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A CONVERGENCE STUDY OF A DECOMPOSITION METHOD FOR SOME NONLINEAR DIFFUSION EQUATIONS

NGARKODJE NGARASTA, BENJAMIN MAMPASSI and OUMAR TRAORE

Département de Mathématiques et Informatique
Faculté des Sciences Exactes et Appliqées
Université de Ndjamena
Chad
e-mail: ngarkodje@yahoo.fr

Université Cheikh Anta Diop de Dakar
Senegal
e-mail: mampassi@yahoo.fr
mampassi@hotmail.com

Ufr/SEA-Département de Mathématiques
Université de Ouagadougou
Burkina Faso
e-mail: ou_traoret@yahoo.fr
traore.oumar@univ-ouaga.bf

Abstract

This paper aims at studying a two level iterative scheme for solving nonlinear parabolic partial differential equations. This scheme combines two basic ideas, the fixed iterative and the so called Adomian decomposition method. We establish the convergence of this scheme and prove its efficiency throughout numerical study of some relevant examples.

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1. Introduction

Let Ω be a bounded domain of $\mathbb{R}^n$ $(n > 0)$ and $T > 0$ be a fixed number. Set $Q =]0, T[\times\Omega$ and $\Sigma =]0, T[\times\partial\Omega.$ In the present work, we are interested in solving the following reaction-diffusion model with homogeneous Dirichlet boundary value conditions:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = N(u), & \text{in } Q, \\ u(t, x) = u_1(t, \sigma), & \text{on } \Sigma, \\ u(0, x) = u_0(x), & \text{in } \Omega, \end{cases} \tag{1}$$

where N is a nonlinear operator and u_0 is a given function. In the following, we will sometime set $u(t) = u(t, .)$.

We assume throughout that N is locally Lipschitzian over $L^2(0, T; H^1(\Omega)),$ and u_1 is smooth enough. We further need following assumptions:

(H_1) If $u \in L^2(0, T; H^1(\Omega)),$ then $N(u) \in L^2(Q).$

(H_2) There exists $V \subset L^2(0, T; H^1(\Omega))$ a vicinity of u_0 and a constant μ depending of V such that $0 < \mu < 1$ and

$$\| N(v) - N(w) \|_{L^2(Q)} \le \mu \| v - w \|_{L^2(0, T; H^1(\Omega))}, \quad \forall v, w \in V. \tag{2}$$

Note that such a problem often occurs in many practical situations involving diffusion phenomena [6, 7, 11, 12, 15]. We also notice that various authors have previously studied problems of type (1). Notably, many of them have proved that for some convenable assumptions on the operator N, there exists a unique solution u in $C^1(0, T; H_0^1(\Omega)).$ We refer to [5, 8, 11] for further information about the existence and uniqueness of parabolic equation type. Subsequently, we agree to call *solution of the system* (1), any function $u \in L^2(0, T; H^1(\Omega))$ that satisfies

$$\int_\Omega u(T)\phi + \int_Q \nabla u \cdot \nabla\phi = \int_\Omega u_0\phi + \int_Q N(u)\phi, \quad \forall\phi \in H_0^1(\Omega). \tag{3}$$

Among various numerical methods for differential equations, the Adomian decomposition method has shown good skill for determining the solutions as polynomial series forms. This method has become as an efficient one for nonlinear ordinary differential equations. In [3], the authors have proposed a new methodology combining decomposition method with fixed point iterative ones to solve nonlinear differential equations. But even in [3], the authors provided many illustrated examples, they did not study the convergence property of the resulting numerical scheme. In this present paper, we shall mainly pay our attention in establishing a convergence result.

The outline of this paper is as follows. In Section 2, we briefly recall the numerical scheme developed in [3]. In Section 3, we discuss the convergence of the scheme and in Section 4, we give some illustration examples.

2. The Numerical Scheme

Let us now recall the numerical scheme developed in [3] for the following system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = L(u) + N(u), & t_0 < t < T, \\ u(t_0) = u_0, \end{cases} \tag{4}$$

where L is a linear differential operator and N is a nonlinear operator. By the basic idea of the successive iterative method we can approach the problem (4) as follows: Find the sequence of functions $(u^k)_{k \geq 0}$ such that

$$\begin{cases} \dfrac{\partial u^k}{\partial t} = L(u^k) + N(u^{k-1}), & t_0 < t < T, \\ u^k(t_0) = u_0, & k = 1, 2, \ldots, \end{cases} \tag{5}$$

where the first term u^0 of the sequence is arbitrary chosen. Thus, after determining the sequence satisfying (5), the solution of the problem (4) will be formally obtained as the limit of this sequence:

$$u = \lim_{k \to \infty} u^k. \tag{6}$$

Here the basic idea consists in calculating the solution of (4) in each iteration as Adomian polynomial series. To this end let us consider the following operators:

$$L_1 = \frac{\partial}{\partial t} \quad \text{and} \quad L_1^{-1}(.) = \int_{t_0}^{t} (.)\, ds, \tag{7}$$

then integrating the equation (5) yields the Adomian canonical form [1, 13, 15, 17], that is,

$$u^k = u^k(t_0) + L_1^{-1}L(u^k) + L_1^{-1}N(u^{k-1}) \tag{8}$$

which provides the following Adomian algorithm:

$$\begin{aligned} u_0^k &= u^k(t_0) + L_1^{-1}N(u^{k-1}), \\ u_1^k &= L_1^{-1}(L(u_0^k)), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ u_n^k &= L_1^{-1}(L(u_{n-1}^k)) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{9}$$

From which, the kth iterative solution is expressed as an Adomian series

$$u^k = \sum_n u_n^k \tag{10}$$

so that we have formally

$$u = \lim_{k \to \infty} \sum_n u_n^k. \tag{11}$$

It should be noticed that numerically the first term u^0 of iteration must be chosen to permit a simplification of calculations and to obtain the Adomian algorithm which leads to higher accuracy. We note that a large number of papers are concerned with the convergence of the Adomian algorithm. See for example [2, 15, 17]. In particular, in the linear case, the convergence of the Adomian algorithm is obvious.

3. The Convergence Study

Let us consider the following iterative scheme associated with the system

$$\begin{cases} \dfrac{\partial u^{k+1}}{\partial t} = \Delta u^{k+1} + N(u^k), & \text{in } Q, \\ u^{k+1}(0) = u_0, & \text{in } \Omega, \\ u^{k+1} = u_1, & \text{on } \Sigma. \end{cases} \tag{12}$$

We need to establish the existence of the sequence $(u^k)_{k\geq 0}$ defined in (12) that converges to the solution of the original problem (1). Let us start by re-calling the following classical result [8].

Theorem 1. *Let us consider the following parabolic problem*:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = f, & \text{in } Q, \\ u(0) = g, & \text{in } \Omega, \\ u = u_1, & \text{on } \Sigma. \end{cases} \tag{13}$$

If $f \in L^2(0, T; H^{-1}(\Omega))$ *and* $g \in L(\Omega)$, *then there exists a unique* $u \in L^2(0, T; H_0^1(\Omega)) \cap C^0(0, T; H_0^1(\Omega))$ *solution of* (13). *Moreover we have*

$$\int_\Omega u(T)\phi + \int_Q \nabla u \cdot \nabla\phi = \int_\Omega g\phi + \int_Q f\phi + \int_\Sigma u_1\phi, \quad \forall\phi = H_0^1(\Omega). \tag{14}$$

We can now state the main result of this paper.

Theorem 2. *For all* $u^0 \in V$, *there exists a unique sequence* $(u^k)_{k\geq 0}$ *in* $L^2(0, T; H_0^1(\Omega))$ *whose terms satisfy the iterative scheme* (12) *such that* $u^k \to u$ *in* $L^2(0, T; H_0^1(\Omega))$, *where u is solution of* (2).

Proof. (i) **Existence and unicity.** By hypothesis (H_1), if $u^0 \in V$, then $N(u^0) \in L^2(0, T; L^2(\Omega))$. Thus, applying Theorem 1 by setting $f = N(u^0)$, there exits a unique solution named $u^1 \in L^2(0, T; H_0^1(\Omega))$ satisfying (12) for $k = 0$. Suppose that there exists a unique subsequence $(u^k)_{0\leq k\leq n} \subset L^2(0, T; H_0^1(\Omega))$ whose terms satisfy (12). Then, applying Theorem 1 by setting $f = N(u^k)$, there exits a unique solution named $u^{k+1} \in L^2(0, T; H_0^1(\Omega))$ satisfying (12). Consequently, by the induction hypothesis, the result follows.

(ii) **Convergence of the sequence** $(u^k)_{k\geq 0}$**.** Let us set $w^k = u^{k+1} - u^k$. Then we have

$$\begin{cases} \dfrac{\partial w^k}{\partial t} = \Delta w^k + N(u^k) - N(u^{k-1}), & \text{in } Q, \\ w^k(0) = 0, & \text{in } \Omega, \\ w^k = 0, & \text{on } \Sigma. \end{cases} \tag{15}$$

By Theorem 1, we have

$$\int_\Omega w^k(T)\phi + \int_Q \nabla w^k \cdot \nabla\phi = \int_Q [N(u^k) - N(u^{k-1})]\phi, \quad \forall \phi \in H_0^1(\Omega). \tag{16}$$

Setting $\phi = w^k$, it follows

$$\int_\Omega |w^k(T)|^2 + \int_Q |\nabla w^k|^2 = \int_Q [N(u^k) - N(u^{k-1})] w^k \tag{17}$$

thus, we deduce

$$\begin{aligned} \int_Q |\nabla w^k|^2 &\le \int_Q [N(u^k) - N(u^{k-1})] w^k \\ &\le \|N(u^k) - N(u^{k-1})\|_{L^2(Q)} \|w^k\|_{L^2(Q)}. \end{aligned} \tag{18}$$

Knowing that $\|w^k\|^2_{L^2(0,\, T;\, H_0^1(\Omega))} = \int_Q |\nabla w^k|^2$ and using Poincare inequality we obtain

$$\|w^k\|_{L^2(0,\, T;\, H_0^1(\Omega))} \le \|N(u^k) - N(u^{k-1})\|_{L^2(Q)} \tag{19}$$

that we rewrite as

$$\|u^{k+1} - u^k\|_{L^2(0,\, T;\, H_0^1(\Omega))} \le \|N(u^k) - N(u^{k-1})\|_{L^2(Q)}. \tag{20}$$

Noticing that, since $u^0 \in V$, $u^k(t) \in V$, for all k, it follows due to the assumption (H_2) that

$$\|N(u^k) - N(u^{k-1})\|_{L^2(Q)} \le \mu \|u^k - u^{k-1}\|_{L^2(0,\, T;\, H_0^1(\Omega))} \tag{21}$$

with $0 < \mu < 1$. From where we deduce

$$\|(u^{k+1}) - u^k\|_{L^2(0,\, T;\, H_0^1(\Omega))} \le \mu \|u^k - u^{k-1}\|_{L^2(0,\, T;\, H_0^1(\Omega))}. \tag{22}$$

This shows that $(u^k)_{k\geq 0}$ is the Cauchy sequence in $L^2(0, T; H_0^1(\Omega))$. Consequently, $u^k \to u$ in $L^2(0, T; H_0^1(\Omega))$. To finish the demonstration we have by Theorem 1,

$$\int_\Omega u^{k+1}(T)\phi + \int_Q \nabla u^{k+1} \cdot \nabla\phi = \int_Q u^{k+1}\phi + \int_Q N(u^k)\phi, \quad \forall\phi \in H_0^1(\Omega), \tag{23}$$

then passing to the limit and using the fact that N is locally Lipschitzian, it follows

$$\int_\Omega u(T)\phi T + \int_Q \nabla u \cdot \nabla\phi = \int_\Omega u\phi + \int_Q N(u)\phi, \quad \forall\phi \in H_0^1(\Omega) \tag{24}$$

thus, u is the solution of (1).

4. Numerical Examples

To gain insights into the convergence issue of the numerical scheme presented in this paper, we have chosen two test examples. All of them concern nonlinear parabolic equations from which analytical solutions are very difficult to compute.

4.1. Example 1

Consider the following nonlinear PDE

$$\begin{cases} \dfrac{\partial u}{\partial t} = \lambda\dfrac{\partial^2 u}{\partial x^2} + u + \left(u\dfrac{\partial u}{\partial x}\right)^3 + \left(u\dfrac{\partial^3 u}{\partial x^3}\right)^3, & \text{in } Q, \\ u(t, x) = 0, & \text{on } \Sigma, \\ u(0, x) = \cos(x) - \sin(x) & \text{in } \Omega \end{cases} \tag{25}$$

with $\Omega = (-5\pi/4, \pi/4)$. Let set

$$N(u) = \left(u\frac{\partial u}{\partial x}\right)^3 + \left(u\frac{\partial^3 u}{\partial x^3}\right)^3. \tag{26}$$

As seen in Section 2, the kth order iterative Adomian canonical form for this system is

$$u^k = \cos(x) - \sin(x) + \lambda\int_0^t \frac{\partial^2 u^k}{\partial x^2}(s)ds + \int_0^t u^k(s)ds + \int_0^t N(u^{k-1}(s))ds. \tag{27}$$

Let us choose the first term u^0 as constant. Then we have $N(u^0) = 0$. We have successively:

- $k = 1$:

$$u_0^1 = \cos(x) - \sin(x) + \int_0^t N(u^0) = \cos(x) - \sin(x),$$

$$u_1^1 = \lambda \int_0^t \frac{\partial^2 u_0^1}{\partial x^2}(s)\, ds + \int_0^t u_0^1(s)\, ds = (1-\lambda)t(\cos(x) - \sin(x)),$$

$$u_2^1 = \lambda \int_0^t \frac{\partial^2 u_1^1}{\partial x^2}(s)\, ds + \int_0^t u_1^1(s)\, ds = (1-\lambda)^2 \frac{t^2}{2}(\cos(x) - \sin(x)),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$u_n^1 = \lambda \int_0^t \frac{\partial^2 u_{n-1}^1}{\partial x^2}(s)\, ds + \int_0^t u_{n-1}^1(s)\, ds = (1-\lambda)^n \frac{t^n}{n!}(\cos(x) - \sin(x)). \tag{28}$$

Thus,

$$u^1 = \sum_{n \geq 0} (1-\lambda)^n \frac{t^n}{n!}(\cos(x) - \sin(x)) = (\cos(x) - \sin(x))e^{(1-\lambda)t}. \tag{29}$$

- $k \geq 2$:

It is easy to establish that

$$u^1 = u^2 = \cdots = u^p = \cdots = (\alpha \cos(x) + \beta \sin(x))e^{(1-\lambda)t}. \tag{30}$$

In fact, if $u^{k-1} = (\cos(x) - \sin(x))e^{(1-\lambda)t}$, then we have $N(u^{k-1}) = 0$. Consequently, $u_n^k = u_n^1$, thus $u^k = (\cos(x) - \sin(x))e^{(1-\lambda)t}$. Then

$$u = \lim_{k \to \infty} u^k (\cos(x) - \sin(x))e^{(1-\lambda)t} \tag{31}$$

which is the exact solution of Example 1.

4.2. Example 2

Our second test example is the Allen-Cahn equation

$$\frac{\partial u}{\partial t} = \varepsilon \frac{\partial^2 u}{\partial x^2} + u - u^3, \tag{32}$$

where ε is a parameter that is assumed to be small enough. This is an example of a nonlinear reaction-diffusion equation. This type of equation provides a phenomenon known as metastability. As in [18], we consider the following boundary and initial conditions

$$\begin{cases} u(t,\ -1) = -1, \quad u(t,\ 1) = 1, \\ u(0,\ x) = 0.53x + 0.47\sin(-1.5\pi x). \end{cases} \tag{33}$$

Let us note that in this example the boundary condition is not homogeneous but the numerical scheme developed in this paper is also applicable. The *k*th order iterative Adomian canonical form for this system is given by

$$\begin{aligned} u^k &= 0.53x + 0.47\sin(-1.5\pi x) \\ &\quad + \varepsilon\int_0^t \frac{\partial^2 u^k}{\partial x^2}(s)\,ds + \int_0^t u^k(s)\,ds + \int_0^t N(u^{k-1}(s))\,ds. \end{aligned} \tag{34}$$

Approximated solutions for various values of order iterative Adomian canonical k with *n*th truncated Adoinian series are pictured in Figures 1 and 2. All these solutions are obtained using Maple. For our computation we have taken as the first term, $u^0 = 1.$ Thus, $N(u^0) = 0.$ The approximated analytical solution for $k = 3$ and $n = 2$ is given by

$$\begin{aligned} &u(t, x) \\ &\approx \frac{53}{100}x - \frac{47}{100}\sin\left(\frac{3\pi x}{2}\right) \\ &\quad + t\left(-\left(\frac{53}{100}x - \frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3 + \frac{53}{100}x - \frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right) + \frac{423}{400}t\varepsilon\pi^2\sin\left(\frac{3\pi x}{2}\right). \end{aligned} \tag{35}$$

This solution is pictured in Figure 1. For $k = 5$ and $n = 5,$ the solution is given by the following

$$\begin{aligned} &u(t,\ x) \\ &\approx t^4\left(\left(-\left(\frac{53}{100}x - \frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3 + \frac{53}{100}x - \frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3\right. \end{aligned}$$

$$-\frac{1269}{1600}\pi^2\varepsilon\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^2\sin\left(\frac{3\pi x}{2}\right)\Bigg)$$

$$+t^3\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)\right)\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3\right.$$

$$\left.+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^2-\frac{423}{200}\pi^2\varepsilon\sin\left(\frac{3\pi x}{2}\right)\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)$$

$$\times\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)\Bigg)$$

$$+t^2\left(-\frac{3}{2}\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^2\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3\right.\right.$$

$$\left.+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)+\varepsilon\left(-3\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)\right.$$

$$\times\left(\frac{53}{100}-\frac{141\pi}{200}\cos\left(\frac{3\pi x}{2}\right)\right)^2-\frac{1269\pi^2}{400}\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^2\sin\left(\frac{3\pi x}{2}\right)$$

$$\left.\left.+\frac{423\pi^2}{400}\sin\left(\frac{3\pi x}{2}\right)\right)-\frac{47}{200}\sin\left(\frac{3\pi x}{2}\right)+\frac{53}{200}x-\frac{1}{2}\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3\right)$$

$$+t\left(-\left(\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)\right)^3+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right)+\frac{423\pi^2\varepsilon}{400}\sin\left(\frac{3\pi x}{2}\right)\right)$$

$$+\frac{53}{100}x-\frac{47}{100}\sin\left(\frac{3\pi x}{2}\right). \tag{36}$$

This solution is pictured in Figure 2 and is very close to the exact ones (see [18]). We see the rapid convergence to the exact solution with a small value of the truncated parameter of the Adomian series as well as the iterative order parameter.

5. Conclusion

The Adomian method has proved reliable for linear or nonlinear ordinary differential equations. However this method in its original form is less effective for partial derivative equations (PDEs). In a systematic way, it does not take into account all boundary and initial conditions. Several attempts were made without success to adapt it to PDE. In this paper, we have combined the fixed points iterative technique with that of the Adomian method for obtaining a two steps numerical scheme which not only take into account all the boundary and initial conditions but make it possible to accelerate convergence towards the exact solution. Examples 1 and 2 are illustrations. It is necessary to underline, a considerable advantage by the scheme presented in this work is at the level of stability. The stability issue is indeed very characteristic for classical numerical schemes applied to the nonlinear parabolic equations. However, one of the difficulties of this scheme lies in the determination of the calculation of the analytical expression of the solution. On the other hand, this numerical scheme probably may be a good alternative for the numerical resolution of the parabolic PDE being able to present blow up phenomena.

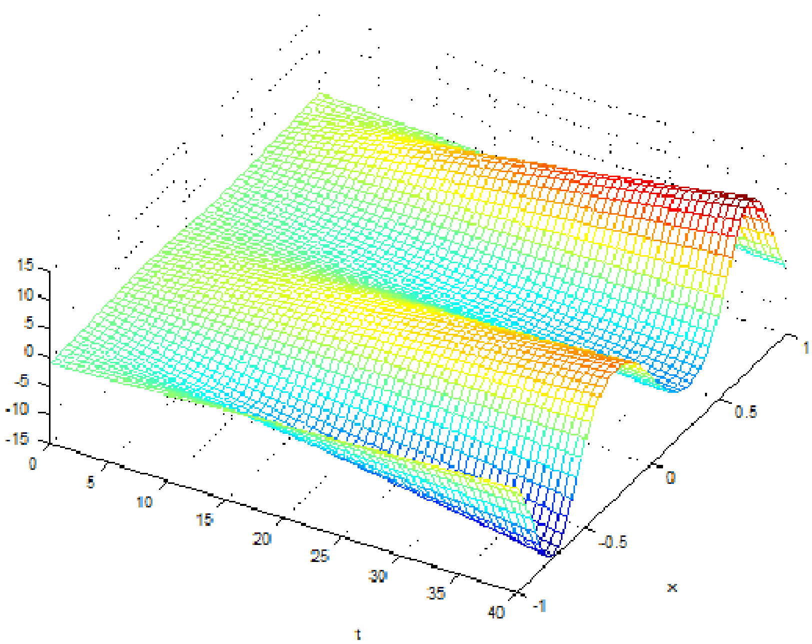

Figure 1. The approximated solution for truncated parameters $k = 3$ and $n = 2$ with $\varepsilon = 10^{-2}$.

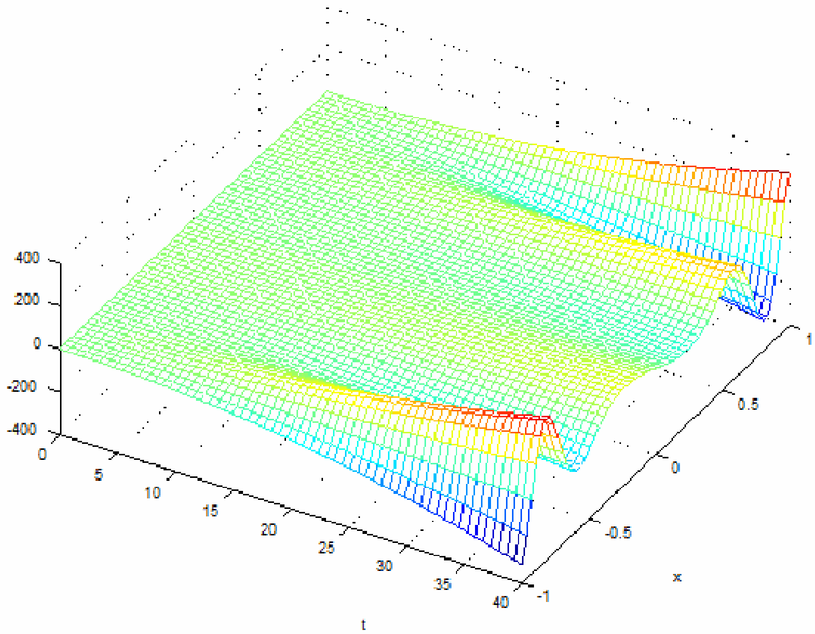

Figure 2. The approximated solution for truncated parameters $k = 5$ and $n = 5$ with $\varepsilon = 10^{-2}$.

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