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FUZZY PROGRAMMING APPROACH FOR PORTFOLIO OPTIMIZATION

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Abstract

Nowadays, industrial companies are frequently faced with the problem of where the capital should be spent and which combination of projects should be selected from several possible project mixes. Traditional methods of portfolio selection, that is how to select a combination of possible projects, are index-ranking and linear programming. In recent years, we have been able to observe that these methods are insufficient, particularly in long-term programming. The nature of real-world problems requires taking into account uncertainty of the input data and it is very difficult to clearly know all information in deterministic parameters. In this paper, we consider the problem of portfolio selection in an oil and gas

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company to motivate our study. We model this problem as a stochastic programming problem and develop a fuzzy programming approach for solving such a problem.

1. Introduction

Nowadays, industrial companies have to face with many challenges such as which combination of projects gives them the maximum benefit. Consider, for example, a typical large oil and gas company. Such a company operates many explorations and production projects, which involve several billion dollars every year. Here the company has the difficult task of portfolio selection from a large number of competing exploration and production projects for immediate or future operation under limited amount of investments [7].

Companies have traditionally used index-ranking method for portfolio selection. In this method, they first rank their projects based on economic metrics such as net present value, internal rate of return, period of payback among others. They then select the projects beginning at the top of this ranking and continue until a certain financial has been met. Unfortunately, this method is rarely adequate, because in practice a company measures their performance against more than one economic metric and has many constraints. This is where optimization models come into play. In fact, the natural solution to overcome the shortcoming of index-ranking method is to use linear programming models. This approach has been used by Lessard [6] for portfolio optimization in the energy sector. Here the task is to select a portfolio so that maximize/minimize one or more value in which satisfies the constraints in the form of annual corporate goals and asset dependencies.

However, in the energy sector there are many challenges in implementing portfolio selection. Let us consider the portfolio selection in oil and gas industry which is a fundamental subject of capital budgeting in the energy sector. Oil and gas companies have the difficult task of portfolio selection from a large number of competing exploration and production projects for immediate or future operation under limited amount of investments. The difficulties arise from the fact that exploration and production projects face both local uncertainties involving the discovery and production of oil at a given site, and global uncertainties involving prices, politics etc. In particular, the nature of these problems requires taking into account uncertainty of the input data such as oil price, oil production, capital etc., and it is very difficult to clearly know all information in deterministic parameters.

The traditional way to evaluate any uncertainty in the parameters is through a post-optimization analysis, with the help of sensitivity analysis and parametric programming. However, none of these methods is suitable for overall analysis of the effect of imprecision in parameters. Another way to handle uncertainty is to use stochastic programming according to the probability theory which is the context of this paper. In particular, the aim of this paper is to formulate the portfolio selection problem in oil and gas industry as a stochastic programming problem and propose a fuzzy programming approach for finding an efficient portfolio.

This paper is organized as follows. In Section 2, the portfolio selection of oil and gas assets is modeled as a stochastic programming problem. In Section 3, an approach is presented for solving the resulting problem based on the well-known work of Bellman and Zadeh [1] in fuzzy programming. This approach is applied in Section 4 to solve a problem of Gama Petroleum, a hypothetical oil and gas company in Brazil. In Section 5, conclusions and extensions are presented.

2. Problem Description

Given a set of n assets, the aim is to maximize the net present value (NPV), subject to a capital limit stating that we can spend no more than C_i million US dollars (MMUS$) and a production limit stating that we must produce at least P_i million barrels (MMbbl) over each of the next L year. Assume that npv_j represents the NPV of jth asset and $p_{i,j}$ and $c_{i,j}$ represent the production and capital for the jth asset in the ith year, respectively. It is known that NPV of a portfolio is the sum of equity-proportioned NPV of each asset and similarly for annual values of production or capital. This problem is formulated as the following linear programming problem:

$$\begin{aligned} \text{LP}: \max NPV &= \sum_{j=1}^{n} npv_j x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} p_{i,j} x_j \geq P_i, \quad i = 1, 2, \ldots, L, \\ & \sum_{j=1}^{n} c_{i,j} x_j \leq C_i, \quad i = 1, 2, \ldots, L, \\ & 0 \leq x_j \leq 1, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where x_j represents the equity interest of the jth asset. Here all problem data must be well defined and precise, which is often impossible in petroleum industry. For example, let j^* th asset be an oil field development. Thus, its NPV depends on oil price in future years and other parameters. Obviously, oil price will vary during the next years. In other words, NPV as well as annual values of production or capital are uncertain and not deterministic.

The traditional way to handle the uncertain parameters of a linear programming model is to perform post-optimization analysis or parametric programming. In this approach usually parameters are analyzed separately, which is not suitable for an overall analysis of the effect of imprecision in parameters. Therefore, since the single parameter sensitivity analysis is not appropriate when there are many uncertain parameters, the other approaches such as robust optimization [2] or stochastic programming [4] are used in order to investigate the overall effect of all uncertain parameters simultaneously. Thus, it is reasonable to model the portfolio selection problem as follows:

$$\text{SLP} : \max \widehat{NPV} = \sum_{j=1}^{n} \widehat{npv}_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \hat{p}_{i,j} x_j \geq \hat{P}_i, \quad i = 1, 2, \ldots, L,$$

$$\sum_{j=1}^{n} \hat{c}_{i,j} x_j \leq \hat{C}_i, \quad i = 1, 2, \ldots, L,$$

$$0 \leq x_j \leq 1, \quad j = 1, 2, \ldots, n. \tag{2}$$

Here and subsequently, the presence of a hat above a parameter is used to indicate that it is a random variable. Each random parameter can have any probability distribution, but for the sake of simplicity, we assume that these random parameters follow normal probability distributions as given below:

$$\widehat{npv}_j \sim N(m(\widehat{npv}_j), \sigma(\widehat{npv}_j)), \quad j = 1, \ldots, n,$$

$$\hat{p}_{i,j} \sim N(m(\hat{p}_{i,j}), \sigma(\hat{p}_{i,j})), \quad j = 1, \ldots, n, \; i = 1, \ldots, L,$$

$$\hat{c}_{i,j} \sim N(m(\hat{c}_{i,j}), \sigma(\hat{c}_{i,j})), \quad j = 1, \ldots, n, \; i = 1, \ldots, L,$$

$$\hat{P}_i \sim N(m(\hat{P}_i), \sigma(\hat{P}_i)), \quad i = 1, \ldots, L,$$

$$\hat{C}_i \sim N(m(\hat{C}_i), \sigma(\hat{C}_i)), \quad i = 1, \ldots, L,$$

where $m(.)$ and $\sigma(.)$ denote mean and standard deviation of the distribution, respectively. Moreover, we assume that the random parameters are independent (so that the covariance matrix is an identity matrix).

The SLP model (2) is a stochastic linear programming problem and one can use the existing methods of stochastic programming for solving it. Two main approaches to stochastic programming are recognized as chance constrained programming [3] and two-stage programming [5]. However, we develop a fuzzy programming approach as discussed in the next section.

3. Solution Approach

Bellman and Zadeh [1] presented an application of the fuzzy set theory in decision making problems. Specifically, they introduced three basic concepts: fuzzy goal (G), fuzzy constraint (C), and fuzzy decision (D) associated with a decision making problem in a fuzzy environment on a universe set X. The fuzzy goal and fuzzy constraint are represented by fuzzy sets G and C, respectively, and fuzzy decision D is defined as the intersection of the fuzzy goal and the fuzzy constraint, that is $D = G \cap C$. In particular, D is characterized by its membership function as:

$$\mu_D(x) = \min\{\mu_G(x), \mu_C(x)\}, \quad x \in X.$$

The decision rule is to select the solution having the highest membership of D. Therefore, the decision making problem reduces to the following problem.

$$\max_{x \in S} \mu_D(x) = \max_{x \in X} \min\{\mu_G(x), \mu_C(x)\}.$$

In the context of stochastic optimization and hence our portfolio selection problem, since the objective function is random-valued and there are no universal concepts of optimal solutions to be accepted widely, it is an important task to define the concept of optimal portfolios and investigate their properties. In this section, we define an efficient portfolio and present an algorithm for finding such a solution based on the seminal work of Bellman and Zadeh [1]. We begin the discussion by introducing the concept of fuzzy constraint.

Definition 3.1 (Fuzzy constraint)**.** Let X represent the set of all possible portfolios of n assets, $\hat{P}_i(x)$ and $\hat{C}_i(x)$ denote the left-hand side of ith production and capital constraints of model (2), respectively. Corresponding to a given portfolio $x \in X$:

(1) The fuzzy production constraint corresponding to the ith production constraint of model (2), denoted by $\mu_{Pro,\, i}$, is defined by its membership function as:

$$\mu_{Pro,\, i}(x) = Prob(\hat{P}_i(x) \geq \hat{P}_i),$$

where $\hat{P}_i(x) = \sum_{j=1}^{n} \hat{p}_{i,\, j} x_j$. The fuzzy production constraint corresponding to the model (2) is given by

$$\mu_{Pro}(x) = \min_{1 \leq i \leq L} \{\mu_{Pro,\, i}(x)\}, \quad x \in X.$$

The fuzzy capital constraint μ_{Cap} is similarly defined.

(2) The fuzzy constraint corresponding to the model (2) is defined as the intersection of the fuzzy production constraint and the fuzzy capital constraint as

$$\mu_C = \mu_{Pro} \cap \mu_{Cap}.$$

Using Min operator, the fuzzy constraint is characterized by membership function

$$\mu_C(x) = \min\{\mu_{Pro}(X),\, \mu_{Cap}(X)\}, \quad x \in X.$$

(3) A portfolio $x \in X$ is said to be α-feasible $(0 < \alpha \leq 1)$ if $\alpha \leq \mu_C(x)$. We refer to α as the degree of feasibility of x.

Lemma 1. *Let* $\alpha \in (0, 1]$. *The portfolio* $x = (x_1, \ldots, x_n)$ *is* α*-feasible if and only if*

$$m(\hat{P}_i) - \sum_{j=1}^{n} m(\hat{p}_{ij}) x_j + \phi^{-1}(\alpha) \left[\sigma^2(\hat{P}_i) + \sum_{j=1}^{n} \sigma^2(\hat{p}_{ij}) x_j^2 \right]^{1/2} \leq 0, \quad i = 1, \ldots, L, \quad (3)$$

$$\sum_{j=1}^{n} m(\hat{c}_{ij}) x_j - m(\hat{C}_i) + \phi^{-1}(\alpha) \left[\sum_{j=1}^{n} \sigma^2(\hat{c}_{ij}) x_j^2 + \sigma^2(\hat{C}_i) \right]^{1/2} \leq 0, \quad i = 1, \ldots, L, \quad (4)$$

where ϕ *represents the cumulative distribution function of the standard normal variable* $N(0, 1)$.

Proof. Obviously $\mu_C(X) \geq \alpha$ if and only if $\mu_{Pro,i}(X) \geq \alpha$ and $\mu_{Cap,i}(X) \geq \alpha$ for all $i = 1, \ldots, L.$ We show that $\mu_{Pro,i}(X) \geq \alpha$ for $i = 1, \ldots, L$ is equivalent to (3). By Definition 3.1, we have

$$Prob(\hat{P}_i(X) \geq \hat{P}_i) \geq \alpha, \quad i = 1, \ldots, L.$$

Set $h_i = \hat{P}_i - \hat{P}_i(X),$ then we can write

$$Prob(h_i \leq 0) \geq \alpha, \quad i = 1, \ldots, L$$

or equivalently

$$Prob\left[\frac{h_i - E(h_i)}{\sigma(h_i)} \leq \frac{-E(h_i)}{\sigma(h_i)}\right] \geq \alpha, \quad i = 1, \ldots, L. \tag{5}$$

It is clear that $\dfrac{h_i - E(h_i)}{\sigma(\alpha_i)}$ is a standard normal random variable with mean zero and variance one. Hence, we have

$$\phi\left[\frac{-E(\alpha_i)}{\sigma(\alpha_i)}\right] \geq \alpha, \quad i = 1, \ldots, L.$$

The cumulative distribution function ϕ is an increasing function. Thus, the last inequality implies

$$\frac{-E(h_i)}{\sigma(h_i)} \geq \phi^{-1}(\alpha), \quad i = 1, \ldots, L. \tag{6}$$

For each $j = 1, \ldots, n,$ $i = 1, \ldots, L,$ we know that $\hat{p}_{ij}$ is normally distributed random variables and so is h_i for $i = 1, \ldots, L$ with

$$E(h_i) = m(\hat{P}_i) - \sum_{j=1}^{n} m(\hat{p}_{ij}) x_j, \tag{7}$$

and

$$\sigma(h_i) = \left[\sigma^2(\hat{P}_i) + \sum_{j=1}^{n} \sigma^2(\hat{p}_{ij}) x_j^2\right]^{1/2}. \tag{8}$$

Therefore substituting (7) and (8) into (6) implies that

$$m(\hat{P}_i) - \sum_{j=1}^{n} m(\hat{p}_{ij}) x_j + \phi^{-1}(\alpha)\left[\sigma^2(\hat{P}_i) + \sum_{j=1}^{n} \sigma^2(\hat{p}_{ij}) x_j^2\right]^{1/2} \leq 0, \quad i = 1, \ldots, L, \quad (9)$$

which establishes the first inequality of the lemma.

Similarly, we can show $\mu_{Cap,i}(X) \geq \alpha$ for $i = 1, \ldots, L$ is equivalent to (4). □

The next step is to introduce the concept of fuzzy goal. To do this, we consider the following bi-objective mathematical programming problem:

$$E\text{-model} : \max E(\widehat{NPV}) = \sum_{j=1}^{n} E(\widehat{npv}_j) x_j$$

$$\max \alpha$$

$$\text{s.t. constraints (3) and (4) hold,}$$

$$0 \leq x_j \leq 1, \quad j = 1, \ldots, n, \quad (10)$$

where E denotes the mathematical expectation. This model is a non-linear multiple objective model in general and is named E-model, hereafter. The E-model (10) is a crisp non-linear multiple objective problem that simultaneously computes the maximum possibilistic mean value of each single objective and degree of feasibility over all possible portfolios x. However, for some fixed value of α, it reduces to a single objective nonlinear programming problem that can be easily solved. It is clear that finding the best value α^* is equivalent to determining maximum value of $\alpha \in (0, 1]$ so that the following problem has a feasible solution

$$\max E(\widehat{NPV}) = \sum_{j=1}^{n} E(\widehat{npv}_j) x_j$$

$$\text{s.t. constraints (3) and (4) hold,}$$

$$0 \leq x_j \leq 1, \quad j = 1, \ldots, n. \quad (11)$$

This approach indicates that maximizing degree of feasibility is much more important than maximizing possibilistic mean value of each objective. In fact, in solving the E-model (10) we have implicitly made use of the lexicographic method for solving multiple objective linear programming problems [9].

Since $0 < \alpha \leq 1,$ we can present a bi-section algorithm for solving E-model as follows:

Bi-section Algorithm 1 (BA1)

Step 1. Set $\alpha = 0,$ $\alpha_L = 0$ and $\alpha_U = 1,$ where α_L and α_U are the lower and upper bounds for the best value $\alpha^*,$ respectively.

Step 2. Test model (11). If model (11) is infeasible, then stop and conclude model (2) is also infeasible. Otherwise set $\alpha = 1$ and go to **Step 3**.

Step 3. Test model (11). If model (11) has a feasible optimal solution, say $x^* = (x_1^*, ..., x_n^*),$ then $(\alpha^* = 1, x^*)$ is an optimal solution for model (2) and stop. Otherwise go to **Step 4**.

Step 4. Set $\alpha = (\alpha_L + \alpha_U)/2$ and solve model (11). If model (11) has a feasible optimal solution then update the value of α_L as $\alpha_L = \alpha.$ Otherwise set $\alpha_U = \alpha.$ Go to **Step 5**.

Step 5. (Stopping Condition) If the difference between two consecutive values α is less than ε, then the algorithm is finished, where ε is an acceptable tolerance; otherwise go to **Step 4**.

Now we can define fuzzy goal.

Definition 3.2 (Fuzzy goal)**.** Let $E^*(\widehat{NPV})$ be the maximum expected value of NPV computed by model (11) and X be the set of all possible portfolios of n assets.

(1) The *fuzzy goal* corresponding to the model (2) is defined by its membership function as:

$$\mu_G(x) = Prob\left(\sum_{j=1}^{n} \widehat{npv}_j x_j \geq E^*(\widehat{NPV})\right), \quad x \in X.$$

(2) A portfolio $x \in X$ is said to be β-*optimal* $(0 \leq \beta \leq 1)$ for model (2) if $\beta \leq \mu_G(x).$ We refer to β as the degree of optimality of x.

Lemma 2. *Let* $\beta \in (0, 1]$. *Then x is* β*-optimal if and only if*

$$E^*(\widehat{NPV}) - \sum_{j=1}^{n} m(\widehat{npv}_j)x_j + \phi^{-1}(\beta)\left[\sum_{j=1}^{n}\sigma^2(\widehat{npv}_j)x_j^2\right]^{1/2} \leq 0. \tag{12}$$

Proof. We have

$$\mu_G(X) = Prob\left(\sum_{j=1}^{n}\widehat{npv}_j x_j \geq E^*(\widehat{NPV})\right) = Prob(\widehat{NPV} \geq E^*(\widehat{NPV})) \geq \beta$$

if and only if

$$1 - Prob(\widehat{NPV} \leq E^*(\widehat{NPV})) \geq \beta$$

or equivalently

$$Prob(\widehat{NPV} \leq E^*(\widehat{NPV})) \leq \overline{\beta},$$

where $\overline{\beta} = 1 - \beta$. This relation can be rewritten as

$$Prob\left(\frac{\widehat{NPV} - E(\widehat{NPV})}{\sigma(\widehat{NPV})} \leq \frac{E^*(\widehat{NPV}) - E(\widehat{NPV})}{\sigma(\widehat{NPV})}\right) \leq \overline{\beta},$$

where $\dfrac{\widehat{NPV} - E(\widehat{NPV})}{\sigma(\widehat{NPV})}$ is a standard normal variable. By using cumulative distribution function, the later relation is stated as

$$\phi\left(\frac{E^*(\widehat{NPV}) - E(\widehat{NPV})}{\sigma(\widehat{NPV})}\right) \leq \overline{\beta}.$$

From the above inequality and the fact that ϕ is an increasing function, we have

$$\frac{E^*(\widehat{NPV}) - E(\widehat{NPV})}{\sigma(\widehat{NPV})} \leq \phi^{-1}(\overline{\beta}).$$

Since $\phi^{-1}(\overline{\beta}) = -\phi^{-1}(\beta)$, and $\widehat{npv}$ is a normally distributed random variable with

$$E(\widehat{NPV}) = \sum_{j=1}^{n} m(\widehat{npv}_j)x_j,$$

$$\sigma(\widehat{NPV}) = \left[\sum_{j=1}^{n} \sigma^2(\widehat{npv}_j)\, x_j^2\right]^{1/2},$$

we obtain the desirable inequality. □

So far, we defined fuzzy constraint and fuzzy goal corresponding to model (2). Now we can define fuzzy decision.

Definition 3.3 (Fuzzy decision)**.** Let μ_C and μ_G be the membership functions of fuzzy constraint and fuzzy goal, respectively, of model (2).

(1) We define the fuzzy decision corresponding to model (2) as follows:

$$\mu_D = \mu_C \cap \mu_G.$$

Using Min operator, fuzzy decision is characterized by membership function

$$\mu_D(x) = \min\{\mu_C(x), \mu_G(x)\}, \quad x \in X.$$

(2) A portfolio $x \in X$ is said to be γ-efficient $(0 \le \gamma \le 1)$ for model (2) if $\gamma \le \mu_D(x)$. We refer to γ as the degree of efficiency of x.

Following Bellman and Zadeh [1], the decision rule is to select the solution having the highest membership of the fuzzy decision. This leads to the following definition.

Definition 3.4 (Efficient portfolio)**.** The portfolio $x^* \in X$ is said to be an efficient portfolio for model (2) if there is no portfolio $x \in X$ such that $\mu_D(x^*) < \mu_D(x)$.

By introducing the auxiliary variable λ, finding an efficient portfolio reduces into the following problem:

$$\begin{aligned} &\max \lambda \\ &\text{s.t. } \lambda \le \mu_D(x), \\ &\quad 0 \le x_j \le 1, \quad j = 1, \ldots, n. \end{aligned} \tag{13}$$

Notice that if (x^*, λ^*) is an optimal solution of model (13), then λ^* represents the degree of efficiency of optimal portfolio x^*. By using Lemmas 1 and 2, we can rewrite model (13) to

$$
\begin{aligned}
&\max \lambda \\
&\text{s.t.}\quad m(\hat{P}_i) - \sum_{j=1}^{n} m(\hat{p}_{ij}) x_j + \phi^{-1}(\lambda)\left[\sigma^2(\hat{P}_i) + \sum_{j=1}^{n} \sigma^2(\hat{p}_{ij}) x_j^2\right]^{1/2} \le 0, \\
&\sum_{j=1}^{n} m(\hat{c}_{ij}) x_j - m(\hat{C}_i) + \phi^{-1}(\lambda)\left[\sum_{j=1}^{n} \sigma^2(\hat{c}_{ij}) x_j^2 + \sigma^2(\hat{C}_i)\right]^{1/2} \le 0, \\
&E^*(\widehat{NPV}) - \sum_{j=1}^{n} m(\widehat{npv}_j) x_j + \phi^{-1}(\lambda)\left[\sum_{j=1}^{n} \sigma^2(\widehat{npv}_j) x_j^2\right]^{1/2} \le 0, \\
&0 \le x_j \le 1, \quad j = 1, \ldots, n.
\end{aligned}
\tag{14}
$$

We can easily establish the following result.

Lemma 3. *If model* (14) *is feasible* (*infeasible*) *for some fixed values of* λ, *then it is also feasible* (*infeasible*) *for all values less* (*greater*) *than* λ.

It is important to note that model (14) is a nonlinear programming model. However, by Lemma 3 and the fact that λ satisfies $0 \le \lambda \le 1$, we can introduce the following method for solving model (14).

Bi-section Algorithm 2 (BA2)

Step 1. Set $\lambda = 1$, $\lambda_L = 0$ and $\lambda_U = 1$, where λ_L and λ_U are the lower and upper bounds for optimal value of λ^*, respectively.

Step 2. Test the existence of a feasible solution satisfying the constraints of model (14). If a feasible solution exists, say $x^* = (x_1^*, \ldots, x_n^*)$, then $(\lambda^* = 1, x^*)$ is an efficient portfolio for model (2). Otherwise go to **Step 3**.

Step 3. Set $\lambda = (\lambda_L + \lambda_U)/2$ and test the existence of a feasible solution satisfying the constraints. If it has a feasible solution then update the value of λ_L as $\lambda_L = \lambda$. Otherwise set $\lambda_U = \lambda$.

Step 4 (Stopping Condition)**.** If the difference between two consecutive values λ is less than ε, which is an acceptable tolerance, then the algorithm is finished otherwise go to **Step 3**.

4. A Numerical Example

In this section, we consider an example of portfolio optimization of 25 potential oil production projects. Table 1 presents the main characteristics of these projects including total oil production, total capital, and NPV. The data is due to Gama Petroleum, a hypothetical oil and gas company in Brazil, and is taken from Lima et al. [8].

The problem parameters of Table 1 are precise and deterministic. Due to uncertainty arising from inherent natural of oil production projects, we assume that the parameters are normal random variables with the following means and variances:

$$\widehat{npv}_j \sim N(npv_j,\ 0.12npv_j),$$

$$\hat{p}_{i,j} \sim N(p_j,\ 0.10p_j),$$

$$\hat{c}_{i,j} \sim N(c_j,\ 0.6c_j),$$

for $j = 1, \ldots, 25.$ Moreover, we assume that the random parameters are independent.

There is a capital limit stating that we can spend no more than C MMUS$ and a production limit stating that we must produce at least P MMbbl. We suppose that C and P are also normal random parameters with the following characteristics:

$$\hat{C} \sim N(19000,\ 1000),$$

$$\hat{P} \sim N(25000,\ 1500).$$

The problem now is to select a combination of potential projects so that NPV is maximized and the capital and production limitations are satisfied. This can be modelled as model (2). In what follows, we present the results of applying our approach for solving this problem.

The first step is to solve E-model (10). By applying Bi-section Algorithm 1 with tolerance error less than 0.001 (i.e., $\varepsilon = 0.001$), we get

$$\alpha^* = 0.959, \quad x^* = (1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 0.508, 1, 1, 1, 1, 0, 1)$$

with maximum expected value $E^*(\widehat{NPV}) = 3793340.78.$ The next step is to solve the model (14) by applying Bi-section Algorithm 2. This leads to the following combination of 25 projects

$$x^* = (1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0.846, 0.17, 1, 1, 1, 1, 0, 1)$$

with degree of efficiency $\lambda^* = 0.948$ and expected NPV $E^*(\widehat{NPV}) = 3872475.76.$

Table 1. Main characteristics of 25 potential projects

Projects (j)	Production (p_j) (MMbbl)	Capital (c_j) (MMUS$)	Net present value (npv_j) (MUS$)
1	695.210	1669.30	46278.96
2	999.400	1466.86	346050.80
3	748.127	1157.37	28562.40
4	214.750	1087.52	140172.27
5	643.200	580.00	339119.55
6	499.706	1020.93	48354.53
7	389.305	1022.00	130332.73
8	305.300	1263.00	22314.76
9	410.790	660.94	535741.33
10	703.391	1790.83	243595.91
11	193.272	992.42	126638.26
12	268.444	1083.44	233245.61
13	335.833	1226.00	85887.38
14	350.375	914.58	155555.96
15	287.555	454.96	459233.76
16	449.731	910.62	93419.94
17	514.560	518.67	254179.93
18	625.690	1560.45	84482.13
19	773.730	1980.91	187566.08
20	897.740	1557.64	19360.56

21	832.834	1524.64	186017.33
22	1199.20	1686.77	41867.82
23	1399.16	2002.00	474417.09
24	214.750	1050.35	145824.82
25	710.790	855.10	497372.86

5. Conclusion and Extension

We have proposed a stochastic programming model for the portfolio optimization problem in petroleum industry in which the profit coefficients as well as the capital and production coefficients of assets are random variables with normal distributions. Further, based on the fuzzy decision making theory a solution approach is suggested. However, another way to handle imprecision in the parameters is to consider the knowledge of experts about parameters as fuzzy numbers and model portfolio selection problem as a fuzzy programming problem, which gives us the conceptual and theoretical framework for dealing with complexity, imprecision and vagueness [10]. It is worth noting that the approach of this paper can be extended to this case without any difficulties.

We conclude the paper by mentioning that although the focus of this paper was on the portfolio optimization problem of oil and gas assets, our theoretical results can be applied for many instances of practical applications. Moreover, the approach of this paper can be applied to find an efficient solution to any stochastic linear programming problem whose all parameters are random variables with normal distributions.

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