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COMPRESSION OF IMAGES USING TYPE-2 FUZZY THRESHOLDING OF BANDLET TRANSFORM COEFFICIENTS

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Abstract

Images have to be compressed in order to store and transmit them efficiently. In this paper, we propose a hybrid image compression technique which uses bandlet transform and type-2 fuzzy thresholding. Bandlet transform is used to efficiently represent the anisotropic regularity of edge structures in images and fuzzy thresholding retains the important coefficients based on the measure of ultrafuzziness. The thresholded bandlet transform coefficients are then compressed using a variable length coding technique, viz., arithmetic coding. The proposed method is applied to two-dimensional (2D) images. The results are compared with type-2 fuzzy thresholded wavelet compression. Test results show that the proposed method using bandlet transform gives better results in terms of Peak-Signal-to-Noise Ratio (PSNR) and compression ratio compared to wavelet transform and type-2 fuzzy thresholding.

I. Introduction

Digital image compression is one of the important issues as there is a huge

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increase in the volume of digital images generated by the use of digital technology in various fields like photography, medical imaging, video conferencing, remote sensing. The objective of image compression techniques is to reduce the redundancy of image data in order to reduce the requirements of memory and bandwidth for storage and transmission of data. Image compression techniques can be lossless or lossy depending on whether the data to be compressed is highly important or not. Using lossless image compression techniques we can reconstruct images without any information loss. On the other hand, using lossy image compression techniques we can reconstruct images with a varying degree of information loss.

Transform coding is used to convert spatial image pixel values to transform coefficient values, which are then coded. Transform based image compression methods have been widely used in the state-of-art image compression. Transform methods like Discrete Cosine Transform (DCT) [1], Discrete Wavelet Transform (DWT) [2] can be used to represent the image as a set of coefficients. These transforms exploit correlation between pixels in the image data. They represent the image data as a set of less correlated coefficients and thus the coefficients are packed into a specific area of the transform domain. This property makes these transform techniques to be widely used in various image compression schemes [3-7]. Some of them are Joint Photographic Experts Group (JPEG) [8], JPEG-LS [9], JPEG2000 [10], Embedded Zerotree Wavelet (EZW) coding [11], Set Partitioning in Hierarchical Trees (SPIHT) coding [12]. The coefficients can be encoded using Huffman coding [13] and arithmetic coding [14], which are popular variable length entropy coding techniques.

Standard wavelet bases can efficiently represent local image regularity using a few coefficients. But, they cannot exploit the geometrical directional regularity efficiently. Several new bases have been developed to exploit this anisotropic regularity of image structures. Curvelets [15], contourlets [16], wedgelets [17] and bandlets [18-19] are some of them. In this work we have used bandlet transform which has the advantage of efficiently representing the regularity of edge structures. The bandlet decomposition [19-22] is computed with a geometric orthogonal transform that is applied on orthogonal wavelet coefficients. Moreover, type-2 fuzzy sets can handle uncertainty/vagueness much better than type-1 fuzzy sets. Hence in this paper we propose a compression scheme, which performs type-2 fuzzy thresholding of the bandlet coefficients to compress images. The thresholded coefficients are further compressed using arithmetic coding.

The rest of the paper is organized as follows. Section II gives an overview of bandlet transform. Section III provides a detailed description of the proposed compression scheme for compressing images. Section IV provides the result of compressing various 2D images using the proposed compression scheme. Section V discusses the conclusion.

II. Review of Bandlet Transform

In bandlet transform the geometry of the image is characterized using a geometric flow of vectors. These vectors give the directions of regions where the image has regular variations. Orthogonal bandlet bases are developed by partitioning the image support into regions inside which the geometric flow is parallel [18]. The bandlet decomposition [19-22] is computed with a geometric orthogonal transform that is applied on orthogonal wavelet coefficients. Wavelet transform, when applied to an image of *N* pixels, computes the set of *N* dot products

$$\langle f, \psi^{s}_{jn} \rangle \text{ for } 2^{-J} \leq 2^{-j} < \sqrt{N} \text{ and } 0 \leq n_1, n_2 < 2^{-j},$$

$$\langle f, \phi^{s}_{Jn} \rangle \text{ for } 0 \leq n_1, n_2 < 2^{-J}, \tag{1}$$

where the projection on ϕ^{s}_{Jn} functions produces a coarse approximation at scale 2^J. The scale 2^J represents the level at which we stop the wavelet transform. Those values can be conveniently stored in an array of *N* pixels. A dyadic square is a square obtained by recursively splitting the original wavelet transformed image f^s_j into four subsquares of equal sizes. Let the width of the squares be *L* pixels with $4 \leq L \leq 2^{-j/2}$. For each dyadic square *S* at a given scale 2^j and orientation *s* of the wavelet transform 1D reordering of the grid points is performed. The possible number of 1D reordering may be equal to the number of directions *d* joining pairs of points in square *S* of width *L*. 1D reordering is done by projecting the sampling location along *d* and sorting the resulting 1D points from left to right. To the resulting 1D discrete signal, f_d, 1D wavelet discrete wavelet transform is performed. For a given threshold *T*, the direction *d*, which generated the less approximation error, is selected. Let b_k denote the coefficients of 1D wavelet transform of f_d, and R_B be the number of bits needed to code the quantized coefficients $QT(b_k)$. To select the best geometry, the direction *d* that minimizes the

Lagrangian

$$\xi(f_d, R) = \| f_d - f_{dR} \|^2 - \lambda T_2 (R_G - R_B), \tag{2}$$

where f_{dR} is the signal recovered from the quantized coefficients and R_G is the number of bits needed to code the geometric parameter *d* with an entropy coder, λ is taken as 3/28 [23].

III. Proposed Compression Scheme

A. Type-2 fuzzy thresholding in bandlet domain

Fuzzy set theory and Fuzzy logic [24] offer us powerful tools to represent and process knowledge represented as fuzzy if-then rules. Type-2 fuzzy sets [25] overcome the disadvantages of type-1 fuzzy sets by efficiently representing uncertainties like the meaning of the words, the measurements which are noisy, etc., which could not be efficiently represented by type-1 fuzzy sets. One of the simple ways of creating type-2 fuzzy sets is by using linguistic hedges like dilation and concentration.

Using fuzzy set theory, bandlet thresholding can be expressed as fuzzy bandlet thresholding. Let $| b_{s,d}(i, j) |$ be the absolute value of the bandlet coefficient, $b_{s,d}(i, j)$, at location (i, j) for the scale *s* and direction *d* and *T* be the threshold value. Any one of the membership functions μ is selected and α is initialized. The upper and lower membership degrees μ_U and μ_L of the membership function are given by

$$\mu_U(x) = [\mu(x)]^{1/\alpha}, \tag{3}$$

$$\mu_L(x) = [\mu(x)]^{\alpha}. \tag{4}$$

We have taken the value of α as 2, i.e.,

$$\mu_U(x) = [\mu(x)]^{0.5}, \tag{5}$$

$$\mu_L(x) = [\mu(x)]^{2}. \tag{6}$$

In [26], it is mentioned that $\alpha \in (1, \infty)$ and also that $\alpha >> 2$ is usually not meaningful for image data. Any membership function can be selected. It is also mentioned in [26] that it is not possible to say which membership function is the best one. We use a standard *S*-membership function and its type-2 fuzzy set is shown in Figure 1 where α is 2. The frequency histogram for the bandlet coefficients, $|b_{s,d}(i, j)|$, is computed. The threshold is moved from 0 to $\max(|b_{s,d}(i, j)|)$ and in each position the amount of ultrafuzziness [26] is computed. The measure of ultrafuzziness is given by:

$$\gamma_B = \frac{1}{MN} \sum_{g=0}^{\max(|b_{s,d}(i,j)|)} h(g) \min(\mu_U(g), 1 - \mu_L(g)), \tag{7}$$

where *M*, *N* is the size of the 2D image, $h(g)$ is the histogram value and $\mu(g)$ is the membership value of the bandlet coefficient *g*. The position where the amount of ultrafuzziness is maximum, is used as the threshold. Let this threshold value be denoted by *T*. The bandlet coefficients whose absolute values are greater than *T* will remain as such and the remaining coefficients will be set to 0.

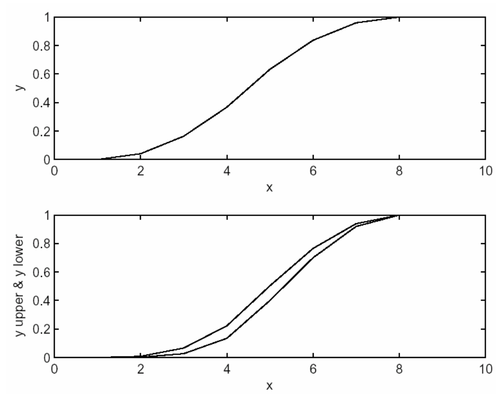

Figure 1. (a) *S*-membership function (smf in Matlab) with $P = [1 \;\; 8]$ and (b) its linguistic hedges with $\alpha = 2$.

B. Compression scheme

The proposed compression scheme consists of three steps. In the first step the 2D image $I_{x,y}$ is forward bandlet transformed to obtain the bandlet coefficients. Let the coefficients of 2D discrete bandlet transform of image $I_{x,y}$ be represented as $F_{x,y}$. Orthogonal bandlets use an adaptive segmentation and a local geometric flow and are thus able to capture the anisotropic regularity of edge structures. The second step is to perform type-2 fuzzy bandlet thresholding as discussed in the previous section. The histogram of the bandlet coefficients is computed. The threshold is moved and in each position the measure of ultrafuzziness is computed. The threshold T where the measure of ultrafuzziness is maximum is used as the threshold. The bandlet coefficients which are greater than T remain as such and the remaining coefficients are set as zero. Let $FT_{x,y}$ be the fuzzy bandlet thresholded image. The third step is to perform lossless arithmetic entropy coding of the thresholded coefficients. This step can use any of the variable length coding techniques such as Huffman coding or arithmetic coding. We have used arithmetic coding.

Decompression is the inverse of the compression stage. Arithmetic decoding is done to extract the fuzzy thresholded bandlet coefficients, $F_{x,y}$. Next step is to perform inverse bandlet transform to obtain the reconstructed image $IR_{x,y}$. The block diagram of this scheme is given in Figure 2.

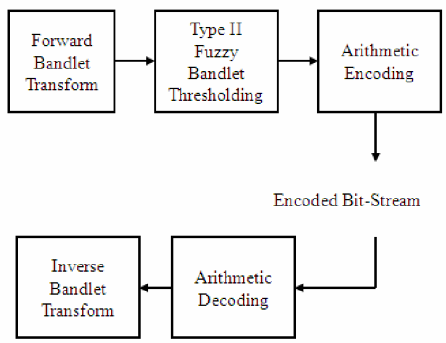

Figure 2. Block diagram of the proposed compression scheme with bandlet transform and type-2 fuzzy thresholding.

IV. Experiments and Results

We have used three 2D images whose dimension is 512∗512. Type-2 fuzzy thresholding of bandlet coefficients and type-2 fuzzy thresholding of wavelet coefficients are performed and the results are compared. Both the wavelet transform and bandlet transform are based on the ‘CDF’ biorthogonal filter. Mean-Squared Error (MSE) and Peak-Signal-to-Noise Ratio (PSNR) are calculated for various compression ratios (CR). Various compression ratios can be obtained by adjusting the extremes of the sloped portion of S-membership function.

The results of the compression of 2D images using the proposed compression scheme are shown in Table 1. The results for PSNR values versus compression ratio are plotted in Figures 3, 4 and 5 respectively, for Lena, Barbara and fingerprint images.

It can be seen that the proposed scheme using bandlet and type-2 fuzzy thresholding technique is better than the wavelet and type-2 thresholding in terms of PSNR and compression ratio.

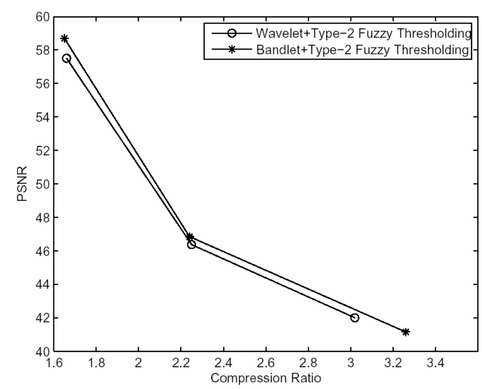

Figure 3. Compression ratio and PSNR for Lena.

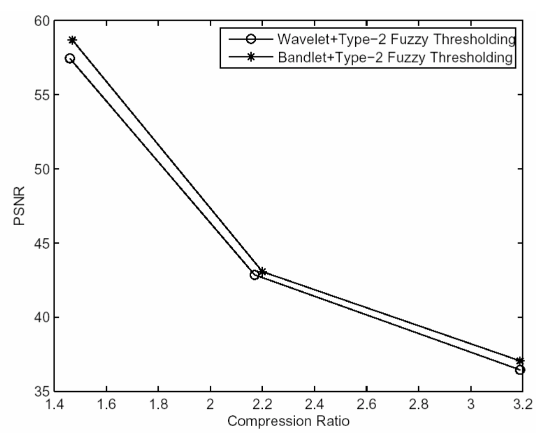

Figure 4. Compression ratio and PSNR for Barbara.

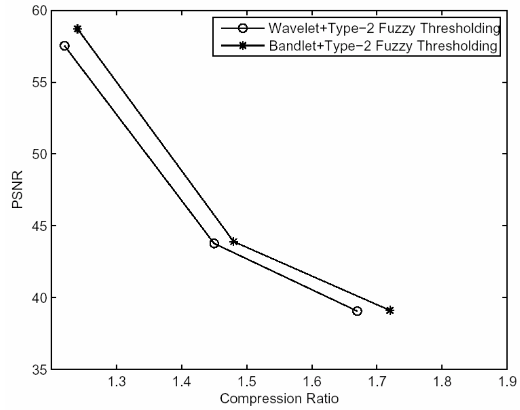

Figure 5. Compression ratio and PSNR for fingerprint.

Table 1. PSNR, MSE and compression ratio for 2D images using bandlet transform and type-2 fuzzy thresholding

Image		Wavelet Transform+ Type-2 Fuzzy Thresholding	Bandlet Transform+ Type-2 Fuzzy Thresholding
Lena	CR	1.66	1.66
	MSE	0.12	0.09
	PSNR	57.51	58.70
	CR	2.25	2.24
	MSE	1.50	1.34
	PSNR	46.37	46.85
	CR	3.02	3.26
	MSE	4.10	4.99
	PSNR	42.00	41.15
Barbara	CR	1.46	1.47
	MSE	0 .12	0.09
	PSNR	57.45	58.68
	CR	2.17	2.20
	MSE	3.36	3.21
	PSNR	42.86	43.07
	CR	3.19	3.19
	MSE	14.71	12.79
	PSNR	36.45	37.06

Fingerprint	CR	1.22	1.24
	MSE	0.11	0.09
	PSNR	57.54	58.71
	CR	1.45	1.48
	MSE	2.73	2.65
	PSNR	43.77	43.90
	CR	1.67	1.72
	MSE	8.09	7.98
	PSNR	39.05	39.11

V. Conclusion

This paper presents type-2 fuzzy thresholded bandlet transform for image compression. The power of bandlet transform to capture the geometric regularity along edges and the ability of type-2 fuzzy sets to handle uncertainty/vagueness makes the compression better. The proposed scheme gives better results for 2D images in terms of PSNR and compression ratio compared to wavelet transform and type-2 fuzzy thresholding.

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References

[1] W. H. Chen and W. K. Pratt, Scene adaptive coder, IEEE Transactions on Communication 32 (1984), 225-232.

[2] S. Mallat, A Wavelet Tour of Signal Processing, Academic Press, San Diego, 1998.

[3] Yung Gi Wu, Medical image compression by sampling DCT coefficients, IEEE Transactions on Information Technology in Biomedicine 6(1) (2002), 86-94.

[4] Yen Yu Chen, Medical image compression using DCT based subband decomposition and modified SPIHT data organization, International Journal of Medical Informatics 76(10) (2007), 717-725.

[5] D. Taubman and M. W. Marcellin, JPEG2000: Image Compression Fundamentals: Standards and Practices, Kluwer Academic Publishers, 2002.

[6] A. Munteanu, J. Cornelis and P. Cristea, Wavelet based lossless compression of coronary angiographic images, IEEE Transactions on Medical Imaging 18 (1999), 272-281.

[7] L. Zeng, C. Jansen, M. Unser and P. Hunziker, Extension of wavelet compression algorithms to 3D and 4D image data: exploitation of data coherence in higher dimensions allows very high compression ratios, Proc. SPIE 4478 (2001), 427-433.

[8] W. B. Pennebaker and J. L. Mitchell, JPEG Still Image Data Compression Standard, Chapman and Hall, New York, 1993.

[9] M. J. Weinberger, G. Seroussi and G. Shapiro, The LOCO-I lossless image compression algorithm: principles and standardization into JPEG-LS, IEEE Trans. Image Process. 9(8) (2000), 1309-1324.

[10] Information Technology – JPEG 2000 Image Coding System, Final Committee Draft Version 1.0, ISO/IEC JTC1/SC29/WG1, N1646R, March 2000.

[11] J. M. Shapiro, Embedded image coding using zero trees of wavelet coefficients, IEEE Trans. Signal Processing 41 (1993), 3445-3462.

[12] A. Said and W. A. Pearlman, A new, fast and efficient image codec based on set partitioning in hierarchical trees, IEEE Transactions on Circuits and Systems for Video Technology 6 (1996), 243-250.

[13] D. A. Huffman, A method for the construction of minimum redundancy codes, Proceedings of the IRE 40 (1951), 1098-1101.

[14] N. Abramson, Information Theory and Coding, McGraw-Hill, New York, 1963.

[15] E. Candes and D. Donoho, Curvelets: A Surprisingly Effective Nonadaptive Representation of Objects with Edges, Vanderbilt University Press, 1999.

[16] M. N. Do and M. Vetterli, The contourlet transform: an efficient directional multiresolution image representation, IEEE Trans. Image Process. 14(12) (2005), 2091-2106.

[17] D. Donoho, Wedgelets: nearly-minimax estimation of edges, Annals of Statistics 27 (1999), 353-382.

[18] E. Le Pennec and S. Mallat, Sparse geometrical image approximation with bandelets, IEEE Trans. Image Process. 14(4) (2004), 423-438.

[19] G. Peyré and S. Mallat, Surface compression with geometric bandelets, ACM Transactions on Graphics 24(3) (2005), 601-608.

[20] S. Mallat and G. Peyré, Discrete bandelets with geometric orthogonal filters, IEEE International Conference on Image Processing, 11-14 September 2005, Vol. 1, pp. 65-68.

[21] S. Mallat and G. Peyré, A review of bandlet methods for geometrical image representation, Numerical Algorithms 44 (3) (2007), 205-234.

[22] S. Mallat and G. Peyré, Orthogonal bandlet bases for geometric images approximation, Comm. Pure Appl. Math. 61(9) (2008), 1173-1212.

[23] E. Le Pennec and S. Mallat, Sparse geometrical image approximation with bandelets, IEEE Trans. Image Process. 14(4) (2004), 423-438.

[24] R. R. Yager and L. A. Zadeh, An Introduction to Fuzzy Logic Applications in Intelligent Systems, Kluwer, 1991

[25] J. M. Mendel and R. I. Bob John, Type-2 fuzzy sets made simple, IEEE Trans. Fuzzy Systems 10(2) (2002), 117-127.

[26] H. R. Tizhoosh, Image thresholding using type II fuzzy sets, Pattern Recognition 38 (2005), 2363-2372.