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CONSTRUCTION OF CHAIN SAMPLING PLAN-2 INDEXED THROUGH CONVEX COMBINATION OF AOQL AND MAAOQ

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Abstract

In chain sampling plan, the sampling inspection has the criteria for acceptance and non-acceptance of the lot depends in part on the results of the inspection of immediately preceding lot. Chain sampling plans allow significant reduction in sample size under conditions of a continuing succession of lots from a stable supplier. In this paper, a procedure for constructing a Chain Sampling Plan-2 (ChSP-2) indexed through a parameter AOQ_{cc} which is a convex combination of AOQL (Average Outgoing Quality Limit) and MAAOQ (Maximum Allowable Average Outgoing Quality) with gain parameter λ is given. This plan may safeguard the interests of both producer as well as consumer by properly choosing a right combination using the gain parameter λ. Tables are also constructed for the easy selection of the plan.

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1. Introduction

The proportion defective corresponding to the inflection point of the OC curve is interpreted as the Maximum Allowable Percent Defective (MAPD). The desirability of developing a set of sampling plans indexed through p^* (MAPD) has been explained by Mandelson [1] and Soundararajan [11]. AOQL is defined as the worst average quality that the consumer will receive in the long run, when the defective items are replaced by non-defective items. Suresh and Ramkumar [12] suggested a procedure for constructing the sampling plans indexed through MAAOQ. Radhakrishnan [2] constructed continuous, chain, double and link sampling plans and established that the sample size is less in the inspection when MAAOQ is used as an indexed parameter than AOQL. Sekkizhar [9] constructed sampling plans using MAAOQ with intervened random effect Poisson distribution. Sampathkumar [8] constructed mixed sampling plan with chain sampling plans as attribute plan indexed through AOQL, MAPD, MAAOQ and emphasized the superiority of MAAOQ. Radhakrishnan and Mallika [3-6] constructed single, double, chain sampling plan-1 and link sampling plans indexed through AOQ_{cc} which is the convex combination of AOQL and MAAOQ.

Shankar et al. [10] developed a new chain sampling plan of the type ChSP-2 in which a lot even with two defects in its sample can still be accepted if last defect was found far enough back in the history of products.

2. Terminologies

2.1. Definition of MAPD

The MAPD is the value of fraction defective $(p = p^*)$ at which

$$d^2\text{Pa}(p)/dp^2 = 0 \quad \text{and} \quad d^3\text{Pa}(p)/dp^3 \neq 0.$$

It is also the inflection point of the Operating Characteristic (OC) curve and $\text{Pa}(p)$ is the probability of acceptance at the quality level p fraction defective.

2.2. Definition of AOQL

The AOQL is defined as the *worst average quality* that the consumer will receive in the end, when the defective items are replaced by non-defective items. It is obtained by maximizing Average Outgoing Quality (AOQ), $\text{AOQ} = p \cdot \text{Pa}(p)$.

2.3. Definition of MAAOQ

The MAAOQ of conditional double sampling plan is defined by the *Average Outgoing Quality* (AOQ) at MAPD, i.e., $\text{AOQ} = p \cdot \text{Pa}(p)$.

Thus $\text{MAAOQ} = \text{AOQ}$ at $p = p^*$. This can be written as $\text{MAAOQ} = p^* \cdot \text{Pa}(p^*)$.

2.4. Definition of AOQ_{cc}

It is the convex combination of AOQL and MAAOQ with gain parameter λ,

$$\text{AOQ}_{\text{cc}} = \lambda\text{AOQL} + (1 - \lambda)\,\text{MAAOQ}.$$

3. Operating Procedure of ChSP-2

1. Select a random sample of size '*n*' from a lot size '*N*' and count the number of non-conformities in the sample.

2. Accept the lot, if

(a) No non-conformities are found in the sample.

(b) One or two non-conformities are found in the sample but no non-conformities were found in the previous '*i*' samples.

3. Reject the lot, otherwise.

4. Glossary of Symbols

N	-lot size
n	-sample size
d	-number of non-conformities
λ	-gain parameter, $0 < \lambda < 1$
p	-submitted lot quality of lot or process
p^*	-Maximum Allowable Percent Defective (MAPD)
$\text{Pa}(p)$	-probability of acceptance for given quality p
AOQ_{cc}	-convex combination of AOQL and MAAOQ
i	-number of previous samples from which the decision of lot is made.

5. Operating Characteristic Function of ChSP-2

Under Poisson model, the OC function of ChSP-2 plan is given by

$$\text{Pa}(p) = P_0 + P_2 P_0^i + \frac{P_1 P_0^i}{1 - P_1},$$

where $P_0 = e^{-np}$, $P_1 = npe^{-np}$ and $P_2 = \frac{(np)^2 e^{-np}}{2}$.

6. Construction of ChSP-2 Indexed through AOQ_{cc}

The general procedure for designing a ChSP-2 indexed through a parameter which is a convex combination of AOQL and MAAOQ using Poisson distribution as base line distribution is given below.

Step 1. Determine nMAPD, nMAAOQ and nAOQL for ChSP-2 for various values of *i*, find $R_1 = n\text{AOQL}/n\text{MAPD}$ and $R_2 = n\text{MAAOQ}/n\text{MAPD}$.

Step 2. Find $n\text{AOQ}_{\text{cc}} = \lambda n\text{AOQL} + (1 - \lambda) n\text{MAAOQ}$ for various values of λ and find $R_3 = n\text{AOQ}_{\text{cc}}/n\text{MAPD}$.

Step 3. Present the results of Steps 1 and 2 for various values of λ (from 0.1 to 0.9) in Table 1.

7. Selection of the Plan

Table 1 is used to construct the plan when the MAPD and AOQ_{cc} are specified. We can find the ratio $R_3 = \text{AOQ}_{\text{cc}}/\text{MAPD}$ and locate the value in Table 1 under the column R_3 (for a fixed value of λ) and the corresponding values of *i* and np^* are noted. The value of *n* is determined using $n = np^*/\text{MAPD}$ and hence the parameters, *n* and *i* are determined.

Example 1. For a specified $\text{AOQL} = 0.0078$ and $\text{MAPD} = 0.010$ compute the ratio $R_1 = \text{AOQL}/\text{MAPD} = 0.7800$ which is nearer to the value 0.7729 and the associated values of i and *n* from Table 1 are $i = 1$ and $n = np^*/\text{MAPD} = 84$. Thus the sampling plan for specified $\text{AOQL} = 0.0078$ is $n = 84$ and $i = 1$.

For a specified MAAOQ = 0.0080 and MAPD = 0.010 compute the ratio R_2 = MAAOQ/MAPD = 0.8000 which is nearer to the value 0.8046 and the associated values of i and n from Table 1 are $i = 5$, $n = np^*/\text{MAPD} = 31$. Thus the sampling plans for specified MAAOQ = 0.0080 are $n = 31$ and $i = 5$.

For a specified value of AOQL = 0.0078, MAAOQ = 0.0080, MAPD = 0.010 and $\lambda = 0.3$, the value of AOQ_{cc} = 0.00794. Find the ratio $R_3 = \text{AOQ}_{cc}/\text{MAPD}$ = 0.794 and locate the value in R_3 corresponding to $\lambda = 0.2$ which is nearer to the ratio 0.7704 and the values of i and n from Table 1 are $i = 2$ and $n = np^*/\text{MAPD}$ = 59. Thus the sampling plans for specified AOQ_{cc} = 0.00794 are $n = 59$ and $i = 2$.

Explanation. In a transistor manufacturing company, if the producer fixes the quality level MAPD = 0.010 (10 non-conformities out of 1000) and the consumer fixes the quality level AOQ_{cc} = 0.00794 (794 non-conformities out of 1,00,000) select a sample of 59 transistors from a lot and count the number of non-conformities (d). If $d = 0$, accept the lot and if $d > 2$, reject the lot and inform the management for improving the quality of the product. If $d \leq 2$, accept the lot if no non-conformities are found in the immediately preceding 2 samples.

Example 2. For a specified AOQL = 0.0040 and MAPD = 0.005 compute the ratio R_1 = AOQL/MAPD = 0.8000 which is nearer to the value 0.8056 and the associated values of i and n from Table 1 are $i = 2$ and $n = np^*/\text{MAPD} = 118$. Thus the sampling plans for specified AOQL = 0.0078 is $n = 118$ and $i = 2$.

For a specified MAAOQ = 0.0044 and MAPD = 0.005 compute the ratio R_2 = MAAOQ/MAPD = 0.8800 which is nearer to the value 0.8832 and the associated values of i and n from Table 1 are $i = 11$, $n = np^*/\text{MAPD} = 32$. Thus the sampling plans for specified MAAOQ = 0.0044 are $n = 32$ and $i = 11$.

For a specified value of AOQL = 0.0040, MAAOQ = 0.0044, MAPD = 0.005 and $\lambda = 0.1$, the value of AOQ_{cc} = 0.00436. Find the ratio $R_3 = \text{AOQ}_{cc}/\text{MAPD}$ = 0.872 and locate the value in R_3 corresponding to $\lambda = 0.1$ which is nearer to the ratio 0.8802 and the values of i and n from Table 1 are $i = 5$ and $n = np^*/\text{MAPD} = 62$. Thus the sampling plans for specified $\text{AOQ}_{\text{cc}} = 0.00436$ are $n = 62$ and $i = 5$.

Explanation. In a courier service providing company, if the management fixes the quality level $\text{MAPD} = 0.005$ (5 non-conformities out of 1000) and the customer fixes the quality level $\text{AOQ}_{\text{cc}} = 0.00436$ (436 non-conformities out of 1,00,000) select a sample of 62 services processed in a day and count the number of non-conformity services (d). If $d = 0$, accept the quality of the service provided in that day and if $d > 2$, reject the quality of the service provided in that day and inform the management for improving the quality of the service. If $d \leq 2$, accept the quality of the service provided in that day and no non-conformity services were found in the immediately preceding 2 days services.

The operating characteristic curves for the plans given in Example 1 are presented in Figure 1 and their AOQ curves are presented in Figure 2.

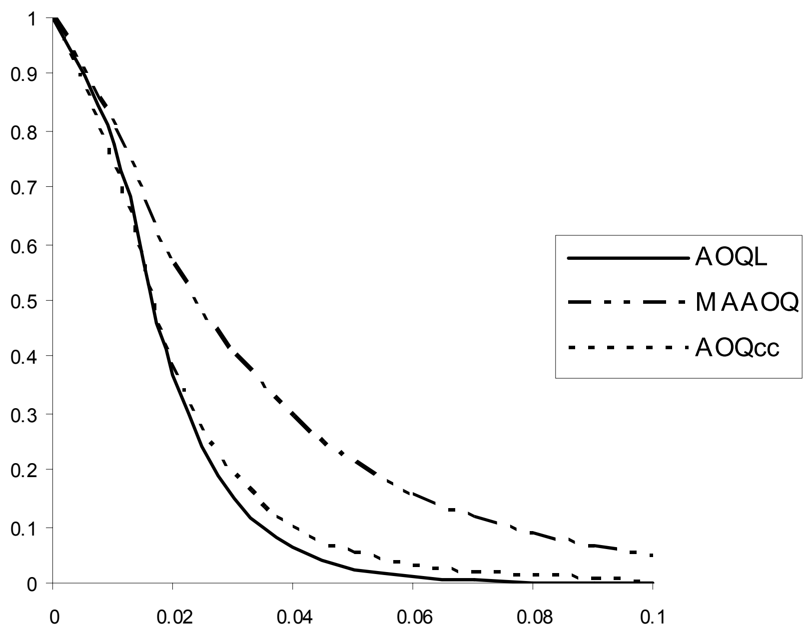

Figure 1. OC curves for $n = 84$ and $i = 1$(AOQL); $n = 31$ and $i = 5$(MAAOQ); $n = 59$ and $i = 2(\text{AOQ}_{\text{cc}})$.

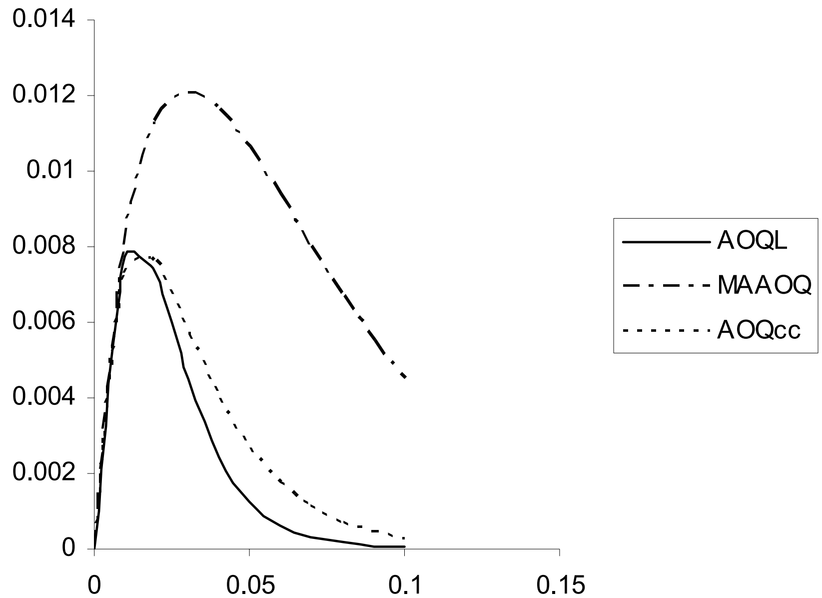

Figure 2. AOQ curves for $n = 84$ and $i = 1(\text{AOQL})$; $n = 31$ and $i = 5(\text{MAAOQ})$; $n = 59$ and $i = 2(\text{AOQ}_{\text{cc}})$.

Conclusion

In this paper, a procedure for the construction and selection of ChSP-2 indexed through gain parameter λ which is a convex combination of AOQL and MAAOQ is stated. Tables are also constructed for the easy selection of the plans when the indexing parameters and gain parameter are known. The engineers after knowing the minds of both producer and consumer can search for the quality level AOQ_{cc} and select the appropriate plan. It is also concluded that the sample size required for the ChSP-2 indexed through (MAPD, AOQ_{cc}) is lesser than the sample size indexed through (MAPD, AOQL) and also satisfies the interests of both the producer as well as the consumer, which can be understood from the OC and AOQ curves. Readymade tables are also provided in this paper for the engineers to take quick decisions on the nature of the sampling plan when the quality level of the producer and consumer are known. This study can be extended for constructing other sampling plans and efficiency of these plans can also be compared with the plans indexed through other parameters.

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Table 1. Parameters for chain sampling plan -2

i	np*	n AOQL	R_1	nMAAOQ	R_2	λ=0.1		λ=0.2		λ=0.3		λ=0.4	
						$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3
1	0.8434	0.6519	0.7729	0.6250	0.7411	0.6302	0.7472	0.6347	0.7526	0.6387	0.7573	0.6421	0.7613
2	0.5913	0.4764	0.8056	0.4331	0.7324	0.4417	0.7470	0.4491	0.7594	0.4556	0.7704	0.4611	0.7799
3	0.4536	0.4114	0.9069	0.3429	0.7559	0.3562	0.7853	0.3678	0.8108	0.3779	0.8332	0.3865	0.8521
4	0.3674	0.3842	1.0456	0.2872	0.7818	0.3061	0.8331	0.3225	0.8777	0.3367	0.9165	0.3489	0.9497
5	0.3086	0.3735	1.2105	0.2483	0.8046	0.2716	0.8802	0.2920	0.9462	0.3099	1.0043	0.3255	1.0548
6	0.2661	0.3698	1.3899	0.2192	0.8238	0.2460	0.9243	0.2697	1.0134	0.2907	1.0923	0.3094	1.1626
7	0.2339	0.3686	1.5757	0.1964	0.8398	0.2259	0.9658	0.2520	1.0773	0.2756	1.1783	0.2969	1.2691
8	0.2087	0.3681	1.7635	0.1781	0.8534	0.2094	1.0032	0.2374	1.1378	0.2631	1.2606	0.2866	1.3734
9	0.1885	0.3680	1.9520	0.1630	0.8648	0.1962	1.0411	0.2263	1.2007	0.2540	1.3474	0.2797	1.4836
10	0.1719	0.3679	2.1399	0.1503	0.8746	0.1847	1.0746	0.2162	1.2578	0.2455	1.4284	0.2730	1.5879
11	0.1580	0.3679	2.3282	0.1395	0.8832	0.1753	1.1093	0.2081	1.3173	0.2391	1.5133	0.2681	1.6967
12	0.1461	0.3679	2.5178	0.1301	0.8907	0.1670	1.1433	0.2012	1.3771	0.2336	1.5986	0.2640	1.8070
13	0.1360	0.3679	2.7048	0.1220	0.8972	0.1599	1.1758	0.1952	1.4355	0.2289	1.6832	0.2606	1.9161
14	0.1372	0.3679	2.6811	0.1239	0.9030	0.1536	1.1194	0.1900	1.3847	0.2248	1.6385	0.2577	1.8781
15	0.1195	0.3679	3.0782	0.1085	0.9082	0.1480	1.2387	0.1854	1.5512	0.2213	1.8522	0.2552	2.1352

Table 1. Parameters of chain sampling plan- 2 (Continued)

i	np*	λ=0.5		λ=0.6		λ=0.7		λ=0.8		λ=0.9	
		$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3	$nAOQ_{cc}$	R_3
1	0.8434	0.6450	0.7648	0.6474	0.7676	0.6492	0.7698	0.6506	0.7714	0.6515	0.7724
2	0.5913	0.4657	0.7876	0.4696	0.7941	0.4725	0.7991	0.4746	0.8026	0.4759	0.8048
3	0.4536	0.3939	0.8683	0.3999	0.8816	0.4046	0.8919	0.4081	0.8997	0.4104	0.9048
4	0.3674	0.3594	0.9782	0.3679	1.0013	0.3747	1.0199	0.3796	1.0333	0.3829	1.0421
5	0.3086	0.3390	1.0985	0.3503	1.1352	0.3596	1.1654	0.3667	1.1883	0.3715	1.2038
6	0.2661	0.3258	1.2242	0.3400	1.2776	0.3518	1.3220	0.3610	1.3566	0.3672	1.3799
7	0.2339	0.3160	1.3509	0.3327	1.4223	0.3468	1.4829	0.3579	1.5302	0.3654	1.5624
8	0.2087	0.3079	1.4754	0.3270	1.5667	0.3431	1.6439	0.3557	1.7046	0.3644	1.7459
9	0.1885	0.3031	1.6082	0.3240	1.7189	0.3415	1.8119	0.3553	1.8848	0.3644	1.9332
10	0.1719	0.2980	1.7336	0.3204	1.8639	0.3393	1.9738	0.3540	2.0595	0.3638	2.1162
11	0.1580	0.2948	1.8657	0.3185	2.0156	0.3383	2.1413	0.3537	2.2386	0.3638	2.3023
12	0.1461	0.2920	1.9988	0.3167	2.1678	0.3374	2.3095	0.3533	2.4184	0.3638	2.4900
13	0.1360	0.2897	2.1303	0.3153	2.3184	0.3367	2.4756	0.3530	2.5955	0.3637	2.6744
14	0.1372	0.2877	2.0970	0.3141	2.2896	0.3360	2.4489	0.3527	2.5706	0.3636	2.6502
15	0.1195	0.2860	2.3932	0.3130	2.6193	0.3354	2.8068	0.3524	2.9491	0.3635	3.0420