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A SIMULATION OF THE MADRIGAL n.1 OF THE BOOK III OF CARLO GESUALDO DA VENOSA

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Abstract

Some vocal forms, which are common in the Renaissance period, have been analyzed in the present paper through an algorithmic approach. More precisely we simulate the Madrigal n.1 of the Book III of Gesualdo da Venosa by means of a standard software of electronic composition. The motivation of our choice is due to the fact that Gesualdo summarized the most important harmonic techniques in order to write a Renaissance Madrigal. We have translated his criteria in a suitable mathematical language. Then we have listened the results in a virtual orchestra.

1. Madrigals and Vocal Style

It is a common opinion in the History of the Music that the *vocal style* anticipates the *instrumental style*. Before we sing a musical piece, then we play it with suitable instruments. We need to replace the voices with the instruments because of motivations of spaces or availabilities. In this passage it is originated the

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musical writing and the corresponding rules. If we look at the Western Music, then we may find many symbols and conventions which originated from this passage. Some examples can be found in the use of the *points* and of the *legato*: they transform in the instrumental style some aesthetic practices of the vocal style. It is clear that we are dealing with a complicate process which is widely described in Harmony and Composition. The reader may refer to [1, 2, 3, 6, 10, 12, 13, 14, 15] for further details. Here, we are interested to study *polyphonic* musical forms, that is, musical forms in which there are a finite number of instruments (or voices), playing (or singing) a same melody. A polyphonic musical form in which the melody is transformed by symmetries is called an *imitatio*. From the Latin the word "imitation" is correlated to mimicking. Chorals, Fugues and Canons are instrumental musical forms which are polyphonic and have a rigorous imitatio. In a certain sense this simplifies the simulation of the Chorals, Fuges and Canons, giving by default a good algorithm. If we know the symmetries which are correlated to the imitatio, then we have the algorithm and so a mathematical model of composition. Literature can be found in [10, 12, 14, 15, 16].

It seems harder the simulation of the vocal forms, because we cannot find a rigorous imitation for these. The vocal forms have both text and music. The text determines often the structure of the composition, without modifying its aesthetic nature. The reader may find details in [1, 6]. For instance, a vocal Fugue solves such a problem with a repetition of the text, which has a metric very close to the theme of the Fugue. Still we have symmetries. Now, we will recall briefly some information on the musical form which we are going to simulate in the present work.

The *Mottet* and the *Madrigal* are special Renaissance vocal forms: they have not a corresponding pure instrumental version, since the Text-Music relation is very hard to separate. A Mottet is a vocal form constructed on a liturgic text, which has not *Cantus Firmus* in choral books. Recall that the Cantus Firmus is a main theme which can be found in some classic books of ancient religious tradition. See [6] for details. Roughly speaking, a Mottet consists of elaborating successive *incipit* of voices together with verses in order to obtain an edifying atmosphere for the listener. See [6]. When the verse does not contain a religious theme, we obtain the *Madrigal*. Then a Madrigal is a musical composition, written with the same harmonic rules of a Mottet, whose Cantus Firmus does not deal with liturgic text. The Renaissance and Baroque Madrigals are different and contain many harmonic difficulties, which test new instruments and techniques. Note that a Madrigal shows a special taste for the description, i.e., they talk about nature, rain, sun, love, death. See [6, 4]. The introduction of the VI book of Madrigals of C. Monteverdi as soon as that of the I book of Madrigals of Gesualdo da Venosa are classic references. They explain the taste of those years.

From the point of view of the electronic composition, we have many examples in [1, 2, 3, 10, 12, 19] of simulation of polyphonic vocal pieces. There are many softwares which allow us to get in real-time the results of a singing voice. See [1, 2, 3, 12, 19]. In each case, the numerical treatment of the sound signal is based on a filter process, which allows to listen a virtual example of what is happening. There is wide field of research which is devoted to such a topic both in Acoustics and Theory of Signals and Fourier's analysis. See [7, 9, 11, 17, 18]. We have used cSounds for the simulation of Madrigal n.1 of the Book III of Carlo Gesualdo da Venosa. This is a well-known software in Electronic Music and details can be found in [5]. The additive synthesis has been adopted, since this is the easiest mathematical model for describing the timbre of a sound signal.

Section 2 describes the time-frequency diagrams from which we obtain the numerical data and the corresponding simulation in Section 3. We will see in Section 3 that our results can be generalized without efforts to an arbitrary Madrigal of the Renaissance and Baroque period.

2. Linear Systems and Time-frequency Diagrams

It is possible to compose by symmetries of the euclidean plane. The linear systems are involved in order to describe the imitatio among the different voices. This was studied for the Canons of the Musical Offer of J. S. Bach in [14, 16] and then generalized in [13] to the Madrigal n.1 of the Book III of Carlo Gesualdo da Venosa. Here, we will recall some essential points of [13].

Introducing a monometric orthogonal frame in the euclidean plane, we can visualize the temporal evolution of a hand which plays the ascending scale of *C* major in the central octave of the piano. Fixed a unity of time, for instance the quaver, we grade the orizzontal axis (*Duration Axis* or *duration*) with integers which are multiple of the fixed unity. On the vertical axis (*High Axis* or *height*) we grade according to the frequencies of the white tastes of the central octave of the piano. Roughly speaking, each sound is characterized by height, intensity and timbre, in particular the height is an acoustic size which is measured in Hertz. The range which competes to the white tastes of the central octave of the piano goes from 264 to

520 Hz in the sequence do = C = 264Hz, re = D = 297Hz, mi = E = 330Hz, fa = F = 352Hz, sol = G = 396Hz, la = A = 440Hz, si = B = 495Hz, do = C = 520Hz. In this easy way we may introduce a *time-frequency diagram* and have a geometric model of a common exercise of piano: the ascending scale of C major, played by a single hand in the central octave. A reflection with respect to the height axis gives another common exercise of piano: the descending and ascending scale of C major, played by a single hand in the central octave. A translation which goes up of the distance of 3 white (a third) tastes can be visualized with another common exercise of piano: the ascending scale of C major by thirds, played simultaneously by the left and the right hand in the central octave. We may proceed with many reflections and translations of the scale in the euclidean plane so we find most of the usual exercises of piano. Consider for a moment the ascending scale of C major by thirds, played simultaneously by the left and the right hand in the central octave. In mathematical language we have just translated an imitatio with 2 voices in which the melody played by right hand in the piano (2nd-voice) is transformed by a reflection in the melody played by the left hand in the piano (1st-voice) with respect to the height. Details can be found in [14].

The natural question is to see what happens when 2 or more instruments or voices are playing, describing translations in their melody. This is an intuitive idea of what is happening when we listen a Canon or a Madrigal.

The following image shows the time-frequency diagram of the voice of the Tenor which sings in the Madrigal n.1 of the Book III of Carlo Gesualdo da Venosa. See [4] for the score.

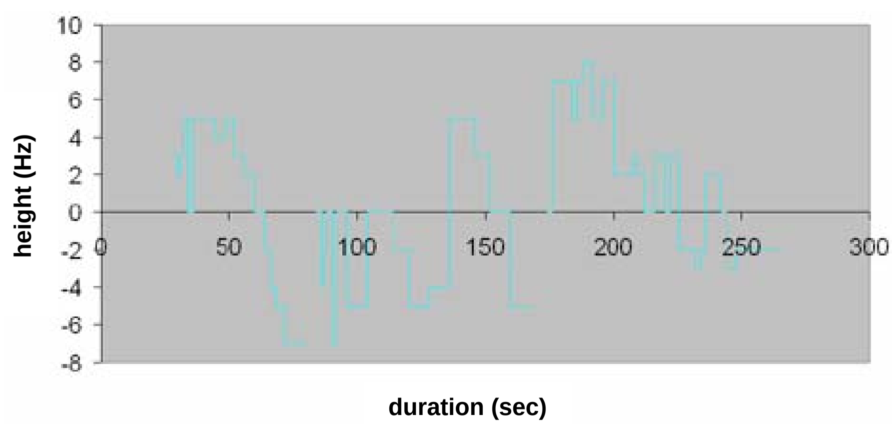

Figure 1. Madrigal n.1 Vol. III of Carlo Gesualdo Tenor (4th-voice).

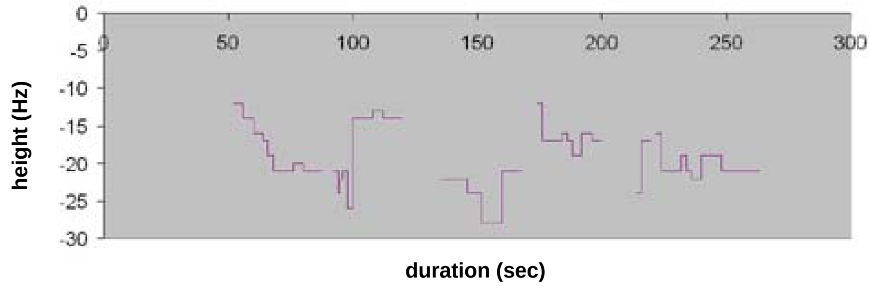

Figure 2. Madrigal n.1 Vol. III of Carlo Gesualdo Basso (5th-voice).

We can analyze the Madrigal n.1 Vol. III of Carlo Gesualdo, using the time-frequency diagrams. More details are in [2, 13]. It is useful to recall that a phrase of a Madrigal is harmonically defined by the melody which is singing between two moments of silence. It is a common practice to consider the start and the end of a piece as moments of silence.

Text of the Madrigal n.1 *Vol*. III *of Carlo Gesualdo*:

Voi volete ch'io mora,

N mi togliete ancora,

questa misera vita,

E non mi date incontr'a morte a ita.

We do not show all the diagrams of the voices of Soprano, Mezzosoprano, Contralto, Tenore and Basso, which sing this piece. They are in [2, 13]. The final result is the following.

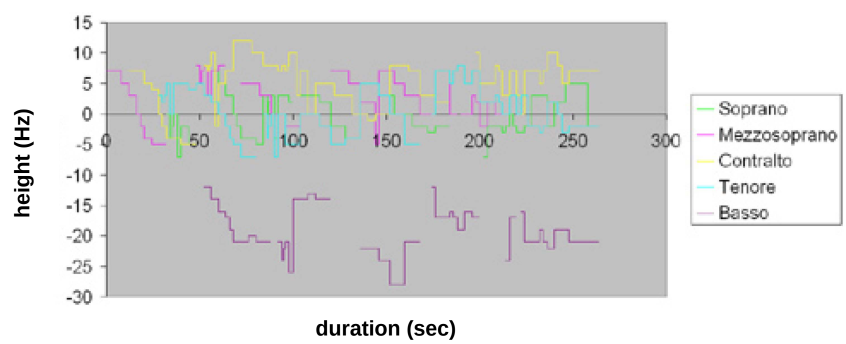

Figure 3. Madrigal n.1 Vol. III dl Gesualdo da Venosa.

We write only the linear system which governs the imitatio between the Cantus Firmus of Tenore and the Cantus Firmus of Basso. This is described by

$$\begin{cases} d_5 = d_4 + 24, \\ h_5 = h_4 - 15, \end{cases} \tag{2.1}$$

where the d_5, h_5 are the duration and height of the Cantus Firmus of Basso (5th-voice); d_4, h_4 are the duration and height of the Cantus Firmus of Tenore (4th-voice). Here Basso start after 24 semiquaver pauses in a more deep range of height with respect to Tenore. This difference is measured by 15 semipitches in the time-frequency diagram. Similar linear systems among the remaining voices can be found in [2, 13].

We recall the idea which is behind (2.1). We have a Madrigal (or a Canon or a musical piece with imitatio) with an arbitrary number of voices (n-voices for $n \geq 1$) and $1 \leq i,\ j \leq n.$ We may fix one voice as the ith-voice, then we write $n - 1$ equations similar to (2.1) in their general form:

$$\begin{cases} d_j = ad_i + b, \\ h_j = ch_i + d, \end{cases} \tag{2.2}$$

where a, b, c, d are real numbers, d_j is the duration and h_j is the height of the jth-voice. The values d_i and h_i are assigned and show the duration and height of the ith-voice. The recognition of isometries and affinities in the equation (2.2), thanks to the structural theorem on the isometries of the euclidean plane (see [14, 15]) allows us to classify harmonically the Madrigal. On the other hand, it is interesting to note that a similar approach to Madrigals gives a method to compose. The assignment of the coefficients a, b, c, d and the values d_i and h_i in (2.2) allows us to construct a polyphonic piece, resolving the associated linear system. A treatment with linear system which we have described can be solved by elementary algorithms (Gauss algorithm).

3. Orchestra and Score: Computational Data

The acoustic features of the singing voice can be found in [19], when we are going to consider it as a sound signal. This process is an acoustic approximation to the human voice. Details are in [7, 8, 9, 19]. This allows us to do the following assumptions with respect to the timbre:

$$Tenore \to Violino \ 695\,\text{Hz}, \tag{3.1}$$

$$Basso \to Contrabasso \ 98\,\text{Hz}, \tag{3.2}$$

$$Mezzosoprano \to Violino \ 294\,\text{Hz}, \tag{3.3}$$

$$Contralto \to Clarinetto \ 466\,\text{Hz}, \tag{3.4}$$

$$Soprano \to Flauto \ 392\,\text{Hz}. \tag{3.5}$$

We recall some general information of cSounds. cSounds uses 2 distinct sampling frequencies. One is audio (sr) and another is for the control signals (kr).

We have 2 files which allow us to simulate a musical piece. The first file has a .sco extension and it represents the score of the musical piece. There is a suitable syntax for such a file and further details are in [5]. The second file has a .orc extension and it represents the timbre's features of the instruments, which are playing the musical piece.

Before reading the file .sco, cSounds makes some preliminary processes in order to check the corresponding syntax. Then it reads the file .sco and generates some numerical values which are correlated to the waves forms. Then cSounds begins to read the notes. Here we need of the numerical values for the score. Equations (2.1) and (2.2) are involved in this point. Successively we have a syntactic translation which allows us to have frequencies and amplitudes and so we have the sound signals. The parameter sr/kr is correlated to the sampling of these sound signals.

We recall the parameters ksmps and nchnls, which are used by cSounds in the file .orc. These are global value assignments, made at the beginning of an orchestra, before any instrument block is defined. Their function is to set certain reserved symbol variables that are required for performance. Once set, these reserved symbols can be used in expressions anywhere in the orchestra.

pvadd reads from a phase vocoder analysis file and uses the data to perform additive synthesis using an internal array of interpolating oscillators. The user supplies the wave table (usually one period of a sine wave), and can choose which analysis bins will be used in the re-synthesis.

The oscil units output periodic control (or audio) signals consisting of the value of kamp (xamp) times the value returned from control rate (audio rate) sampling of a stored function table. The internal phase is simultaneously advanced in accordance with the cps input value. While the amplitude and frequency inputs to the k-rate oscils are scalar only, the corresponding inputs to the audio-rate oscils may each be either scalar or vector, thus permitting amplitude and frequency modulation at either sub-audio or audio frequencies.

We recall the remaining parameters for convenience of the reader.

p_1 is a unitary parameter corresponding to the attac,

p_2 is a unitary parameter corresponding to the duration,

p_3 is a unitary parameter corresponding to the release,

p_4 is a unitary parameter corresponding to the amplitude,

p_5 is a unitary parameter corresponding to the frequency.

In the next lines there are the simulation data of the voice of Tenor: first those of the timbre, then those of the score. We omit the simulation data of the voices of Basso, Contralto, Mezzosoprano and Soprano, since the method which we used is the same.

sr = 44100; kr = 44100; ksmps = 1; nchnls =2

instr 1; timbre of Tenore;

$1\,\mathrm{dB} = 10^{(1/10)};$ (increment of relative amplitude max 1000)

kenv oscili 1, 1/p3,2

iamp1 = p4*24*1.256*700; iamp2 = p4*16*1.256*700; iamp3 = p4*25*1.256*700;
iamp4 = p4*14*1.256*700; iamp5 = p4*21*1.256*700; iamp6 = p4*19*1.256*700;
iamp7 = p4*19*1.256*700; iamp8 = p4*28*1.256*700; iamp9 = p4*18*1.256*700;
iamp10 =p4*12*1.256*700; iamp11 =p4*8*1.256*700; iamp12 =p4*4*1.256*700;
iamp13=p4*2*1.256*700; iamp14 =p4*4*1.256*700; iamp15 =p4*5*1.256*700

ifrq1 = p5; ifrq2 = p5*2; ifrq3 = p5*3; ifrq4 = p5*4; ifrq5 = p5*5;
ifrq6 = p5*6; ifrq7 = p5*7; ifrq8 = p5*8; ifrq9 = p5*9; ifrq10= p5*10;
ifrq11= p5*11; ifrq12= p5*12; ifrq13= p5*13; ifrq14= p5*14; ifrq15= p5*15

```
afond
oscili iamp1, ifrq1 ,1 arm2; oscili iamp2, ifrq2 ,1 arm3;
oscili iamp3, ifrq3 ,1 arm4; oscili iamp4, ifrq4 ,1 arm5;
oscili iamp5, ifrq5,1 arm6; oscili iamp6, ifrq6 ,1 arm7;
oscili iamp7, ifrq7 ,1 arm8; oscili iamp8, ifrq8 ,1 arm9;
oscili iamp9, ifrq9 ,1 arm10; oscili iamp10, ifrq10,1 arm11;
oscili iamp11, ifrq11,1 arm12; oscili iamp12, ifrq12,1 arm13;
oscili iamp13, ifrq13,1 arm14; oscili iamp14, ifrq14,1 arm15;
oscili iamp15, ifrq15,1

additiv = (afond + arm2 + arm3 + arm4 + arm5 + arm6 + arm7 + arm8 + arm9 +
arm10 + arm11 + arm12 + arm13 + arm14 + arm15)/15

outs additiv*kenv, additiv*kenv

endin



f1 0 4096 10 1;
f2 0 4096 5  .001 300 1 3496 1 300 .001;

instr1 Tenore; p1 p2 p3 p4(db) p5(hz) (no values after the comma);


phrase 1

i1 14 1 1 523 ;3   i1 + .5 . 494 ;2   i1 + .5 . 523 ;3
i1  +  1 . 587 ;0   i1 + 1 . 440 ;5    i1 + 4 . 587 ;4
i1  +  2 . 554 ;4   i1 + 2 . 587 ;5    i1 + 2 . 523 ;3
i1  +  2 . 494 ;2   i1 + 2 . 440 ; 0   i1 + 1 . 392 ; -2
i1  +  1 . 349 ; -4 i1 + 2 . 330 ; -5  i1 + 4 . 294 ; -7;


phrase 2

i1 42 1 1 440 ; 0  i1 + .5 . 349 ; -4   i1 + .5 . 392 ; -2
i1  +  1 . 440 ; 0  i1 + 1 . 294 ; -7   i1 + 2 . 440 ; 0
i1  +  4 . 330 ; -5 i1 + 5 . 440 ; 0    i1 + 3 . 392 ; -2
i1  +  4 . 330 ; -5 i1 + 4 . 349 ; -4   i1 + 5 . 587 ; 5
i1  +  3 . 523 ; 3  i1 + 4 . 440 ; 0    i1 + 4 . 330 ; -5
```

phrase 3

i1 87 1 1 440 ; 0	i1 + 4 . 659 ; 7	i1 + 1 . 587 ; 5
i1 + 1 . 659 ; 7	i1 + 2 . 698 ; 8	i1 + 2 . 587 ; 5
i1 + 2 . 659 ; 7	i1 + 4 . 494 ;2	i1 + 1 . 523 ; 3
i1 + 1 . 494 ; 2	i1 + 2 . 440 ; 0	i1 + 2 . 523 ; 3
i1 + 1 . 440 ; 0	i1 + 2 . 523 ;3	i1 + 3 . 392 ;-2
i1 + 1 . 370 ;-3	i1 + 1 . 392 ;-2	i1 + 3 . 494 ; 2
i1 + 1 . 440 ;0	i1 + 2 . 370 ;-3	i1 + 8 . 392 ;-2;

Our approach has been focused on the Madrigal n.1 of the I Book of Madrigals of Carlo Gesualdo da Venosa, but it can be generalized to every Madrigal, once we know the scores and the timbric features of the singers.

Remark 3.1. Note that a live execution of [4] is more complicated to simulate with the previous methods. We know from [19] that the human voice changes timbre for each vocal and for each consonant. Furthermore, the body's structure of each singer contributes to variate the timbre in each singer. These characteristics cannot be translated in a mathematical model by means of a simple Fourier's analysis of the phenomenon. Actually, there are many dispersions and diffusions contributions which an additive synthesis does not consider. □

Remark 3.2. Note that the acoustics features of the hall, in which we are listening the musical piece, gives an indirect contribution on some indices, i.e., clarity and depth which modify the sound signal from the part of the listener. Such a topic is widely investigated in [7, 8, 9, 17, 18]. Our model does not consider these psyco-acoustic characteristics of the phenomenon. □

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