Far East Journal of Mathematical Sciences (FJMS)
Volume 39, Number 1, 2010, Pages 49-58
Published Online: April 26, 2010
This paper is available online at http://www.pphmj.com

NUMERICAL INVESTIGATION OF MHD BOUNDARY LAYER FLOW OF AN UPPER-CONVECTED MAXWELL FLUID OVER A RIGID PLATE

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Abstract

In this paper, the problem of magnetohydrodynamics (MHD) boundary layer flow of an upper-convected Maxwell fluid over a rigid surface is investigated, numerically. The governing boundary layer equation is reduced into ordinary differential equation by a similarity transformation. The transformed equation is then solved numerically using an implicit finite-difference scheme known as the Keller-box method. The effects of the Deborah number and magnetic parameter on the velocity profiles and the skin friction coefficients are computed, analyzed and discussed.

1. Introduction

From the perspective related to fluid, it is found that Newtonian fluid is categorized as the simplest type of fluid to form constitutive equations and the

2010 Mathematics Subject Classification: 76D10, 76M20, 76W05.

Keywords and phrases: boundary layer, finite-difference, magnetohydrodynamics, numerical solution, upper-convected Maxwell fluid.

Received February 22, 2010

governing equation for such a fluid is the Navier-Stokes equation [10]. However, the other type of fluid, namely the non-Newtonian fluid is more relevant towards the technological applications [9], for example, in bio-engineering, drilling operations and food industry [11]. Meanwhile, the solution for the problem of non-Newtonian fluid is rather complex and the classic Navier-Stokes theory is inadequate to deal with this type of problem. As such, various models for non-Newtonian fluid have been studied and solved by the researchers in order to fulfill the needs of the industries. The type of fluid considered in this paper concerns the Maxwell fluid, which is one of the non-Newtonian fluids having the properties of elasticity and viscosity [16]. Several studies have considered this kind of fluid such as those by Fetecau and Fetecau [5, 6, 7] and Fetecau et al. [8]. To be more specific, the present study investigates the upper-convected Maxwell fluid, which is a generalisation of the Maxwell fluid for the case of large deformations using the upper-convected time derivative [16].

The case of the hydrodynamic boundary layer flow of an upper-convected Maxwell fluid over a rigid surface has been studied by Sadeghy et al. [13] by solving the problem via perturbation method and two numerical schemes, namely the Runge-Kutta and the finite-difference methods. Similar problem has been considered by Hayat and Sajid [11] by adding the effect of magnetic field and they have solved the problem analytically by using a semi-analytical method, namely the homotopy analysis method (HAM).

In this paper, we revisit and investigate numerically the MHD boundary layer flow of an upper-convected Maxwell fluid over a rigid surface. By using the similarity transformation, the governing boundary layer equation is then reduced into ordinary differential equation before it is solved numerically by an efficient implicit finite-difference scheme known as the Keller-box method (see Cebeci and Bradshaw [1, 2]).

2. Basic Equations

The equation used to describe the Maxwell fluid model starts from the Cauchy stress tensor $\mathbf{T}$, which is given by (see [3, 4, 12, 14, 17]),

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{1}$$

where $-p\mathbf{I}$ denotes the indeterminate spherical stress. The extra stress tensor $\mathbf{S}$ is given by (see [11]),

$$\mathbf{S} + \lambda\left(\frac{d\mathbf{S}}{dt} - \mathbf{LS} - \mathbf{SL}^T\right) = \mu\mathbf{A}_1, \tag{2}$$

which $\mathbf{L}$ is the velocity gradient, μ is the dynamic viscosity, λ is the relaxation time, and the first Rivlin-Ericksen tensor $\mathbf{A}_1$ is defined by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T. \tag{3}$$

Now, consider the MHD Sakiadis flow with a uniform magnetic field, $\mathbf{B}_0$, is imposed along the y-direction. After neglecting the induced magnetic field, the steady flow is then governed by the equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

with the momentum equation along the x-direction is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\left(-\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u\right), \tag{5}$$

and the momentum equation along the y-direction is

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\left(-\frac{\partial p}{\partial y} + \frac{\partial S_{yx}}{\partial x} + \frac{\partial S_{yy}}{\partial y}\right), \tag{6}$$

where u and v are the velocities in the x- and y- directions, respectively, ρ is the fluid density, σ is the electrical conductivity, p is the pressure and S_{xx}, S_{xy}, S_{yx} and S_{yy} are the components of the extra stress tensor. By using the boundary layer approximations (see [13] and [15]),

$$u = O(1), \quad v = O(\delta), \quad x = O(1), \quad y = O(\delta), \tag{7}$$

$$\frac{S_{xx}}{\rho} = O(1), \quad \frac{S_{xy}}{\rho} = O(\delta), \quad \frac{S_{yy}}{\rho} = O(\delta^2), \tag{8}$$

where δ is the boundary layer thickness (see [17]), the flow without the influence of pressure gradient is now governed by (4) and (see [11]),

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u. \tag{9}$$

The boundary conditions are given by

$$u = U, \quad v = 0 \quad \text{at} \quad y = 0,$$
$$u \to 0 \quad \text{as} \quad y \to \infty. \tag{10}$$

By introducing the stream function ψ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{11}$$

equation (4) is identically satisfied and equation (9) becomes

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}$$
$$= \nu \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho}\frac{\partial \psi}{\partial y} - \lambda\left[\left(\frac{\partial \psi}{\partial y}\right)^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} + \left(\frac{\partial \psi}{\partial x}\right)^2 \frac{\partial^3 \psi}{\partial y^3} - 2\left(\frac{\partial \psi}{\partial y}\right)\left(\frac{\partial \psi}{\partial x}\right)\frac{\partial^3 \psi}{\partial x \partial y^2}\right]. \tag{12}$$

By using the following similarity transformation (see [11] and [13]):

$$\eta = \sqrt{\frac{U}{\nu x}}\, y, \quad \psi = \sqrt{\nu x U}\, f(\eta), \tag{13}$$

where U is the withdrawal velocity and ν is the kinematic viscosity, (12) becomes

$$2f''' - 2M^2 f' + ff'' - \beta(2ff'f'' + f^2 f''' + \eta f'^2 f'') = 0, \tag{14}$$

where the magnetic parameter $M^2 = \sigma B_0^2 x / \rho U$ and the Deborah number $\beta = \lambda U / 2x$, and prime $(')$ denotes the differentiation with respect to η. The boundary conditions (10) are transformed into

$$f = 0, \quad f' = 1 \quad \text{at} \quad \eta = 0,$$
$$f' \to 0 \quad \text{as} \quad \eta \to \infty. \tag{15}$$

3. Numerical Procedure

Equation (14) subject to the boundary conditions (15) is solved numerically using an implicit finite-difference scheme known as the Keller-box method, as described by Cebeci and Bradshaw [1, 2]. The method has the following four main steps:

(i) Reduce equation (14) to a first order equation.

(ii) Write the difference equations using central differences.

(iii) Linearize the resulting algebraic equation by Newton's method and write in matrix-vector form.

(iv) Use the block-tridiagonal-elimination technique to solve the linear system.

4. Results and Discussion

Equation (14) subject to the boundary conditions (15) is solved numerically [1, 2]. The numerical value of the skin friction coefficient at the wall $f''(0)$ is given in Table 1 for various values of the Deborah number β and the magnetic parameter *M*. It is observed that the magnitude of the skin friction coefficient decreases with the increment of Deborah number β when the magnetic parameter *M* is fixed, except for the case of $\beta = 2.0.$ The effect is the opposite for the variation in magnetic parameter *M*, namely the magnitude of the skin friction coefficient increases with the increment of *M* when β is fixed.

Table 1. Values of $f''(0)$ for various values of β and *M*

β	$M = 0.00$	$M = 0.25$	$M = 0.50$	$M = 1.00$
0.0	–0.443903	–0.503783	–0.657239	–0.859284
0.4	–0.408902	–0.467862	–0.622057	–0.828268
0.8	–0.363669	–0.425801	–0.586407	–0.798052
1.2	–0.300159	–0.366485	–0.543091	–0.766302
1.6	–0.238798	–0.305375	–0.524208	–0.754476
2.0	–0.400956	–0.471673	–0.620967	–0.808917

Figures 1 and 2 illustrate the effects of the Deborah number β on the velocity profiles $f'(\eta)$ and $f(\eta)$ in the *x*-direction, respectively, for the case of the hydrodynamic flow. It is shown that as β increases, both $f'(\eta)$ and $f(\eta)$ profiles decrease. On the other hand, Figures 3 and 4 illustrate the effect of the presence of a uniform imposed magnetic field in the same flow as Figures 1 and 2, and it is observed that the magnetic field has strengthened the thinning effect. Figures 5 and 6 show the effects of the magnetic parameter *M* on the velocity profiles $f'(\eta)$ and $f(\eta).$ It is observed that the increasing of magnetic parameter will cause the decreasing of both $f'(\eta)$ and $f(\eta)$ profiles and the increasing of the boundary layer thickness. Besides that, the boundary layer thickness increases for the velocity at the *x*-component and decreases for the *y*-component. Most of the numerical solutions obtained in the present study are in good agreement with the analytical solutions obtained by Hayat and Sajid [11] who solved the problem via homotopy analysis method.

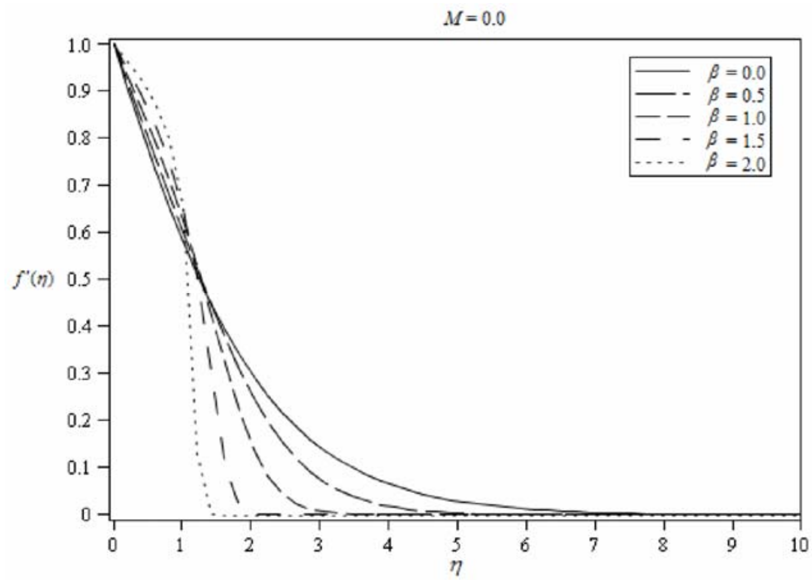

Figure 1. Effect of Deborah number β on the velocity fields $f'(\eta)$ when $M = 0.0$.

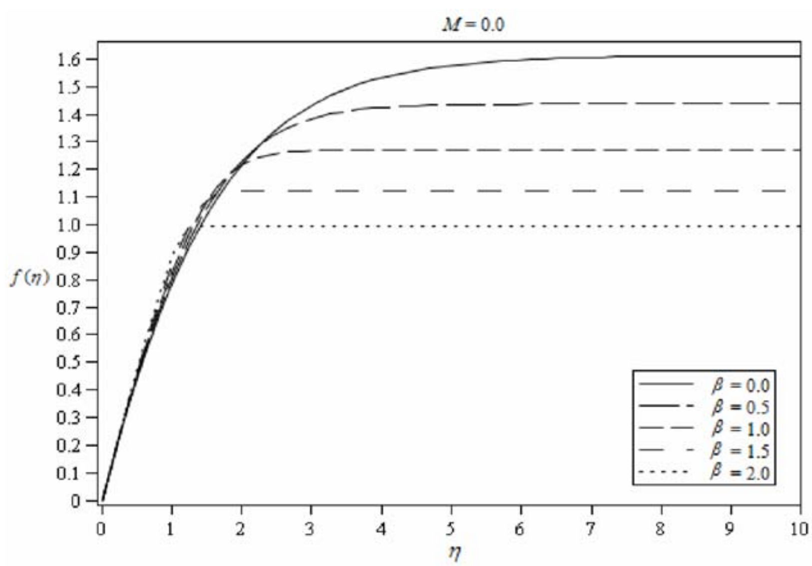

Figure 2. Effect of Deborah number β on $f(\eta)$ fields when $M = 0.0$.

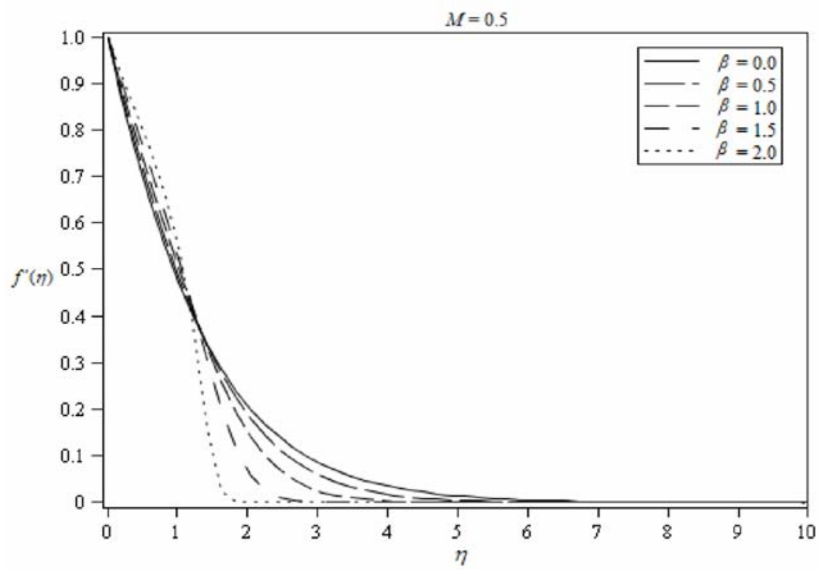

Figure 3. Effect of Deborah number β on the velocity fields $f'(\eta)$ when $M = 0.5$.

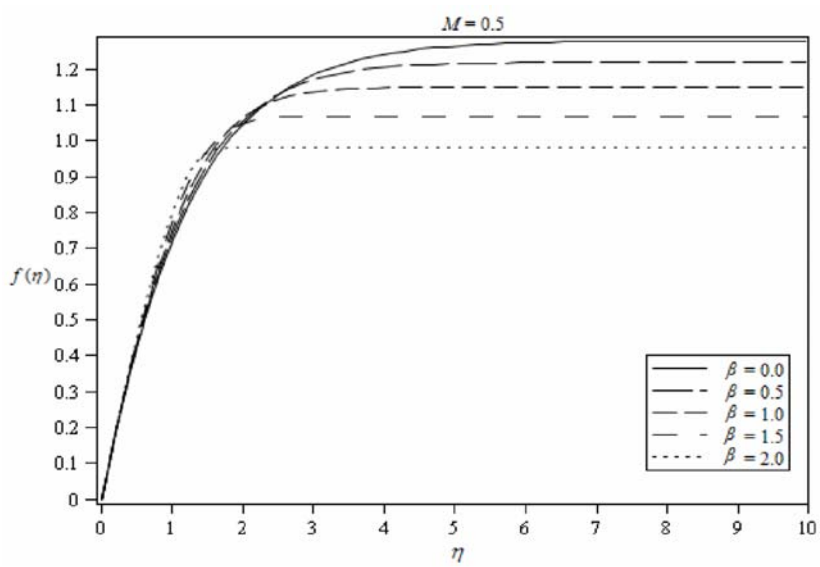

Figure 4. Effect of Deborah number β on $f(\eta)$ fields when $M = 0.5$.

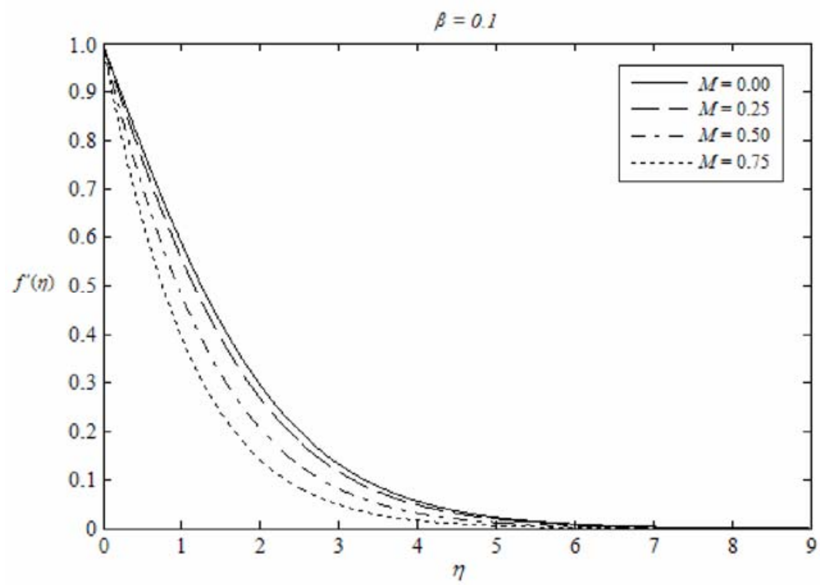

Figure 5. Effect of magnetic parameter M on the velocity fields $f'(\eta)$ when $\beta = 0.1$.

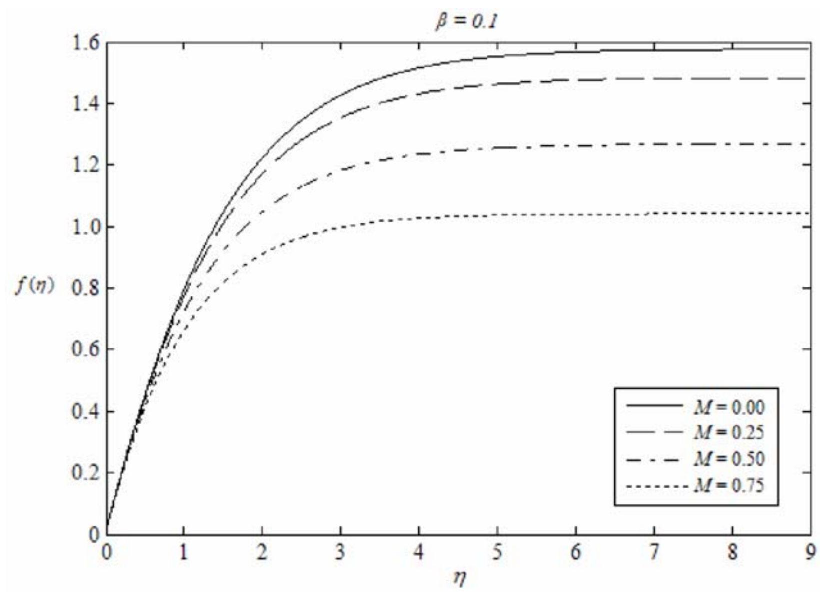

Figure 6. Effect of magnetic parameter M on $f(\eta)$ fields when $\beta = 0.1$.

5. Conclusion

A numerical study is performed to solve the problem of magnetohydrodynamics boundary layer flow of an upper-convected Maxwell fluid over a rigid plate. Effects of Deborah number and MHD parameter on the skin friction coefficient and velocity fields are observed and discussed and it is found that the numerical solutions obtained in the present study are in good agreement with the analytical solutions obtained in [11].

Acknowledgement

The authors would like to acknowledge the financial support received from the Universiti Kebangsaan Malaysia and the Ministry of Higher Education, Malaysia under the fundamental research grant (UKM-ST-07-FRGS0036-2009).

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