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SIMPLE PARAMETRIC PREDICTIVE CONTROL ALGORITHMS BASED ON NEIGHBORING EXTREMAL

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Abstract

When the true plant evolution deviates significantly from the plant predicted by the model, the performance of a model predictive controller (MPC) is typically poor. So a robust MPC approach considering model uncertainty is explicitly needed. For the range of uncertainty considered, multiple input profiles that each profile is for each realization of the uncertainty, was determined. Unfortunately, it is extremely expensive in computation such that its practical implementation is difficult. This paper proposes two approaches based on neighboring extremal, which are

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described as triple-mode and quadruple-mode prediction structures, respectively, to improve the performance of the closed-loop system. Furthermore, these algorithms demonstrate how an underexplored prediction class can be combined with parametric programming to generate far simpler solutions with a little cost to optimality. The idea is illustrated via the simulation of a continuous stirred tank reactor.

1. Introduction

Model predictive control (MPC) has become an effective control algorithm for dealing with multivariable constrained control problems that are encountered in the process industries: At each sampling time, MPC uses an explicit process model and information about input and output constraints to compute process inputs so as to optimize future plant behavior over the prediction horizon. Although more than one input move is computed, the controller implements only the first computed input and repeats these calculations at the next sampling time.

Over the last decade, a solid theoretical foundation for MPC has emerged so that in real life, large-scale MIMO application controllers with non-conservative stability guarantees can be designed routinely and with ease [10]. The big drawback of the MPC is the relatively formidable on-line computational effort, which limits its applicability to relatively slow and/or small problems.

A key element in predictive control is the extensive use of the dynamic process model, but model is only an approximation of the real process. When the difference between the predicted and the true plant evolutions is significant, standard predictive control may be unable to conduct the desired performance [8, 12]. So it is extremely important to re-cast MPC to be robust for model uncertainty.

Standard robust predictive control computes an input that represents a compromise solution for the range of uncertainty considered [2, 6, 7], it requires a modest amount of online computation and introduces extra free degree to enlarge the volume of the relevant invariant set and improve closed-loop response. To prove robust stability, it is important to guarantee that the final state is within some bounded set [9]. When the dispersion of the open-loop predicted state is large, especially in the case of unstable processes, it may not be possible to find a feasible solution to the robust optimization problem. So, the state dispersion should be controlled, and the inherent state feedback needs to be incorporated in the robust predictive control formulation [6].

We can express the inherent state feedback by two ways: One is using multiple input profiles [8, 13], i.e., at each sample interval, the predictive control law can be constructed in more than one phase: one phase is a transient phase of first n_c steps; the others are the asymptotic phases thereafter. At the next sampling interval, the same step is started again. This approach has flexibility but its computational load is large; the other way is using the neighboring extremal approach to approximate the inherent state feedback [1, 2]. For small deviation away from the optimal solution, the theory of neighboring extremals provides a closed-form solution to the optimization problem. Thus, the optimal input can be obtained using state feedback, which approximates the feedback provided by explicit numerical re-optimization.

In [5], an approach is proposed which embeds the NE-based approach into robust optimization with multiple input profiles, i.e., the optimization scheme optimizes the multiple profiles, and the nominal one is exactly obtained via explicit optimization and all the perturbed ones are obtained approximately via the NE approach. Since only one input profile is optimized explicitly, the computational complexity of the problem is reduced considerably, while keeping the advantages of the robust optimization scheme with multiple input profiles. This scheme gives a first order approximation of the optimal input.

Note that the main emphasis is not only in reducing the computational complexity of a general MPC problem as in [14], but also in the context of robust predictive control. In [11], based on parametric programming for MPC, a simple algorithm showed how to move all the computations necessary for the implementation of MPC offline, while preserving all its other characteristics. Moreover, such an explicit form of the controller provides additional insight for a better understanding of the control policy of MPC.

We know, the validity of the first order approximation is not good, when the solution deviates great away from the optimal solution. In this paper, as the expansion of the robust predictive control based on the NE, two algorithms, which embedding neighboring extremal (NE) approach into robust predictive control with multiple input profiles, are proposed. These algorithms introduce some adjustable parameters to improve the performance of the closed loop system. Furthermore, they demonstrate how an underexplored prediction class can be combined with parametric programming to generate far simpler solutions with a little cost to optimality. The simulation results of applying these algorithms to a continuous stirred tank reactor verify the effectiveness of the algorithm proposed in this paper.

2. Preliminaries

2.1. Standard predictive control

Consider the nonlinear dynamic process:

$$\dot{x} = F(x, u, \theta), \quad x(0) = x_0, \tag{1}$$

where the state x and the input u are vectors of dimensions n and m, respectively. x_0 represents the initial conditions, θ denotes the vector of uncertain parameters that are assumed to lie in the admissible region Θ, and F is the process dynamics.

In predictive control, the following optimization problem is solved repeatedly at discrete time instants:

$$\min_{u([t_k, t_k + T_f])} J = \frac{1}{2} x(t_k + T_f)^T P x(t_k + T_f) + \frac{1}{2} \int_{t_k}^{t_k + T_f} \left(x(\tau)^T Q x(\tau) + u(\tau)^T R u(\tau) \right) d\tau$$

$$\text{s.t. } \dot{x} = F(x, u, \theta), \quad x(t_k) = x_k, \quad x(t_k + T_f) \in X, \tag{2}$$

where P, Q and R are positive-definite weighting matrices with appropriate dimensions, X is the bounded region of state space where the final state should lie, t_k is the present time at which the optimization is performed, T_f is the prediction horizon, and x_k is the state measured or estimated at time t_k. The optimal input computed by solving (2) is represented by $u^*([t_k, t_k + T_f])$. The importance of having a terminal cost, and also a bounded region for the final state for the sake of stability is discussed in [8].

Let h be the sampling period, which, in general, is a constant. The first part of the optimal input, $u^*([t_k, t_{k+1}])$, is applied to the open loop, and the optimization problem is repeated at the time instant $t_{k+1} = t_k + h$. So, for implementation purposes, the infinite-dimensional input $u([t_k, t_k + T_f])$ is parameterized using a finite number of decision variables, typically a piecewise-constant approximation.

Remark 1. When the controlled system has parameter uncertainties and disturbance, the difference between the model and the true plant is significant, standard predictive control may be unable to provide the desired performance. So we must improve the robustness of predictive control.

2.2. Neighboring extremals controller

Including the dynamic constraints of optimization problem (2), the augmented cost function, $\bar{J}$, can be written as

$$\bar{J} = \Phi(x(t_k + T_f)) + \int_{t_k}^{t_k+T_f} (H - \lambda^T \dot{x}) dt, \tag{3}$$

where $\Phi(x) = \frac{1}{2} x^T P x$, $H = \frac{1}{2}(x^T Q x + u^T R u) + \lambda^T F(x, u, \theta)$, and $\lambda(t) \neq 0$ is the n-dimensional vector of adjoint states or Lagrange multipliers for the system equations.

At the optimal solution, the first variation of $\bar{J}$ is given by [3, p. 64]:

$$\Delta\bar{J} = (\Phi_x - \lambda^T)\Delta x|_{t_k+T_f} + \int [(H_x + \dot{\lambda}^T)\Delta x + H_u \Delta u] d\tau, \tag{4}$$

where $\Delta x(t) = x(t) - x^*(t)$ and $\Delta u(t) = u(t) - u^*(t)$, with x^* and u^* are the optimal state and input, respectively, and the notation $a_b = \frac{\partial a}{\partial b}$ is used.

Upon the convenient choice of the adjoint states, $\dot{\lambda}^T = -H_x$ with $\lambda^T(t_k + T_f)$ $= \Phi_x(t_k + T_f)$, the necessary conditions of optimality that are derived from $\Delta\bar{J}$ $= 0$ are:

$$H_u = u^T R + \lambda^T F_u = 0. \tag{5}$$

The second-order variation of $\bar{J}$ is given by [3, p. 317]:

$$\Delta^2\bar{J} = \frac{1}{2}\Delta x(t_k + T_f)^T P \Delta x(t_k + T_f)$$
$$+ \frac{1}{2}\int_{t_k}^{t_k+T_f} [\Delta x^T \;\; \Delta u^T] \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta u \end{bmatrix} d\tau. \tag{6}$$

Choose Δu to minimize $\Delta^2\bar{J}$ under the linear dynamic constraint

$$\Delta\dot{x} = F_x \Delta x + F_u \Delta u. \tag{7}$$

Equation (6) represents a time-varying linear quadratic regulator (LQR) problem, for which a closed-form solution is available:

$$\Delta u(t) = -K(t)\Delta x(t), \tag{8}$$

$$K = H_{uu}^{-1}(H_{ux} + F_u^T S), \tag{9}$$

$$\dot{S} = -H_{xx} - SF_x - F_x^T S + H_{xu}K + SF_u K,\ S(t_k + T_f) = P. \tag{10}$$

The solution combining (8) with (9) and (10) is optimal solution of LQR problem at optimal point. Gros et al. [5] applied this solution to the neighborhood, and termed it as neighboring extremals controller. Actually, the solution becomes an approximate solution in the neighborhood.

Remark 2. For small deviation away from the optimal solution, the NE approach can provide a first order approximation of the optimal solution. Thus, the optimal input can be obtained using state feedback, which approximates the feedback provided by explicit numerical re-optimization.

In [5], an approach was proposed which embedded the NE-based approach into robust optimization with multiple input profiles, i.e., the optimization scheme optimizes the multiple profiles, and the nominal one is exactly obtained via explicit optimization and all the perturbed ones are obtained approximately via the NE approach. Since only one input profile is optimized explicitly, the computational complexity of the problem is reduced considerably, while keeping the advantages of the robust optimization scheme with multiple input profiles.

It is clear that the first order approximation of a nonlinear function cannot provide a satisfying approximating result, when the nonlinearity of this function is high. So the control approach based on NE theory cannot obtain a satisfying performance under this situation. In next section, we will propose two improved predictive control algorithms based on NE with a system free degree to overcome the drawback of first order approximation of nonlinearity to improve the optimization performance of the closed-loop system.

3. An Improved Algorithms with Multiple Input Profiles and Based on Neighboring Extremals

In this section, we introduce two algorithms which combine the NE approach with multiple input profiles to improve the performance of the closed-loop system and keep the advantages of the predictive control based on NE approach.

3.1. Triple mode prediction structure

Here we will use a modified method based on the theory of neighboring extremal to construct the control increments as a triple mode structure:

$$\Delta u(t) = \begin{cases} 0, & t_k \le t < t_{k+1}, \\ -K(t)\Delta x(t) + c_k, & t_{k+1} \le t < t_{k+n_c-1}, \\ -K(t)\Delta x(t), & t_{k+n_c} \le t < t_k + T_f, \end{cases} \tag{11}$$

where $K(t)$ is defined as formula (9), c_k is an adjustable parameter, $\Delta u(t) = u_\theta(t) - u_{\theta_0}(t)$, and $\Delta x(t) = x_\theta(t) - x_{\theta_0}(t)$, where θ_0 is the nominal parameter vector.

Embedding $\Delta x(t) = x_\theta(t) - x_{\theta_0}(t)$ and $\Delta u(t) = u_\theta(t) - u_{\theta_0}(t)$ into formula (11), we can obtain:

$$u_\theta(t) = \begin{cases} u_{\theta_0}(t), & \text{if } t_k \le t < t_{k+1}, \\ u_{\theta_0}(t) - K(x_\theta - x_{\theta_0}) + c_k, & \text{if } t_{k+1} \le t \le t_{k+n_c-1}, \\ u_{\theta_0}(t) - K(x_\theta - x_{\theta_0}), & \text{if } t_{k+n_c} \le t \le t_k + T_f. \end{cases}$$

Then the optimization problem of model predictive control can be expressed as follows:

$$\min_{u([t_k, t_k+T_f])} F(J_\theta,\ p(\theta)),$$

$$\text{s.t. } \dot{x}_{\theta_0} = F(x_{\theta_0}, u, \theta_0),\ \ x_{\theta_0}(t_k) = x_k;\ \ \dot{x}_\theta = F(x_\theta, u_\theta, \theta),\ \ x_\theta(t_k) = x_k,$$

$$\forall\theta \ne \theta_0,$$

$$x_\theta(t_k + T_f) \in X, \quad \forall\theta \in \Theta, \tag{12}$$

$$u_\theta(t) = \begin{cases} u_{\theta_0}(t), & \text{if } t_k \le t < t_{k+1}, \\ u_{\theta_0}(t) - K(x_\theta - x_{\theta_0}) + c_k, & \text{if } t_{k+1} \le t \le t_{k+n_c-1}, \\ u_{\theta_0}(t) - K(x_\theta - x_{\theta_0}), & \text{if } t_{k+n_c} \le t \le t_k + T_f, \end{cases} \tag{13}$$

$$J_\theta = \frac{1}{2} x_\theta(t_k + T_f)^T P x_\theta(t_k + T_f)$$

$$+ \frac{1}{2}\int_{t_k}^{t_k+T_f} (x_\theta(\tau)^T Q x_\theta(\tau) + u_\theta(\tau)^T R u_\theta(\tau))\, d\tau, \tag{14}$$

where $E(J_\theta,\ p(\theta))$ is a general stochastic objective function that depends on the probability density function $p(\theta)$.

Remark 3. The free degree of the controlled system is added by introducing the adjustable parameter c_k to improve the optimization performance of the closed-loop system in some certain while keep the advantages of the approaches based on NE theory.

3.2. Quadruple mode prediction structure

Although more than one input moves are computed at each sampling time, the controller implements only the first computed input and repeats these calculations at the next sampling time. Since there is inherent feedback between control inputs and states, state $x(t_k)$ which relates to the first input is very important. Hence, the state should be emphatically considered in constructing the controller. Therefore, we construct the control increments as a quadruple mode structure:

$$\Delta u(t) = \begin{cases} 0, & t_k \leq t < t_{k+1}, \\ -K(t)\Delta x(t) + L_k \Delta x_k + c_{k1}, & t_{k+1} \leq t < t_{k+n_{c1}-1}, \\ -K(t)\Delta x(t) + c_{k2}, & t_{k+n_{c1}} \leq t < t_{k+n_{c2}-1}, \\ -K(t)\Delta x(t), & t_{k+n_{c2}} \leq t < t_k + T_f, \end{cases} \tag{15}$$

where L_K, c_{k1}, c_{k2} are adjustable parameters, $K(t)$ is defined as formula (9), and $\Delta u(t) = u_\theta(t) - u_{\theta_0}(t)$, $\Delta x(t) = x_\theta(t) - x_{\theta_0}(t)$, $\Delta x_k = x_\theta(t_k) - x_{\theta_0}(t_k)$. Embedding $\Delta u(t) = u_\theta(t) - u_{\theta_0}(t)$, $\Delta x(t) = x_\theta(t) - x_{\theta_0}(t)$, $\Delta x_k = x_\theta(t_k) - x_{\theta_0}(t_k)$ into (15), then optimization problem can be expressed:

$$\min_{u([t_k, t_k + T_f])} E(J_\theta,\ p(\theta))$$

$$\text{s.t. } \dot{x}_{\theta_0} = F(x_{\theta_0}, u, \theta_0),\ x_{\theta_0}(t_k) = x_k;\ \dot{x}_\theta = F(x_\theta, u_\theta, \theta),\ x_\theta(t_k) = x_k,\ \forall \theta \neq \theta_0,$$

$$x_\theta(t_k + T_f) \in X, \quad \forall \theta \in \Theta, \tag{16}$$

$$u_\theta(t) = \begin{cases} u_{\theta_0}(t), & \text{if } t_k \leq t < t_{k+1}, \\ u_{\theta_0}(t) - K(t)(x_\theta(t) - x_{\theta_0}(t)) \\ \quad + L_k(x_\theta(t_k) - x_{\theta_0}(t_k)) + c_{k1}, & \text{if } t_{k+1} \leq t \leq t_{k+n_{c1}-1}, \\ u_{\theta_0}(t) - K(x_\theta(t) - x_{\theta_0}(t)) + c_{k2}, & \text{if } t_{k+n_{c1}} \leq t \leq t_{k+n_{c2}-1}, \\ u_{\theta_0}(t) - K(x_\theta(t) - x_{\theta_0}(t)), & \text{if } t_{k+n_{c2}} \leq t \leq t_k + T_f, \end{cases} \tag{17}$$

$$J_{\theta} = \frac{1}{2} x_{\theta}(t_k + T_f)^T P x_{\theta}(t_k + T_f)$$

$$+ \frac{1}{2} \int_{t_k}^{t_k + T_f} (x_{\theta}(\tau)^T Q x_{\theta}(\tau) + u_{\theta}(\tau)^T R u_{\theta}(\tau)) d\tau, \tag{18}$$

where $E(J_{\theta},\ p(\theta))$ is a general stochastic objective function that depends on the probability density function $p(\theta)$.

Remark 4. This method further enhances the free degree regulation of the controlled plant by adding adjustable parameters c_{k1}, c_{k2} and L_k to improve the system's performance with less computing complexity than standard robust predictive control.

4. Illustrative Example

In this section, a continuous stirred tank reactor (CSTR) is considered to show the effectiveness of the algorithms proposed in this paper.

The concentration control in a CSTR with constant cooling jacket temperature is considered. There is a single exothermic chemical reaction, $A \to B$, and the manipulated variable is the inlet feed rate. The model equations are:

$$\dot{c}_A = -k_0 c_A e^{\frac{-E}{RT}} + \frac{F}{V}(c_{A_{in}} - c_A),$$

$$\dot{T} = \frac{(-\Delta H)}{\rho c_p} k_0 c_A e^{\frac{-E}{RT}} + \frac{F}{V}(T_{in} - T) + \frac{UA}{V \rho c_p}(T_c - T), \tag{19}$$

where c_A is the concentration of species A; T, T_{in} and T_c are the reactor temperature, the inlet temperature and the cooling jacket temperature, respectively. F is the feed rate of A, k_0 is the pre-exponential factor, E is the activation energy, R is the gas constant, V is the reactor volume, ΔH is the reaction enthalpy, ρ is the density, c_p is the heat capacity, U is the heat transfer coefficient, and A is the heat transfer area. These parameter values, taken from [4], are listed in Table 1.

Table 1. Parameter values

$V = 1.36\,\mathrm{m}^3$	$c_{A,in} = 8008\,\mathrm{mol/m}^3$	$k_0^{nom} = 7.08\times10^7\ \mathrm{l/h}$
$E/R = 8375\,\mathrm{K}$	$T_{in} = 373.3\,\mathrm{K}$	$T_c = 532.6\,\mathrm{K}$
$\rho = 800.8\,\mathrm{kg/m}^3$	$c_p = 3140\,\mathrm{J/(kgK)}$	$U_A = 7.04\times10^6\ \mathrm{J/(hK)}$
$(-\Delta H) = 69{,}775\,\mathrm{J/mol}$		

The uncertainty regards the pre-exponential factor k_0 that can take any value in the range $[5.66\times10^7, 8.5\times10^7]$ with equal probability. The value used for simulating the reality is $k_0^{real} = 8.14\times10^7$ l/h, which is larger than the nominal model value $k_0^{nom} = 7.08\times10^7$ l/h, i.e., the reality is more reactive than that predicted by the model. For a feed rate $F = 5\,[\mathrm{m}^3/\mathrm{h}]$, the model exhibits three equilibrium points. The control objective is to drive the process from the stable equilibrium, Point A $[c_A, T]' = [1586.08\,\mathrm{mol/m}^3, 5.44.72\,\mathrm{K}]'$, to the unstable equilibrium, Point B $[c_A, T]' = [4786.23\,\mathrm{mol/m}^3, 487.79\,\mathrm{K}]'$.

The equilibrium point varies with k_0, i.e., $F = 5[\mathrm{m}^3/\mathrm{h}]$ no longer corresponds to the equilibrium points A and B for the real process. Hence any proportional controller will seek a compromise meeting between $F = 5[\mathrm{m}^3/\mathrm{h}]$ and $c_A = c_{A,ref}$ at steady state. Fortunately, the resulting steady-state error can be eliminated via integral control. For this, an additional state I with the following dynamics is included:

$$\dot{I}(t) = c_{A,ref} - c_A(t),\ I(0) = 0. \tag{20}$$

Integral controller is added in the output channel artificially to eliminate steady-state error.

All control predictive schemes investigated below share the same features as [5]: (i) re-optimization of the input at each sampling instant, with $\lambda = 0.2[\mathrm{h}]$ is the sampling interval, (ii) $T_f = 1[\mathrm{h}]$ is the prediction horizon, (iii) control sequence for

$t_k \leq t \leq t_k + \lambda$ is applied, leaving the rest of the sequence unused, (iv) piecewise-constant parameterization of the input $F(t)$ with time intervals of length *h*, and (v)

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad R = 50.$$

4.1. Robust control with triple-mode input profile based on NE

Firstly, the robust predictive control scheme with dual-mode structure as (15)-(17) is studied. At the re-optimization at time t_k, an optimal input is computed for the nominal model using a piecewise constant input parameterization of length λ. Based on this input, an improved NE controller is designed to generate $u([t_k + \lambda, t_k + (n_c - 1)\lambda])$ and $u([t_k + n_c\lambda, t_k + T_f])$, where $n_c = 2$, $c_k = 0.3$. For the tuning c_k, firstly, take c_k in the interval $|c_k| \leq C$ evenly, where *C* is boundary, and usually $C < 10$. The cost function *J* for all of them is calculated, and then the parameter c_k which has the minimum *J* is chosen. A new NE controller is therefore computed at each re-optimization. Simulation results are displayed in Figures 1-4 (dashed line).

4.2. Robust control with quadruple-mode input profile based on NE

Here the robust predictive control scheme with triple-mode structure as (18)-(20) is studied. Similarly, at the re-optimization at time t_k, an optimal input is computed using a piecewise constant input parameterization of same length λ. Based on this input, giving $n_{c1} = 1$ and $n_{c2} = 2$, control sequences $u([t_k + \lambda, t_k + (n_c - 1)\lambda])$, $u([t_k + n_{c1}\lambda, t_k + (n_{c1} + n_{c2} - 1)\lambda])$, $u([t_k + (n_{c1} + n_{c2})\lambda, t_k + T_f])$ are computed, where $c_{k1} = 1.5$, $c_{k2} = 0.6$, $L_k = 0.1$. The simulation results are shown in Figures 1-4 (solid line).

The simulation results of the method proposed in [5] are given in Figures 1-4 (dotted line).

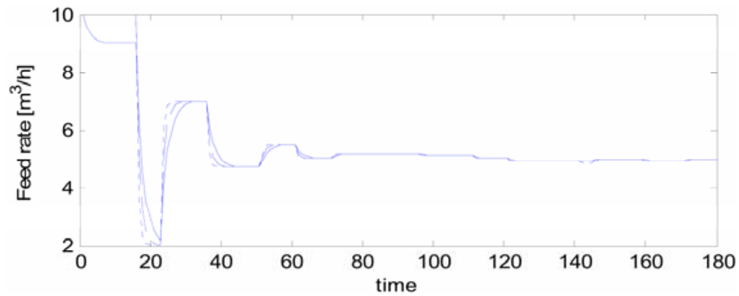

Figure 1. The feed rate F based on NE and triple mode input profile (i) solid line: Quadruple-mode input profile based on NE; (ii) dash line: Triple-mode input profile based on NE; (iii) dot line: Robust control based on NE (proposed in [5]).

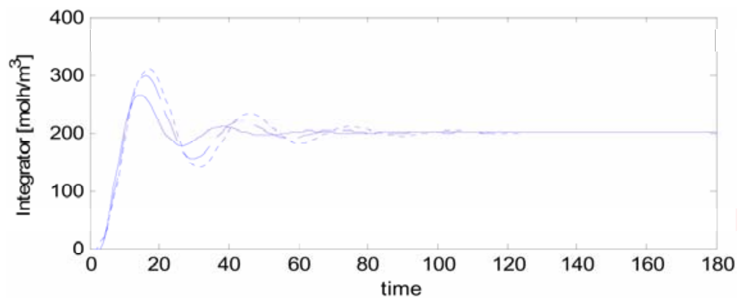

Figure 2. The integrator rate F based on NE and triple mode input profile (i) solid line: Quadruple-mode input profile based on NE; (ii) dash line: Triple-mode input profile based on NE; (iii) dot line: Robust control based on NE (proposed in [5]).

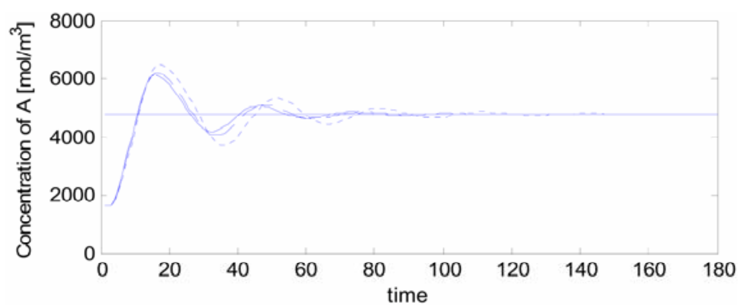

Figure 3. The concentration of A, c_A based on NE and triple mode input profile (i) solid line: Quadruple-mode input profile based on NE; (ii) dash line: Triple-mode input profile based on NE; (iii) dot line: Robust control based on NE (proposed in [5]).

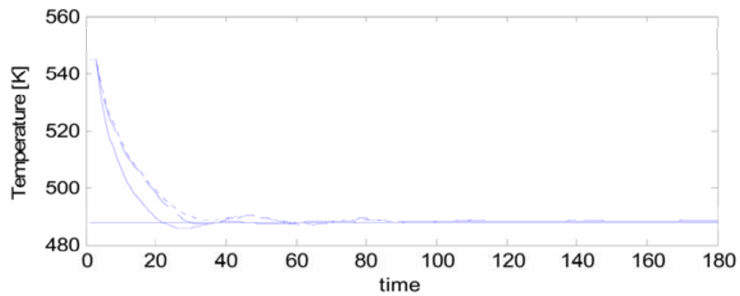

Figure 4. The temperature *T* based on NE and triple mode input profile (i) solid line: Quadruple-mode input profile based on NE; (ii) dash line: Triple-mode input profile based on NE; (iii) dot line: Robust control based on NE (proposed in [5]).

From these results of simulation on the continuous stirred tank reactor, we can see that the triple-mode and quadruple-mode algorithms proposed in this paper conducted the better performances in terms of settling time, peak time and percent overshoot than the algorithm proposed in [5] as illustrated in Figures 1-4.

By adding the free degree in the first order approximation of nonlinearity, the optimization performance is superior to the purely first order approximation, which verifies the validity of the proposed methods.

5. Conclusion

In this paper, two approaches based on neighboring extremal, which are described as triple-mode and quadruple-mode predictive control structures, respectively, were proposed for nonlinear systems. These algorithms can improve the performance of the closed-loop system and have little computational load. The simulation results demonstrated that they can control the nonlinear systems well. But, their stability and performance have not been addressed in this paper yet. These issues, in particular, the validity of the first-order approximation and the effect of the approximation on stability, will form the subject of future research. These algorithms are only used as a fictitious input for computational purposes. Yet, its use for direct implementation is another promising research direction.

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