \leq d_i^- \cdot \end{cases}$$

It is interesting that the objective is similarly the function of degree of dissatisfaction, i.e., the problem FP2 is equivalent to that of minimizing the maximum degree of dissatisfaction in [3]. Obviously, $\mathrm{poss}(C_i(\pi) < \tilde{D}_i)$ is the function of degree of satisfaction.

4. Conclusions

In this paper, a possibilistic approach to sequencing problem with fuzzy interval numbers is proposed. A fuzzy problem is formulated, the algorithm and the computational complexity of it are discussed. It turns out that the problem can be solved in polynomial time using the generalized Lawler's algorithm.

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