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ESTIMATION OF LAG LENGTH IN DISTRIBUTED LAG MODELS: A COMPARATIVE STUDY

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Abstract

There are several techniques to estimate the lag length of a dynamic regression model. However, none of them is completely satisfactory and a wrong choice may imply serious problems in the estimation of regression parameters. This paper presents a review of the main criteria for model selection used in the classical methodology. A Monte Carlo simulation study is conducted in order to compare the performance of significance tests, adjusted R^2, final prediction error, Akaike information criterion, Schwarz information criterion, Hannan-Quinn criterion and corrected Akaike information criterion.

1. Introduction

A dynamic regression model for a variable Y includes lagged values of some variables, including Y itself, as explanatory variables for the response Y_t at time t, and has a general structure for the error component. This class of models is used in economic and financial applications when it is reasonable to think that some variables observed at time t will affect the value of Y_{t+h} for $h = 0, 1, \ldots, p$. Among those models, the simplest ones impose independent and identically normally

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distributed errors and specify that only some subset of covariates has a long-term effect on the response Y at time t.

A Distributed Lag (DL) model belongs to this class, as it assumes independent and identically distributed normal errors, and does not have lagged values of the response variable among regressors. Its relevant characteristic within this class is that it assumes that for one or more values of j, the effect of covariate X_j on Y_t is given as a linear function of its value at time t and at p lagged values of t (Hill et al. [7], Greene [3], Gujarati [4]).

The DL model with p lags and just one explanatory variable X may be given by (1) with p being the lag size of X, α being the intercept and $(\beta_i)_{i=1,\ldots,p}$ the vector of regression coefficients corresponding to $(X_{t-i})_{i=1,\ldots p}$. The errors u_t in equation (1) are assumed to be iid $N(0, \sigma^2)$ variables:

$$Y_t = \alpha + \sum_{i=0}^{p} (\beta_i x_{t-i}) + u_t. \tag{1}$$

The choice of a Distributed Lag Model to describe appropriately the relationship between the variables in a data set, consists of the selection of covariates and their lags length. In classical methodology, there are several methods to accomplish this, none of them being the best in all situations. This is a serious issue since wrong choices will provide bad estimates of the effects of the regressors on the response variable.

The problem of omission of variables in the model may arise if the size p of the lags of some X is assumed to be smaller than its true value, and the consequence is that least squares estimators (LS) will be biased and inconsistent, their disturbance covariance matrix is incorrectly estimated and, consequently, confidence intervals and hypothesis-testing will give misleading results (see Judge et al. [9]). On the other hand, when an irrelevant variable is included in the model, least squares estimators of regression coefficients of the relevant variables are unbiased and consistent although they will have variance larger than when the appropriate model is fitted. The conclusion is that to include irrelevant lags in the model is preferable to the omission of important variables.

Due to the importance of making the most correct specification of the number of lags in a Distributed Lags Model for a data set, this paper shows the results of an extensive Monte Carlo simulation study carried out to compare performances of several criteria for model selection under the Classic approach: a sequence of t-testings of one coefficient at each step, or of a global F test at each step (Judge et al. [9]), the comparison of adjusted R_s^2 associated at each submodel (Judge et al. [9]), the computation and comparison of Final Prediction Errors (Akaike [1]) and Information Criteria as defined by Akaike [2], Schwarz [10], Hannan-Quinn [6] and the computation and comparison of corrected Akaike criterium [8] for the different models.

2. Criteria for Model Selection

The choice of a parsimonious model nested in a full DL model with all available regressors and maximum number of lags fixed is often made through a sequence of t-testings of null hypothesis about one coefficient at each step, corresponding to a sequence of fittings of several available sub DL models. This can be done beginning with the most simple submodel to a more general one or vice-versa.

When fitting a DL model with only one covariate in the *simple to general* way, we begin fitting a model with just this covariate without any lag, and proceeds adding its successive lags to the model sequentially, stopping the process when all regression coefficient have been tested and keeping in the model only the lags whose coefficients showed to be significantly different from zero by the t-test, with a prefixed global probability of Type I error.

In a *general to simple* process of fitting, we begin fitting the full model with p lags of the covariate. If the coefficient of the pth lag shows to be significant in the t-test, then it is kept in the model, otherwise it is deleted from the model. The next step of the process of selection is the test of the coefficient of the $(p-1)$ th lagged value of x and so successively, until all the coefficients have been tested, finishing with a model of order $p_{final} \leq p$. The same kind of sequential procedure can be used with a F-test at each stage.

A second way to choose between two regression submodels is to prefer the model that would "explain" the most part of variation observed in the response Y. The usual coefficient of determination R^2 is not adequate for this purpose since its increase may be solely due to the increase of the number of regressors. The coefficient R^2_{aju} defined in (2) is a penalized version of the usual R^2, and will be useful for comparison of models with different number of covariates, as it does not increase with the number of covariates:

$$R^2_{aju} = 1 - \frac{n-1}{n-k-1}(1-R^2) = 1 - \frac{n-1}{n-k-1}\frac{SSE}{SST} = 1 - \frac{Var_e}{Var_y}. \tag{2}$$

In (2), k is the number of covariates in the model, SSE is the sum of squares due to error, SST is the total sum of squares and $Var_e = \frac{SSE}{n-k-1}$ and $Var_y = \frac{SST}{n-1}$ are estimates of $Var(u)$ and $Var(Y)$ in (1).

The Final Prediction Error (FPE) (Akaike [1]) is an estimate of the asymptotic mean square error of prediction one-step-ahead in an autoregressive model. It can be used to choose a submodel in a class of nested autoregressive models, by finding the submodel with the minimum value of FPE. In our applications in fitting a DL model with a maximum lag length p, submodels with q lag lengths $(q < p)$ are classified according their FPE values.

For a linear regression model of Y on k regression coefficients, with n observations on Y and iid normal errors, FPE is defined by (3), where $\hat{\sigma}^2$ is the Least Squares (LS) estimate of σ^2:

$$FPE = \frac{n+k}{n-k}\hat{\sigma}^2. \tag{3}$$

Akaike [2], defined an information criterion (AIC) to choose the best model within a class of nested parametric regression models for a data set with n observations on the response variable Y and p regression coefficients $(p < n)$. It is defined by $AIC(k) = 2k - 2\log(L)$, where L denotes the maximized value of the likelihood function of a model with $k-1$ regression coefficients $(k-1 < p)$. This criterion is based on Kullback-Leibler information and in asymptotic normality and consistency of maximum likelihood estimators. AIC measures the information loss associated to each submodel relative to the full model. A choice based on minimum AIC is equivalent to choose the submodel with the minimum information loss. For a regression linear model of Y on $k-1$ regression coefficients with iid normal errors with variance σ^2, $-2\log(L)$ is equal to $n\log(\tilde{\sigma}^2) + n + n\log(2\pi)$, where $\tilde{\sigma}^2$ is the maximum likelihood estimator of σ^2 and AIC can be defined by (4), where n is the number of observations on Y:

$$AIC(k) = n\log(\tilde{\sigma}^2) + 2k + n. \tag{4}$$

Schwarz [10] under a Bayesian perspective, adopted a special class of a priori distribution for the parameters, and defined a criterion for selection of models based on the maximum value of a penalized logarithm of the likelihood of the observations. His method of model selection is very similar to Akaike's and consists in choosing the model with order k such that it minimizes SC defined by (5):

$$SC = \log(\tilde{\sigma}^2) + k\frac{\log(n)}{n}. \tag{5}$$

Hannan and Quinn [6] defined the HQ information criterion for the determination of the order of an autoregressive model. Hannan [5] generalized the criterion for a moving average autoregressive model. HQ criterion is one of several criteria that are defined as sum of the maximum value of the log likelihood plus a penalty and is given by (6). A model is chosen among other competitors if it has the minimum value of HQ:

$$HQ = \log(\tilde{\sigma}^2) + k\frac{2\log(\log(n))}{n}. \tag{6}$$

An error-corrected version of AIC (AIC_{cor}) was developed to improve AIC performance in applications in choice of models when the number of observations in Y is small, or when the ratio k/n of number of parameters to the number of observations is large. The reductions on error are done without enlarging the variance as the correction is made through the addition of a non-stochastic term to AIC. The expression of AIC_{cor} is given by (7):

$$AIC_{cor} = AIC + \frac{2(k+1)(k+2)}{n-k-2}. \tag{7}$$

An equivalent expression for AIC_{cor}, when the number of coefficients is k and the number of observations is n, is given by (8):

$$AIC_{cor} = n\log(\tilde{\sigma}^2) + n\frac{1+k/n}{1-(k+2)/n}. \tag{8}$$

3. Simulation

In this section, the results of a simulation study designed to compare the performance of the above described criteria are presented. B realizations of time series with n observations $(n = 20, 50, 100)$ on Y were generated from models given by equation (1). The data generating process involved choice of p, α, $(\beta_i)_{i=1,\ldots,p}$ and simulations of a sample of n observations on the covariate X and on the error term in (1). The covariate X was supposed to have a $N(20, 20)$ distribution and the errors $u_t's$ distribution was $N(0, 10)$. For each series size, $B = 999$ independent realizations of (1) with the given parameters were generated with a choice of a different random seed for each simulation. The models simulated are described below, from equation (9) to equation (11).

Distributed Lags model with $p = 3$ lags of covariate X:

$$Y_t = 10 + 32x_t + 16x_{t-1} + 24x_{t-2} + 10x_{t-3} + u_t. \tag{9}$$

Distributed Lags model with $p = 6$ lags of covariate X:

$$Y_t = 17 + x_t + 3x_{t-1} + 6x_{t-2} + 5x_{t-3} + 7x_{t-4} + 7x_{t-5} + 9x_{t-6} + u_t. \tag{10}$$

Distributed Lags model with $p = 10$ lags of covariate X:

$$\begin{aligned} Y_t = {} & 23 + 70x_t + 30x_{t-1} + 58x_{t-2} + 35x_{t-3} + 48x_{t-4} + 70x_{t-5} \\ & + 42x_{t-6} + 28x_{t-7} + 51x_{t-8} + 19x_{t-9} + 12x_{t-10} + u_t. \end{aligned} \tag{11}$$

In all simulated series, the estimation of a DL model was made by Ordinary Least Squares. A comparison of performance criteria for the model order selection is based on the number of correct choices made with each criterion and on the observed mean value of each criterion. The mean value, ϕ, of each criterion is calculated using the expression

$$\phi = \frac{\sum_{i=1}^{B} \hat{\phi}_i}{B}. \tag{12}$$

In expression (12), $\hat{\phi}_i$ is the observed value of one of the criteria: R^2_{aju}, FPE, AIC, SC, HQ, AIC_{cor} or p-values of t-test for each time series realization.

The estimates of the probabilities of each method making a correct choice of the order p of the model, superestimating the order p of the model, or subestimating the order p of the model, are given below:

Estimate of the probability of a right choice of

$$p = \frac{\text{frequency of the event } \{\hat{p} = p\}}{B}.$$

Estimate of the probability of superestimation of

$$p = \frac{\text{frequency of the event } \{\hat{p} > p\}}{B}.$$

Estimate of the probability of underestimation of

$$p = \frac{\text{frequency of the event } \{\hat{p} < p\}}{B}.$$

In the definitions given above, p is the number of lags of X in each model, $\hat{p}$ its estimate, B is the total number of realizations of each model. In all comparisons, p was assumed to be greater than or equal to 3. For this reason, the probability of underestimation of a $DL(3)$ will be zero, for all criteria.

4. Results

The estimated probabilities of a correct, under and superestimation of the order of a $DL(p)$ model by each criteria, are depicted in Figures 1 to 3. Figure 1(a) corresponds to a $DL(3)$ model. It shows that the percentual frequency of making the right choice of the order of the model increases with the size n of the simulated series. The criteria AIC_{cor}, AIC and R^2_{aju} had the worst performance and Figure 1(b) shows that the percentual frequency of making a superestimation of the model order decreases as size n of the simulated series increases.

Figure 2 shows that, for the simulated Dl(6) series, the most efficient methods for estimation of the lag length are FPE, HQ and AIC_{cor} which have a good performance for the three chosen values of n. AIC and SC superestimate the lag length, for $n = 20$. However, for longer series, the estimate of the probability of a right choice of p is reasonable (near 0,55) for AIC and quite good for SC.

For the simulated series of *DL*(10), Figure 3 shows that R^2_{aju} and FPE present a high percentual frequency of finding the correct value of *p*, the lag length, for all values of *n*. The other criteria also have this behaviour for series size larger than 20. For $n = 20$, those criteria superestimate the true lag length.

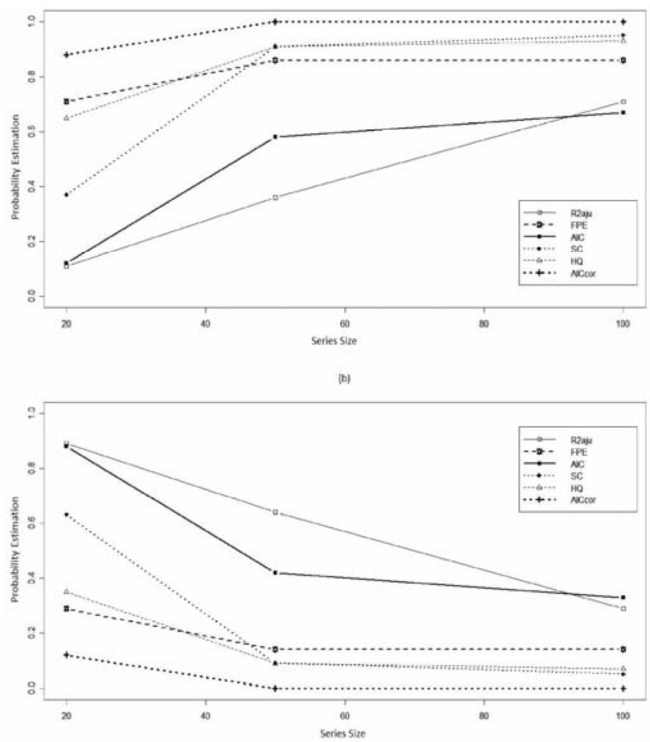

Figure 1. Performance of several criteria to detect the lag length *p* in simulated *DL*(3) series. Observed percentage of: (a) Correct estimation of lag length, $\hat{p} = p$. (b) Superestimation of lag length, $\hat{p} > p$.

Tables from 1 to 9, in Appendix, present the observed mean values of the criteria under study. Tables from 10 to 12, in Appendix, show the estimated lag lengths for each method when the mean value of each criterion (shown in Tables from 1 to 9) are used as a new criterion. When using SC, AIC, HQ, FPE and AIC_{cor}, the lag length corresponding to the least mean is chosen. On the other hand, when using R^2_{aju}, the lag length corresponding to the greatest mean is chosen.

From the tables, one verifies that the smaller means of criteria HQ, FPE and AIC_{cor} are related to the choice of the true model, for any number of lags in it and any size of the simulated series.

As what concerns to *t*-testings in finding the adequate model, at a significance level of 0.05, when using the *general to simple* approach, one must choose the model with the mean of the *p*-values less than 0.05, and, conversely, when using the *simple to general* approach, one must choose the model with the mean of the *p*-values greater than 0.05.

The estimated lag lengths from the means of the *p*-values of the *t*-testing procedure are in Table 13 in Appendix. For the Distributed Lag models, the choice of the lag length by using the *general to simple t*-testing was adequate for any size of simulated series. However, if *t*-testing is used in a *simple to general* way, then there is more chance to obtain a correct estimate of the lag length if the size n of the simulated series is large.

5. Final Comments

The analysis of the simulated time series data allowed the authors to observe important characteristics of some model selection criteria when used to estimate the order of a Distributed Lags model.

SC and AIC are the most popular criteria to choose a submodel fitted by Ordinary Least Squares within a rich class of regression models, being the most used in practical analysis. In the present study, it was possible to verify that for small value of n, the series size, their performance in detecting the correct order of a DL model is worse than the performance of FPE, HQ and AIC_{cor}. FPE is the oldest among the selection techniques used in the present work and it showed the same efficacy in selecting the order of a DL model as more recent techniques.

SC and AIC showed similar behaviors, both techniques choosing the correct order of the DL model only if n is large. For small or medium size number n of observations on Y, SC and AIC overestimate the order of the model. However, the percentage of correct estimation by SC is larger than that of AIC. The AIC_{cor} criterion, as was expected, corrects the choice of AIC when the size n of the series is small. R_{aju}^2 did not show to be a good method for determination of the order of a Distributed Lag model.

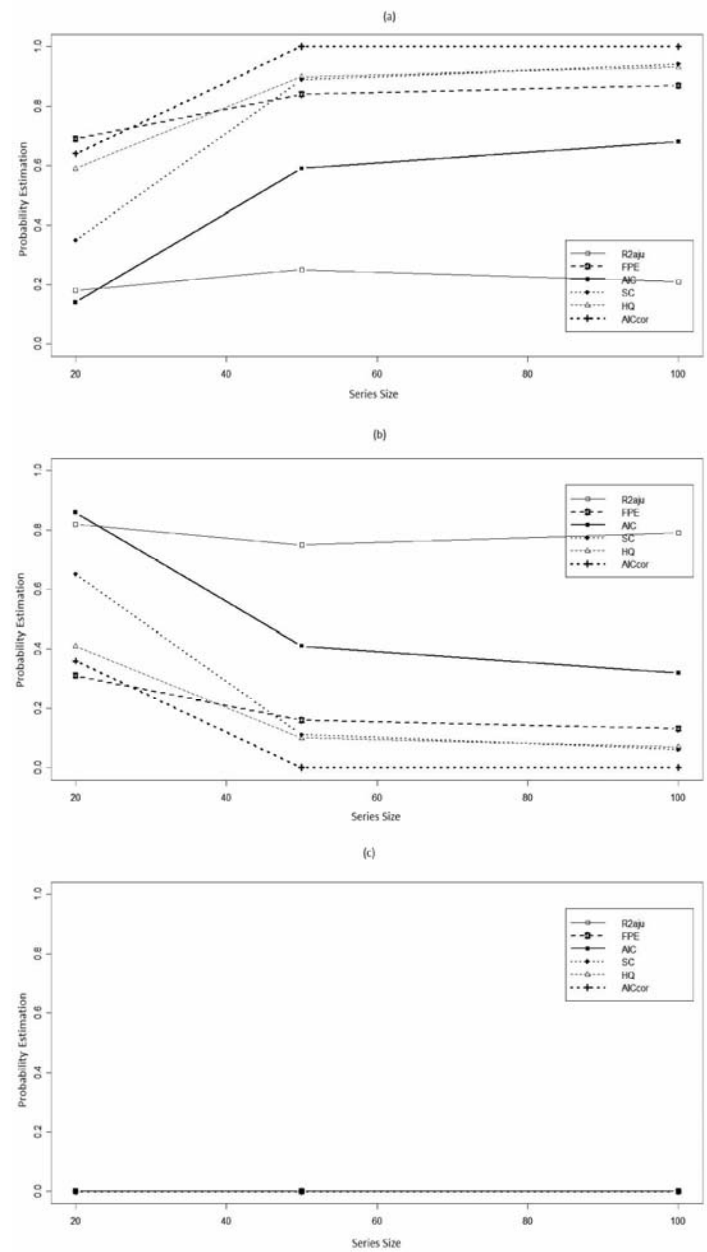

Figure 2. Performance of several criteria to detect the lag length p in simulated $DL(6)$ series. Observed percentage of: (a) Superestimation of lag length, $\hat{p} > p$; (b) Subestimation of lag length, $\hat{p} < p$; (c) Correct estimation of lag length, $\hat{p} = p$.

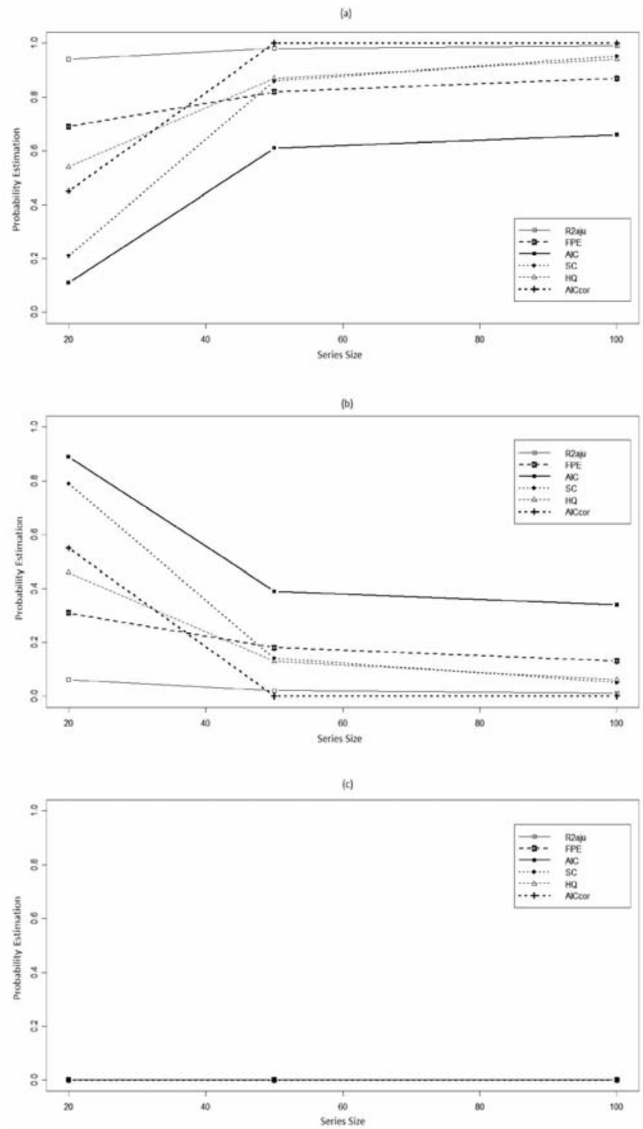

Figure 3. Performance of several criteria to detect the lag length p in simulated $DL(10)$ series. Observed percentage of: (a) Superestimation of lag length, $\hat{p} > p$; (b) Subestimation of lag length, $\hat{p} < p$; (c) Correct estimation of lag length, $\hat{p} = p$.

When using the t-testing procedure to choose the order of the model, it was observed in these simulations, that the *general to simple* approximation has a greater percentage of correct estimates than the other way, *simple to general*.

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Appendix

Tables with the observed mean values of the criteria under study and values of lag length *p* selected by some criteria

This Appendix presents the tables with the observed mean values of the criteria under study, with the estimated lag lengths for each method when the mean value of each criterion is used as a new criterion and with the estimated lag lengths from the means of the p-values of the t-testing procedure. The symbol "-" in Tables from (10) to (13) means that it was not possible to select a lag length through the means of R^2_{aju} or the t means.

Table 1. Means of some criteria when estimating p from simulations of a $DL(3)$ with $n = 20$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	110,66	105,68	2,78	16,72	–26,75	1,00
4	112,12	106,14	2,88	18,57	–20,48	1,00
5	113,51	106,54	2,99	20,74	–15,60	1,00
6	114,69	106,73	3,09	23,28	–11,81	1,00
7	115,76	106,79	3,19	26,26	–8,74	1,00
8	116,67	106,71	3,29	29,89	–6,18	1,00
9	117,35	106,40	3,39	34,43	–4,09	1,00
10	117,40	105,45	3,47	39,41	–2,71	1,00
11	117,34	104,39	3,56	46,39	–1,37	1,00
12	116,84	102,90	3,65	56,01	–0,29	1,00
13	115,83	100,89	3,74	70,00	0,68	1,00
14	112,99	97,06	3,78	89,44	0,50	1,00

Table 2. Means of some criteria when estimating p from simulations of a $DL(3)$ with $n = 50$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	270,35	260,79	2,56	12,26	178,13	1,00
4	273,05	261,58	2,61	12,75	180,76	1,00
5	275,79	262,40	2,66	13,28	183,58	1,00
6	278,52	263,22	2,72	13,84	186,56	1,00
7	281,20	263,99	2,77	14,43	189,68	1,00
8	283,80	264,68	2,82	15,02	192,90	1,00
9	286,38	265,35	2,88	15,65	196,32	1,00
10	288,99	266,04	2,93	16,34	199,99	1,00
11	291,52	266,66	2,99	17,06	203,83	1,00
12	294,08	267,31	3,04	17,83	207,96	1,00
13	296,55	267,87	3,09	18,63	212,31	1,00
14	299,05	268,45	3,15	19,51	216,98	1,00

Table 3. Means of some criteria when estimating p from simulations of a $DL(3)$ with $n = 100$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	530,82	517,79	2,44	11,04	–1270,86	1,00
4	534,35	518,71	2,47	11,27	–1095,88	1,00
5	537,88	519,64	2,50	11,50	–959,77	1,00
6	541,38	520,53	2,54	11,73	–850,91	1,00
7	544,90	521,45	2,57	11,97	–761,81	1,00
8	548,39	522,34	2,60	12,21	–687,57	1,00
9	551,90	523,24	2,63	12,47	–624,73	1,00
10	555,28	524,02	2,66	12,71	–570,98	1,00
11	558,76	524,89	2,69	12,98	–524,30	1,00
12	562,24	525,76	2,72	13,25	–483,43	1,00
13	565,61	526,53	2,75	13,52	–447,47	1,00
14	569,03	527,35	2,78	13,80	–415,44	1,00

Table 4. Means of some criteria when estimating p from simulations of a $DL(6)$ with $n = 20$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	225,27	220,29	8,51	5480,69	197,75	0,11
4	222,19	216,22	8,39	4688,99	197,93	0,27
5	216,34	209,37	8,13	3526,20	196,32	0,47
6	114,86	106,89	3,10	23,52	100,35	1,00
7	115,98	107,02	3,20	26,70	108,66	1,00
8	116,94	106,98	3,30	30,46	119,08	1,00
9	117,59	106,63	3,40	35,02	132,41	1,00
10	117,55	105,60	3,47	40,21	149,83	1,00
11	117,39	104,45	3,56	47,13	174,68	1,00
12	116,58	102,64	3,63	56,80	211,95	1,00
13	114,93	99,99	3,69	69,88	274,29	1,00
14	112,54	96,61	3,76	90,84	400,05	1,00

Table 5. Means of some criteria when estimating p from simulations of a $DL(6)$ with $n = 50$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	561,22	551,66	8,37	4225,39	468,96	0,21
4	549,69	538,22	8,14	3266,92	457,37	0,40
5	530,45	517,07	7,76	2169,84	438,22	0,60
6	278,57	263,27	2,72	13,86	186,61	1,00
7	281,27	264,06	2,77	14,45	189,74	1,00
8	283,91	264,79	2,83	15,05	193,02	1,00
9	286,53	265,50	2,88	15,69	196,47	1,00
10	289,15	266,21	2,94	16,38	200,16	1,00
11	291,66	266,80	2,99	17,09	203,98	1,00
12	294,24	267,47	3,04	17,87	208,13	1,00
13	296,70	268,02	3,10	18,67	212,45	1,00
14	299,20	268,61	3,15	19,56	217,13	1,00

Table 6. Means of some criteria when estimating p from simulations of a $DL(6)$ with $n = 100$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	1116,56	1103,53	8,30	3900,26	927,73	0,25
4	1089,80	1074,17	8,03	2923,04	899,76	0,44
5	1047,63	1029,39	7,60	1880,17	856,43	0,64
6	541,38	520,54	2,54	11,73	349,10	1,00
7	544,91	521,47	2,57	11,97	351,59	1,00
8	548,42	522,37	2,60	12,22	354,13	1,00
9	551,92	523,27	2,63	12,47	356,73	1,00
10	555,33	524,07	2,66	12,72	359,31	1,00
11	558,81	524,94	2,69	12,98	362,03	1,00
12	562,31	525,84	2,72	13,26	364,86	1,00
13	565,68	526,60	2,75	13,53	367,63	1,00
14	569,09	527,41	2,78	13,81	370,53	1,00

Table 7. Means of some criteria when estimating p from simulations of a $DL(10)$ with $n = 20$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	306,09	301,11	12,55	342942,16	278,58	0,30
4	305,42	299,45	12,55	328106,43	281,17	0,37
5	294,85	287,88	12,06	201870,39	274,83	0,64
6	290,65	282,68	11,89	165809,54	276,16	0,72
7	289,76	280,80	11,89	162027,09	282,46	0,75
8	245,88	244,92	10,20	30937,97	257,05	0,96
9	233,55	22,59	9,20	11297,37	248,38	0,99
10	117,76	105,81	3,49	40,20	150,04	1,00
11	117,75	104,81	3,58	47,54	175,04	1,00
12	117,76	103,42	3,67	57,64	212,75	1,00
13	116,09	101,16	3,75	71,88	275,46	1,00
14	113,78	97,85	3,82	94,30	401,28	1,00

Table 8. Means of some criteria when estimating p from simulations of a $DL(10)$ with $n = 50$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	770,40	760,84	12,56	288285,44	678,14	0,38
4	765,09	753,62	12,45	251049,45	672,76	0,47
5	737,34	723,95	11,89	141691,22	645,08	0,71
6	723,15	707,85	11,61	103740,21	631,15	0,79
7	716,50	699,29	11,48	88238,76	624,94	0,82
8	628,85	609,73	9,73	15128,55	537,93	0,97
9	570,73	549,70	8,56	4630,14	480,66	0,99
10	289,21	266,27	2,94	16,40	200,22	1,00
11	291,73	266,88	2,99	17,11	204,05	1,00
12	294,25	267,48	3,04	17,87	208,14	1,00
13	296,79	268,11	3,10	18,69	212,54	1,00
14	299,30	268,71	3,16	19,58	217,23	1,00

Table 9. Means of some criteria when estimating p from simulations of a $DL(10)$ with $n = 100$

Lags/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
3	1539,96	1526,94	12,54	273679,19	–261,77	0,41
4	1525,46	1509,83	12,39	231321,72	–104,81	0,51
5	1467,12	1448,89	11,80	126936,54	–30,58	0,73
6	1434,82	1413,98	11,47	89981,41	42,49	0,81
7	1417,48	1394,03	11,29	73998,78	110,74	0,85
8	1239,84	1213,79	9,51	12343,01	3,84	0,97
9	1119,45	1090,79	8,30	3634,36	–57,21	0,99
10	555,23	523,97	2,66	12,71	–571,02	1,00
11	558,71	524,85	2,69	12,97	–524,33	1,00
12	562,18	525,71	2,72	13,25	–483,47	1,00
13	565,60	526,53	2,75	13,52	–447,46	1,00
14	569,04	527,35	2,78	13,80	–415,42	1,00

Table 10. Values of lag length p selected by some criteria for a $DL(3)$ simulated model

Series size/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
$n = 20$	14	14	3	3	3	-
$n = 50$	3	3	3	3	3	-
$n = 100$	3	3	3	3	3	-

Table 11. Values of lag length p selected by some criteria for a $DL(6)$ simulated model

Series size/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
$n = 20$	14	14	6	6	6	-
$n = 50$	6	6	6	6	6	-
$n = 100$	6	6	6	6	6	-

Table 12. Values of lag length p selected by some criteria for a $DL(10)$ simulated model

Series size/Criteria	SC	AIC	HQ	FPE	AIC_{cor}	R^2_{aju}
$n = 20$	14	14	10	10	10	-
$n = 50$	10	10	10	10	10	-
$n = 100$	10	10	10	10	10	-

Table 13. Values of lag length p selected by t-testing for $DL(3)$, $DL(6)$ and $DL(10)$ simulated models

Series size	Simple to General			General to Simple		
	$DL(3)$	$DL(6)$	$DL(10)$	$DL(3)$	$DL(6)$	$DL(10)$
$n = 20$	3	-	-	3	6	10
$n = 50$	3	6	-	3	6	10
$n = 100$	3	6	10	3	6	10