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ON THE STUDY OF SYSTEMS OF RIGID BODIES

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Abstract

When considering a wide range of problems of modern engineering, it is desirable to simulate these objects as coupled rigid bodies. For example, many devices and instruments aviation, marine and space represents a set of rigid bodies. The systems of coupled rigid bodies might be: gyrostat [1, 2], gyroscopic system [3-9], manipulators [10-13], body rotating on a string or a string suspension [14, 15], wheeled crews [16, 17], etc. The models of the rigid bodies are widely used to describe the dynamics of spacecraft [18, 19].

In practice, the use of systems of rigid bodies, currently defines a number of fields in science dedicated to studying this problem, for example, the motion of a system of rigid bodies with a fixed point. Despite the difficulties that arise for the solution of differential equations that describe the mechanical motion of the object, and a large number of parameters to consider it, there are many works in the field of analytical mechanics. These topics have been studied by well-known people such as A. Bogojavlensky, J. Wittenburg, A. Ishlinsky, D. Klimov, V. Koshlyakov, F. Pfeiffer, A. Savchenko, V. Storozhenko, M. Temchenko, P. Kharlamov

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and N. Chetaev. To simplify the analysis of instruments and devices, these systems are modeled using rigid bodies. In this way researchers seek to maximize the characteristics of the object being studied, and schemes of work for solving the problem. In this sense, between the directions of development of the theory of systems of rigid bodies, we can consider different ways of describing the equations of motion of coupled rigid body systems, as well as the construction of simple solutions of these equations, also the regime of motion of the system, and the property of stability.

1. Equations of Motion of Systems of Coupled Rigid Bodies

The problem on the motion of coupled rigid body systems, even when only two bodies, is more complicated than the classical problem of motion of a body. The first task of any problem is to obtain the equations of motion of the object, and in that sense, we would think it would be appropriate to use standard methods of analytical dynamics, for example, writing the equation in the form of Lagrange equations. But often, we get equations so complex that use or manipulation is almost impossible. In this regard, many scientists suggest that the first step to obtain the equations of motion is to try to write in such a way that highlights the elements of interest for the study.

In this direction, we mention the works of Lurie [20, 21], he obtained the equations of motion of bodies, suitable for studying the movement of this system of coupled rigid bodies (SCRB), in which the movement of some bodies was given in the kinematic form.

In the works of Roberson and Wittenburg [22] and Wittenburg [23], the equations of motion of bodies, which are coupled with different types of hinges, are written in matrix form. This proposed approach to the various SCRB, uses the concepts of graph-theory.

The structural form of the equations of motion of bodies proposed by Kharlamov [24] demonstrated that, when there is a successful choice of coordinate systems and variables, and the union of bodies in groups, we can get a system of n equations of motion of coupled rigid bodies, which in some special cases can be integrated [25-28]. These equations give a real opportunity to study the motion of n Lagrange gyroscopes. In the work of Savchenko et al. [29], it used the method of Kharlamov [24] to write equations of a system of coupled rigid free and non-free bodies. The works of Bolgrabskaya [30, 31], and Bolgrabskaya and Savchenko [32] used this method to the study of a system of rigid bodies, that, these form a half-closed chain.

In problems of stabilization of the angular position of a spacecraft and control of its rotation, it is necessary to know the equation of rotational motion of spacecraft bearer, that, it has a mobile mass. These equations are written in the work of Raushenbakh and Tokar [33]. In this work, the mobile bearers are: the gyroscopic stabilizers, which we used to control the angular position of bearer.

In the work, Kharlamov and Kharlamov [34] proposed a design and develop equations for a new type of joints: nonholonomic hinges and the equations of motion by two articulated bodies were obtained in Lesina and Kharlamov [35], Mozalevskaya [36], Mozalevskaya and Kharlamova [37]. The work of Lesina [38] considers the problem of inertial motion of two bodies coupled. She writes six forms of equations of motion of this problem. In [39], Kharlamov and Kononykhin described the equations of motion in the form of the Euler-Lagrange equation, using the method of nonholonomic mechanics.

The evolution of technology has increased the possibility of finding solutions to many engineering problems. Using equations of motion to study the complex mechanical systems. The works of Burlakova et al. [40, 41] and Gorodetsky et al. [42] show programs in Lagrange functions, and the linearization of the equations of motion around the stationary conditions. The work [30, 31, 43-49] of Bolgrabskaya shows models of elastic rods.

In [50], Savchenko studied the uniform rotation of two heavy non-free Lagrange's gyroscopes in the case when the bodies are coupled by spherical hinge. The variables were selected as p_k, q_k, r_k $(k = 1 \cdots n)$, components of absolute angular velocity of the body S_k, and the components of the vertical vector γ, in a coordinate system associated with the first body.

To study the regular precession [51], Kononykhin introduced Euler's angles θ_k, ψ_k, ϕ_k, determining the position of the body in an inertial space, and the study of uniform rotation [52] Krylov's angles α_k, β_k, γ_k.

Chebanov obtained the equations of system motion of n heavy Lagrange's gyroscopes connected by spherical hinges, in which the position of some bodies relative to fixed base were defined by Euler angles, and other bodies were defined by Krylov angles [53].

In studying the system motion, when the bodies are coupled by universal or cylindrical hinges [29, 54, 55], normally the first body is selected, whose position in the fixed space is defined by Krylov or Euler angles, depending on the purpose of further study, and for the remaining bodies the position is defined by the rotation angles around hinge axes.

2. Stationary Motion of SCRB

As the stationary motions correspond to regimes of working of many of the engineering designs, then, one of the most important problems of analytical mechanics is the selection of these regimes. In [24, 56], it is proved that non-mobile SCRB allows two types of stationary motions: uniform rotation around vertical and regular precession. Non-mobile SCRB and SCRB formed a half-closed chain was studied in [24, 29, 32, 57, 58]. In [24] Kharlamov established the presence of a regular precession on a system of n heavy Lagrange's gyroscopes coupled ideal spherical hinges. It was assumed that the axis of symmetry of all the gyroscopes is in a vertical plane, and the angles rotation of all the bodies are constant, but may be different for each S_k. In [59], Gorr and Birman established some properties of precession motions, when one of the bodies is a Lagrange's gyroscope or Hess's gyroscope, and another-Griol's gyroscope.

In [60], it obtained the conditions of existence of motions of coupled asymmetric bodies. In case where one of the bodies of the system rotates uniformly around vertical, and another body performs precession around vertical. It considers a regular precession of asymmetric bodies coupled by spherical hinge.

3. Study of Stability of Stationary Motions

The next step in studying the dynamics SCRB is to study the stability of stationary regimes, which ensures reliable operation of regimes of workers of simulated objects.

In [61, 62], Temchenko found sufficient conditions for the stability of uniform rotation of a non-mobile system of two balanced Lagrange's gyroscopes. To study these conditions was used graph-analytic method.

In [63], Bolgrabskaya obtained sufficient conditions of stability of an asymmetric non-mobile system of two rigid bodies. In [52], Kononykhin obtained sufficient conditions of stability of a uniform rotation around the vertical of system of *n* heavy Lagrange's gyroscopes with fixed point. In [51], Kononykhin obtained sufficient conditions for stability of regular precession of *n* Lagrange's gyroscopes. Regular precessions around a common axis, of a two Lagrange's gyroscopes, which move by inertia, were studied by Kononykhin and Pozdnyakovich [64]. Method of Chetaev obtained sufficient conditions to study the motion in the variables by parts.

In the work from Robe [65], there is a model of the satellite. There are of two identical asymmetric bodies. The bodies are coupled by weightless rod. In this work was studied the motion of system of non-mobile rigid bodies on orbital system, and the rod is collinear, which joining the center of mass of the Earth to the center of the satellite. The stability of this motion was studied by Routh's method.

Wittenburg in [23] simulated the satellite system of *n* rigid body and gyrostat and determined the stability respect equilibrium itself. The sufficient conditions of relative equilibrium obtained by Wittenburg and Lilov in [66].

Burlakova [67] established necessary and sufficient conditions for stability of stationary motions of the system coupled rigid bodies in a Newtonian field. Interesting studies have been performed by a group of authors [18, 69, 70], studying the equivalence replacement of flexible elastic rod bundle by elastic hinges.

In [54, 55, 68], Rudenko studied the stability of uniform rotation of a system of two bodies coupled by elastic cylindrical, and anisotropic elastic hinge. In [71-73], the elastic hinges are used to simulate the kinetic energy storage. Stationary regimes are determined and studied the stability of system.

4. Conclusions

This work shows an overview of papers related the topic of system rigid bodies, stationary regimes and stability. Searching rational forms of writing the equations of motion, the choice of main variables, enabling a better understanding of the properties the object being studied; finding stationary solutions describing operational modes of the simulated object and study the stability of these decisions.

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