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ANALYSIS OF PRODUCT WARRANTIES BASED ON TWO-SIDED ADVERSE SELECTION

WANG HAIYAN and SUN SHULEI

School of Management Engineering
Nanjing University of Finance and Economics
210046, P. R. China
e-mail: jnwangh@yahoo.com.cn

Abstract

Product warranty is an important marketing tool, which is used in screening and signaling. Based on the game of product warranty between seller and buyer, Soberman's model is improved, single-sided adverse selection when quality is observable and especially two-sided adverse selection when quality is unobservable are analyzed, and the law of mutual actions of two uses of product warranty for screening and signaling is disclosed.

1. Introduction

If two sides in a game possess pre-contractual private information at the same time, then they may encounter adverse selection that leads to inefficiency in the operation of a market and the so-called two-sided adverse selection problem occurs. With many durable products, quality cannot be evaluated by customers prior to purchase and it becomes evident only after prolonged use. When sellers know more about the quality of products than buyers do, we clearly have an adverse selection

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problem for buyers. However, sellers also lack information on the preferences of buyers that is important for contracting. A warranty is a written and/or oral seller's assurance to a buyer that a product or service is or shall be as represented. It may be considered a contractual agreement between buyer and seller that is entered into upon sale of the product or service. The contract specifies product performance, buyer responsibilities, and what the warrantor will do if an item purchased fails to meet the stated performance. A warranty may be implicit or it may be explicitly stated. When consumers do not have the time or expertise to assess the quality of products, they can make useful inferences about a product's quality from the length of its warranty. On the other hand, with heterogeneous consumers, a seller can increase profit by offering different price/warranty combinations and having buyers choose the combination that best suits them. Therefore, product warranty has two uses such as screening and signaling. For warranties to screen, Kubo [1] shows how a monopolist can increase its profits with an optional quality guarantee when consumers are heterogeneous. Matthews and Moore [2] extend this problem to a three variables situation. Tirole [3] also discusses that a firm can extract additional surplus from the consumer by including a repair warranty with the product where the firm has market power. Padmanabhan and Rao [4] show that how customer heterogeneity can arise from risk tolerances, which vary across consumers. Given this heterogeneity, sellers can increase their profits by offering a menu of price/warranty bundles. Spence [5] who finds the amount of coverage offered by sellers is a perfect signal of quality in a competitive market analyzes the use of warranties as signals. Quite simply, when warranties act as signals, a longer warranty signals a better product (Boulding and Kirmani [6]). As an important marketing tool, warranty has two uses of screening and signaling. However, in the context of two-sided adverse selection, can screening and signaling occur simultaneously? Soberman [7] discussed this problem for the first time. Based on Soberman's model, this paper improved some assumption, discussed separating and pooling equilibriums, respectively, and so disclosed the law of mutual actions of two uses of product warranty for screening and signaling.

2. The Model

The market, we consider, is one in which a buyer purchases no more than one unit of product. We further assume that the seller has a degree of price setting ability and is risk neutral. This assumption allows us to analysis on the problem of screening and signaling without incorporating risk-sharing considerations. Product

quality Q can be either premium or high $(Q = 1)$ with probability r or standard and low $(Q = 2)$ with probability $1 - r$, and buyers cannot determine quality prior to buying. Consumers differ in their valuation for warranty coverage and sellers cannot observe these valuations. We assume two different kinds of buyers: one places higher value on warranty $(\theta = \theta_1)$ with proportion v, the other places lower value on warranty $(\theta = \theta_2)$ with proportion $1 - v$. We have $\theta_1 > \theta_2$ and refer $\Delta\theta = \theta_1 - \theta_2$. We assume that the consumer's utility function is

$$U_Q(x) = \theta\left(l_Q x - \frac{x^2}{2}\right) - p, \tag{1}$$

where $Q = 1$ or 2, l_Q is the life limit determined by product quality, with $l_1 > l_2$, x is the length of the warranty, with $x \in [0, l_Q]$ and p is the price of the product with some warranty coverage x. Apparently, $U'_x > 0$, $U''_x < 0$, namely, buyers prefer length warranty coverage. $U'_l > 0$ means that consumer's utility increases with the improving of quality. In consumer's utility function of Soberman's model, warranty coverage is irrelevant with product quality, in reality, warranty coverage is closely combined with a certain product quality and they form an integral part of a market deal. The consumer's function in our model reflects this important character. Other more, it is not appropriate that the life of products with different quality is standard to one.

The seller's expect profit function is given by

$$\pi_Q = v\left(p_{Q,1} - \frac{l_Q^2}{2} - \frac{x_{Q,1}^2}{2}\right) + (1 - v)\left(p_{Q,2} - \frac{l_Q^2}{2} - \frac{x_{Q,2}^2}{2}\right), \tag{2}$$

where the first subscript in variables denotes product quality level, the second subscript denotes product consumer's type. $\frac{l_Q^2}{2}$ and $\frac{x^2}{2}$ is the cost of seller resulted, respectively, by product quality and warranty coverage. The warranty/price bundles $(x_{Q,1}, p_{Q,1})$ and $(x_{Q,2}, p_{Q,2})$ are purchased by high and low valuation buyers, respectively. The warranty length $x_{Q,2}$ can be thought of as the base warranty and $(x_{Q,1} - x_{Q,2})$ is the length of the extended warranty that can be purchased for

$(p_{Q,1} - p_{Q,2})$. In Soberman's model, the cost from product quality is ignored and only the cost from product warranty coverage is considerated. The improved seller's profit function includes two costs at the same time and adopts more reasonable quadratic cost function (Laffont and Martimort [8]).

The time of game between seller and buyer is as follows: In the first stage, seller provides product price/warranty bundles. In the second stage, consumer accepts the contract or not. In the last stage, seller and consumer realize profit and utility severally.

3. Optimal Contracts under Observable Quality

First, we consider the action that a seller would take were product quality observable. Under this condition, there is consumer's type as private information that is valuation for product warranty coverage. Therefore, the seller faces a unilateral adverse selection

$$\text{Max } \pi_Q = v\left(p_{Q,1} - \frac{l_Q^2}{2} - \frac{x_{Q,1}^2}{2}\right) + (1-v)\left(p_{Q,2} - \frac{l_Q^2}{2} - \frac{x_{Q,2}^2}{2}\right)$$

$$\text{s.t. } \theta_1\left(l_Q x_{Q,1} - \frac{x_{Q,1}^2}{2}\right) - p_{Q,1} \geq \theta_1\left(l_Q x_{Q,2} - \frac{x_{Q,2}^2}{2}\right) - p_{Q,2}, \tag{3}$$

$$\theta_2\left(l_Q x_{Q,2} - \frac{x_{Q,2}^2}{2}\right) - p_{Q,2} \geq \theta_2\left(l_Q x_{Q,1} - \frac{x_{Q,1}^2}{2}\right) - p_{Q,1}, \tag{4}$$

$$\theta_1\left(l_Q x_{Q,1} - \frac{x_{Q,1}^2}{2}\right) - p_{Q,1} \geq 0, \tag{5}$$

$$\theta_2\left(l_Q x_{Q,2} - \frac{x_{Q,2}^2}{2}\right) - p_{Q,2} \geq 0. \tag{6}$$

The objective function in this optimization problem is based on buyers selecting the appropriate bundle $(x_{Q,1}, p_{Q,1})$ or $(x_{Q,2}, p_{Q,2})$ depending their type. Equations (3) and (4) are two incentive compatibility constraints, one for each type of buyer.

Equations (5) and (6) are two individual rationality constraints that make both types realize positive surplus by purchasing.

Solving this optimization problem leads to the following results:

$$(x^*_{Q,1},\ p^*_{Q,1}) = \left(\frac{l_Q}{1+\theta_1},\ \theta_1\left(l_Q x^*_{Q,1} - \frac{x^*_{Q,1}2}{2}\right) - \Delta\theta\left(l_Q x^*_{Q,2} - \frac{x^*_{Q,2}2}{2}\right)\right), \tag{7}$$

$$(x^*_{Q,2},\ p^*_{Q,2}) = \left(\frac{(\theta_2 - v\theta_1)l_Q}{(1-v)+(\theta_2 - v\theta_1)},\ \theta_2\left(l_Q x^*_{Q,2} - \frac{x^*_{Q,2}2}{2}\right)\right), \tag{8}$$

where $x^*_{Q,2} \geq 0$ implies $\frac{\theta_2}{\theta_1} \geq v$, that means only if ratio $\frac{\theta_2}{\theta_1}$ of heterogeneous consumers' preferences is lager than the amount v of consumers with type θ_2, seller would screen consumers efficiently through positive product warranty coverage.

From the results, $x^*_{Q,1} > x^*_{Q,2} p^*_{Q,1} > p^*_{Q,2}$ can be obtained, and equations (3) and (5) are binding. Therefore, when product quality is observable, the profit maximizing action for the seller is to offer a menu of price/warranty combination where each buyer-type self selects to a bundle designed for her. Second-degree price discrimination occurs. The profit maximizing menu for the seller has:

(a) a bundle designed for the high valuation buyer with warranty protection of efficient length and a price which leaves her with strictly positive utility, and (b) a bundle for the low valuation buyer with a warranty that is shorter than the efficient length and a price which leaves her indifferent between buying or not.

4. Optimal Contracts under Unobservable Quality

When product quality is unobservable, two-sided adverse selection needs to be addressed under two situations, separating equilibrium and pooling equilibrium.

4.1. Pooling equilibrium for seller

If the types of sellers pool with together, what is certain is that the type of seller with low product quality mimic type of one with high quality. This condition specifies that expect utility of seller with low quality is higher in pooling strategies than that in separating strategies

$$v\left(\bar{p}^*_{2,1}-\frac{l_2^2}{2}-\frac{\bar{x}^*_{2,1}2}{2}\right)+(1-v)\left(\bar{p}^*_{2,2}-\frac{l_2^2}{2}-\frac{\bar{x}^*_{2,2}2}{2}\right)$$

$$\geq v\left(p^*_{2,1}-\frac{l_2^2}{2}-\frac{x^*_{2,1}2}{2}\right)+(1-v)\left(p^*_{2,2}-\frac{l_2^2}{2}-\frac{x^*_{2,2}2}{2}\right). \tag{9}$$

This optimal problem is given by

$$\text{Max } \pi_Q = v\left(\bar{p}_{Q,1}-\frac{l_Q^2}{2}-\frac{\bar{x}^2_{Q,1}}{2}\right)+(1-v)\left(\bar{p}_{Q,2}-\frac{l_Q^2}{2}-\frac{\bar{x}^2_{Q,2}}{2}\right)$$

$$\text{s.t. } \theta_1\left(\bar{l}\bar{x}_{Q,1}-\frac{\bar{x}^2_{Q,1}}{2}\right)-\bar{p}_{Q,1}\geq\theta_1\left(\bar{l}\bar{x}_{Q,2}-\frac{\bar{x}^2_{Q,2}}{2}\right)-\bar{p}_{Q,2}, \tag{10}$$

$$\theta_2\left(\bar{l}x_{Q,2}-\frac{\bar{x}^2_{Q,2}}{2}\right)-\bar{p}_{Q,2}\geq\theta_2\left(\bar{l}x_{Q,1}-\frac{\bar{x}^2_{Q,1}}{2}\right)-\bar{p}_{Q,1}, \tag{11}$$

$$\theta_1\left(\bar{l}\bar{x}_{Q,1}-\frac{\bar{x}^2_{Q,1}}{2}\right)-\bar{p}_{Q,1}\geq 0, \tag{12}$$

$$\theta_2\left(\bar{l}x_{Q,2}-\frac{\bar{x}^2_{Q,2}}{2}\right)-\bar{p}_{Q,2}\geq 0, \tag{13}$$

where $\bar{l}=rl_1+(1-r)l_2$.

Solving this programming problem, we have

$$(\bar{x}^*_{Q,1},\ \bar{p}^*_{Q,1})=\left(\frac{\bar{l}}{1+\theta_1},\ \theta_1\left(\bar{l}\bar{x}^*_{Q,1}-\frac{\bar{x}^*_{Q,1}2}{2}\right)-\Delta\theta\left(\bar{l}\bar{x}^*_{Q,2}-\frac{\bar{x}^*_{Q,2}2}{2}\right)\right), \tag{14}$$

$$(\bar{x}^*_{Q,2},\ \bar{p}^*_{Q,2})=\left(\frac{(\theta_2-v\theta_1)\bar{l}}{(1-v)+(\theta_2-v\theta_1)},\ \theta_2\left(\bar{l}\bar{x}^*_{Q,2}-\frac{\bar{x}^*_{Q,2}2}{2}\right)\right), \tag{15}$$

where $x^*_{Q,2}\geq 0$ also implies $\dfrac{\theta_2}{\theta_1}\geq v$.

When pooling equilibrium for seller occurs, it means effect of screening exceeds that of signaling for high quality seller, which has to tolerate the mimic behaviors of

low quality seller in order to efficiently screen heterogeneous consumers. All types of buyers reach a longer warranty under unobservable quality than one under observable quality. High valuation buyers realize strictly positive utility and low valuation buyers only realize the reservation utility. Compared with the conditions under observable quality, the welfare of all participants except low quality sellers in this game will not be improved but deteriorated.

4.2. Separating equilibrium for seller

If the separating equilibrium for seller occurs, then sellers with low quality type face a programming problems uniform with the situation when quality is observable, and sellers with high quality type face a programming as follows:

$$\text{Max } \pi_1 = v\left(p_{1,1} - \frac{l_1^2}{2} - \frac{x_{1,1}^2}{2}\right) + (1-v)\left(p_{1,2} - \frac{l_1^2}{2} - \frac{x_{1,2}^2}{2}\right)$$

$$\text{s.t. } \theta_1\left(l_1 x_{1,1} - \frac{x_{1,1}^2}{2}\right) - p_{1,1} \geq \theta_1\left(l_1 x_{1,2} - \frac{x_{1,2}^2}{2}\right) - p_{1,2}, \tag{16}$$

$$\theta_2\left(l_1 x_{1,2} - \frac{x_{1,2}^2}{2}\right) - p_{1,2} \geq \theta_2\left(l_1 x_{1,1} - \frac{x_{1,1}^2}{2}\right) - p_{1,1}, \tag{17}$$

$$\theta_1\left(l_1 x_{1,1} - \frac{x_{1,1}^2}{2}\right) - p_{1,1} \geq 0, \tag{18}$$

$$\theta_2\left(l_1 x_{1,2} - \frac{x_{1,2}^2}{2}\right) - p_{1,2} \geq 0, \tag{19}$$

$$v\left(p_{2,1}^* - \frac{l_2^2}{2} - \frac{x_{2,1}^* 2}{2}\right) + (1-v)\left(p_{2,2}^* - \frac{l_2^2}{2} - \frac{x_{2,2}^* 2}{2}\right)$$

$$\geq v\left(p_{1,1} - \frac{l_2^2}{2} - \frac{x_{1,1}^2}{2}\right) + (1-v)\left(p_{1,2} - \frac{l_2^2}{2} - \frac{x_{1,2}^2}{2}\right), \tag{20}$$

$$v\left(p_{1,1} - \frac{l_1^2}{2} - \frac{x_{1,1}^2}{2}\right) + (1-v)\left(p_{1,2} - \frac{l_1^2}{2} - \frac{x_{1,2}^2}{2}\right)$$

$$\geq v\left(\bar{p}_{1,1}^* - \frac{l_1^2}{2} - \frac{\bar{x}_{1,1}^* 2}{2}\right) + (1-v)\left(\bar{p}_{1,2}^* - \frac{l_1^2}{2} - \frac{\bar{x}_{1,2}^* 2}{2}\right). \tag{21}$$

From Kuhn Tucker conditions for this problem, we have

$$(\bar{\bar{x}}^*_{1,1},\ \bar{\bar{p}}^*_{1,1}) = \left(\frac{\theta_1 l_1}{1+\theta_1},\ \theta_1\left(l_1\bar{\bar{x}}^*_{1,1} - \frac{\bar{\bar{x}}^*_{1,1}2}{2}\right) - \Delta\theta\left(l_1\bar{\bar{x}}^*_{1,2} - \frac{\bar{\bar{x}}^*_{1,2}2}{2}\right)\right), \tag{22}$$

$$(\bar{\bar{x}}^*_{1,2},\ \bar{\bar{p}}^*_{1,2}) = \left(\frac{(\theta_2 - v\theta_1)l_1}{(\theta_2 - v\theta_1) - (1-v)},\ \theta_2\left(l_1\bar{\bar{x}}^*_{1,2} - \frac{\bar{\bar{x}}^*_{1,2}2}{2}\right)\right), \tag{23}$$

where $0 \le \bar{\bar{x}}^*_{1,2} \le l_1$ implies $\dfrac{\theta_2}{\theta_1} \le v$, that means which equilibrium comes forth depends on the differentiation degree of consumers and the structure of market.

From these results, when separating equilibrium for seller occurs, all types of buyers reach a shorter warranty under unobservable quality than one under observable quality. It means that effect of signaling will depress the effect of screening. The warranty coverage offered to buyers by low quality seller is equivalent to that under conditions of observable quality. The existence of low quality seller may bring influence to high quality seller in an invisible way to make them weaken screening degree to heterogeneous consumers.

5. Conclusions

The objective of this paper has been to analyze warranty policy that is used to support the sales of durable goods in markets characterized by two-sided adverse selection. In this paper, two conditions of observable quality and unobservable quality are both considered and compared with each other. Analysis of optimal warranty contract under tow-sided adverse selection problem has disclosed the law of screening and signaling. The key insight of the paper is that warranties can be used to screen and signal simultaneously and the two uses may badger and influence with each other when sellers have price setting ability.

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