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ALGORITHMS OF MULTI-OBJECTIVE TRANSPORTATION PROBLEMS WITH UNCERTAIN COEFFICIENTS

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Abstract

In this paper, we discuss the multi-objective transportation problems with uncertain coefficients. The interval numbers and fuzzy numbers programmings of multi-objective transportation problem are proposed, respectively. For the model of interval numbers programming, we find an optimal satisfactory degree solution by introducing the satisfactory degree function. The fuzzy programming with trapezoidal fuzzy number coefficients is reduced to multi-objective linear programming on the basis of α-cut of the fuzzy numbers, the α level optimal solutions are obtained. Compared with existing methods. Finally numerical examples are illustrated the effectiveness and practicality of the proposed algorithms.

1. Introduction

In the real life, the transportation problems are not single objective. Therefore, Diaz developed algorithms for multi-objective transportation problems [2].

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By considering the imprecise nature of human judgments, parameters such as quantity of product, demand and cost associated with transporting a unit of the product are uncertain data (i.e., fuzzy numbers or intervals). Bit et al., Verma et al., etc. applied fuzzy programming technique to solve multi-objective problems [1, 3-5, 7-9].

In this paper, the uncertain parameters of transportation problem are considered as trapezoidal fuzzy numbers and interval numbers. For the model of interval numbers programming, we find an optimal satisfactory degree solution by introducing the satisfactory degree function. But existing papers discussed seldom the interval number programming of multi-objective transportation. A new algorithm of multi-objective transportation problem with trapezoidal fuzzy number is proposed.

2. Interval Number Programming for Multi-objective Transportation Problem

Definition 1. Let $[a, b], [c, d] \in I(R)$, where $I(R) = \{[a, b] \mid a, b \in R, a \leq b\}$. Then the operations of interval number are defined as:

$$[a, b] + [c, d] = [a + c, b + d],$$

$$[a, b] - [c, d] = [a - d, b - c],$$

$$r[a, b] = \begin{cases} [ra, rb] & \text{if } r \geq 0 \\ [rb, ra] & \text{if } r < 0 \end{cases}, \quad r \in R.$$

Definition 2. Let $r \in R$. If $a \leq r \leq b$, then we denote $r \leq [a, b]$.

The model of interval number programming for multi-objective transportation problem can be formulated as follows:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}^k, d_{ij}^k] x_{ij}, \quad k = 1, 2, \ldots, K,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq [a_i, e_i], \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} \leq [b_j, f_j], \quad j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n, \tag{1}$$

where an interval number $[c_{ij}^k, d_{ij}^k]$ is a penalty associated with transportation of a unit of the product from source i to destination j for k-th penalty criterion. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc. An interval number $[a_i^k, e_i^k]$ is product quantity which we want to transport to destination D_j to satisfy the interval number demand $[b_j^k, f_j^k]$ for k-th penalty criterion. The problem is to determine the project which will minimize the total penalty criteria.

By definition,

$$Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}^k, d_{ij}^k] x_{ij}$$

$$= \left[\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}, \; \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^k x_{ij} \right]$$

$$= [Z_k^1, Z_k^2], \; a_i \leq \sum_{j=1}^{n} x_{ij} \leq e_i, \; b_j \leq \sum_{i=1}^{m} x_{ij} \leq f_j.$$

Definition 3. Let $Q_1 = [a, b], Q_2 = [c, d] \in I(R)$. Then we call the function

$$S(Q_1, Q_2) = \begin{cases} 1, & a \leq c, \\ 1 - \dfrac{a - c}{b + d}, & c < a \leq c + b + d, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

to be satisfactory degree of Q_1 smaller than Q_2.

Let

$$Z_k^+ = \min_{x \in G_1} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}, \; Z_k^- = \min_{x \in G_2} \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^k x_{ij}, \; Q_k = [Z_k^+, Z_k^-],$$

where

$$x = (x_{11}, \ldots, x_{1n}, x_{21}, \ldots, x_{2n}, \ldots, x_{m1}, \ldots, x_{mn})$$

$$G_1 = \left\{ x \mid a_i \leq \sum_{j=1}^{n} x_{ij} \leq e_i, \, b_j \right.$$

$$\left. \leq \sum_{i=1}^{m} x_{ij} \leq f_j, \, x_{ij} \geq 0, \, i = 1, 2, \ldots, m, \, j = 1, 2, \ldots, n \right\},$$

$$G_2 = \left\{ x \mid \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^k x_{ij} \geq Z_k^+, \, x \in G_1 \right\}.$$

Assume that $Q_k = [Z_k^+, Z_k^-]$ is expected interval number of the problem (1) for the k-th penalty criterion.

By Definition 3, let S_k be satisfactory degree function of Z_k smaller than Q_k,

$$S_k(Z_k, Q_k) = \begin{cases} 1, & Z_k^1 \leq Z_k^+, \\ 1 - \dfrac{Z_k^1 - Z_k^+}{Z_k^2 + Z_k^-}, & Z_k^+ < Z_k^1 \leq Z_k^+ + Z_k^2 + Z_k^-, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

By using the max-min operator, an equivalent fractional mathematical programming to the problem (1) is formulated as follows:

$$\begin{aligned} & \max \; \beta \\ & \text{s.t.} \quad S_k \geq \beta, \; k = 1, 2, \ldots, K, \\ & \qquad\; x \in G_1, \end{aligned} \tag{4}$$

where $\beta = \min_{x \in G_1} \{S_1, S_2, \ldots, S_K\}$.

The above programming (4) can be further written as

$$\max \beta$$

$$\text{s.t.} \quad \frac{Z_k^1 - Z_k^+}{Z_k^2 + Z_k^-} + \beta \leq 1, \; k = 1, 2, \ldots, K,$$

$$x \in G_1. \tag{5}$$

Theorem 1. *If* y^* *is an optimal solution to the problem* (5), *then* x^* *is an optimal solution to the problem* (1), *its satisfactory degree is* β^*.

We call x^* to be an *optimal satisfactory degree solution* to the problem (1), where

$$y^* = (\beta^*, x_{11}^*, x_{12}^*, \ldots, x_{1n}^*, x_{21}^*, x_{22}^*, \ldots, x_{2n}^*, \ldots, x_{m1}^*, x_{m2}^*, \ldots, x_{mn}^*),$$

$$x^* = (x_{11}^*, x_{12}^*, \ldots, x_{1n}^*, x_{21}^*, x_{22}^*, \ldots, x_{2n}^*, \ldots, x_{m1}^*, x_{m2}^*, \ldots, x_{mn}^*).$$

We call interval number Z_k^* to be the *optimal satisfactory degree interval number* for k-th penalty criterion, where

$$Z_k^* = \left[\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}^*, \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^k x_{ij}^* \right], \; k = 1, 2, \ldots, K.$$

Algorithm 1.

Step 1. Find Z_k^+, Z_k^-, the expected interval numbers $Q_k = [Z_k^+, Z_k^-]$ of the problem (1) are obtained.

Step 2. The satisfactory degree function S_k of Z_k smaller than Q_k is represented by (3).

Step 3. From the results of Step 2, solve the programming (4). The optimal satisfactory degree solution x^* and the optimal satisfactory degree interval number Z_k^* of the problem (1) are obtained.

Example 1.

$$\min Z_1 = [2, 3]x_{11} + [4, 5]x_{12} + [4.2, 5]x_{13} + [4, 5]x_{14} + [5.5, 6.5]x_{21} + [4.5, 6]x_{22} + [3, 4.8]x_{23} + [7.6, 9.2]x_{24}$$

$$\min Z_2 = [1.2, 2.5]x_{11} + [3, 4.2]x_{12} + [2, 3.4]x_{13} + [3.8, 5]x_{14} + [1.5, 2.3]x_{21} + [8, 9.3]x_{22} + [4, 5.6]x_{23} + [3.6, 4.2]x_{24}$$

$$\text{s.t.} \quad \sum_{j=1}^{4} x_{1j} \leq [12, 15],$$

$$\sum_{j=1}^{4} x_{2j} \leq [9, 12.8],$$

$$\sum_{i=1}^{2} x_{i1} \leq [6, 7.5],$$

$$\sum_{i=1}^{2} x_{i2} \leq [5, 6.5],$$

$$\sum_{i=1}^{2} x_{i3} \leq [4, 6],$$

$$\sum_{i=1}^{2} x_{i4} \leq [6, 7.8],$$

$$x_{ij} \geq 0, \; i = 1, 2, \; j = 1, 2, 3, 4. \tag{6}$$

Based on MATLAB 6.1 platform, by Algorithm 1, the obtained solutions are as follows:

$$Z_1^+ = 70.5, \; Z_1^- = 97.2, \; Z_2^+ = 52.7, \; Z_2^- = 74.2,$$

$$Z_1^* = [89.72, 115.52], \; Z_2^* = [67.61, 90.82].$$

3. The α Level Optimal Solution to the Multi-objective Transportation Problem with Fuzzy Parameters

A mathematical programming of the multi-objective transportation problem with fuzzy parameters is formulated as follows:

$$\min \widetilde{Z}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{c}_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, K,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \le \widetilde{a}_i, \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} \le \widetilde{b}_j, \quad j = 1, 2, \ldots, n,$$

$$x_{ij} \ge 0, \; i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n, \tag{7}$$

where $\widetilde{c}_{ij}^{k}$, $\widetilde{a}_i$ and $\widetilde{b}_j$ are fuzzy parameters or fuzzy coefficients.

In this section, by applying α-cut of the trapezoidal fuzzy number and interval number, we describe an algorithm for solving multi-objective transportation problem with trapezoidal fuzzy numbers. Comparison with existing methods [3, 4], this algorithm is effective. Assume that $\widetilde{c}_{ij}^{k}$, $\widetilde{a}_i$ and $\widetilde{b}_j$ are trapezoidal numbers in (7).

Definition 4. The α-cut of the trapezoidal fuzzy number $\widetilde{A} = [\eta, m_1, m_2, \gamma]$ is defined as the ordinary set $(\widetilde{A})_\alpha$ for which the degree of its membership function exceeds the level $\alpha \in [0, 1]$:

$$(\widetilde{A})_\alpha = \{x \mid \mu_{\widetilde{A}} \ge \alpha\} = [\alpha + (m_1 - \eta)/\eta, \; m_2 + \gamma(1 - \alpha)].$$

For the sake of simplicity, denoted by

$$(\widetilde{A})_\alpha^L = \alpha + (m_1 - \eta)/\eta, \; (\widetilde{A})_\alpha^R = m_2 + \gamma(1 - \alpha),$$

then

$$(\widetilde{A})_\alpha = [(\widetilde{A})_\alpha^L, (\widetilde{A})_\alpha^R].$$

First, for a certain degree α, we can find the following parametric programming

$$\min\ Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}^k)_\alpha^L x_{ij},\ \ k = 1, 2, \ldots, K,$$

$$\text{s.t.}\ \ \sum_{j=1}^{n} x_{ij} \le [(\tilde{a}_i)_\alpha^L, (\tilde{a}_i)_\alpha^R],\ \ i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} \le [(\tilde{b}_j)_\alpha^L, (\tilde{b}_j)_\alpha^R],\ \ j = 1, 2, \ldots, n,$$

$$x_{ij} \ge 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{8}$$

By definition, the programming (8) may be written as

$$\min\ Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}^k)_\alpha^L x_{ij},\ \ k = 1, 2, \ldots, K,$$

$$\text{s.t.}\ \ (\tilde{a}_i)_\alpha^L \le \sum_{j=1}^{n} x_{ij} \le (\tilde{a}_i)_\alpha^R,\ \ i = 1, 2, \ldots, m,$$

$$(\tilde{b}_j)_\alpha^L \le \sum_{i=1}^{m} x_{ij} \le (\tilde{b}_j)_\alpha^R,\ \ j = 1, 2, \ldots, n,$$

$$x_{ij} \ge 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{9}$$

Secondly, we can find the problem (9) by using fuzzy programming approach.

Let

$$Z_k^+ = \min_{x\in F_1}\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}^k)_\alpha^L x_{ij},\quad Z_k^- = \min_{x\in F_2}\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}^k)_\alpha^R x_{ij},$$

where

$$F_1 = \Bigg\{x \,|\, (\tilde{a}_i)_\alpha^L \le \sum_{j=1}^{n} x_{ij} \le (\tilde{a}_i)_\alpha^R,\ (\tilde{b}_j)_\alpha^L \le \sum_{i=1}^{m} x_{ij} \le (\tilde{b}_j)_\alpha^R,\ x_{ij} \ge 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n\Bigg\},$$

$$F_2 = \left\{ x \mid \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{c}_{ij}^k)_\alpha^R x_{ij} \geq Z_k^+,\ x \in F_1 \right\}.$$

Assume the membership function corresponding to the k-th objective function of programming (9) in the form

$$\mu_k = \begin{cases} 1, & Z_k \leq Z_k^+ \\ 1 - (Z_k - Z_k^+)/(Z_k^- - Z_k^+), & Z_k^+ < Z_k < Z_k^- \\ 0, & Z_k \geq Z_k^-. \end{cases} \tag{10}$$

By adopting the max-min operator, the programming (9) is equivalent to the following linear programming

$$\max\ \beta$$

$$\text{s.t.}\quad \mu_k \geq \beta,\ \ k = 1, 2, \ldots, K$$

$$(\tilde{a}_i)_\alpha^L \leq \sum_{j=1}^{n} x_{ij} \leq (\tilde{a}_i)_\alpha^R,\ \ i = 1, 2, \ldots, m,$$

$$(\tilde{b}_j)_\alpha^L \leq \sum_{i=1}^{m} x_{ij} \leq (\tilde{b}_j)_\alpha^R,\ \ j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{11}$$

The optimal solution to the problem (11)

$$y^* = (\beta^*, x_{11}^*, x_{12}^*, \ldots, x_{1n}^*, x_{21}^*, x_{22}^*, \ldots, x_{2n}^*, \ldots, x_{m1}^*, x_{m2}^*, \ldots, x_{mn}^*),$$

which means that the possibility level of all fuzzy parameters in y^* is α. β^* is the degree to satisfy all objective functions when the possibility level of all fuzzy parameters is α.

Theorem 2. *Suppose*

$$y^* = (\beta^*, x_{11}^*, x_{12}^*, \ldots, x_{1n}^*, x_{21}^*, x_{22}^*, \ldots, x_{2n}^*, \ldots, x_{m1}^*, x_{m2}^*, \ldots, x_{mn}^*)$$

is an optimal solution to the problem (11) *if the value of* α *is fixed*, *then*

$$x^* = (x_{11}^*, x_{12}^*, \ldots, x_{1n}^*, x_{21}^*, x_{22}^*, \ldots, x_{2n}^*, \ldots, x_{m1}^*, x_{m2}^*, \ldots, x_{mn}^*)$$

is an optimal solution to the problem (7), *and its optimal level is* α.

We call x^* to be the α *level optimal solution.*

Algorithm 2.

Step 1. By using (10), the membership function and the α-cut of trapezoidal fuzzy numbers in programming (7) are given.

Step 2. Find Z_k^+, Z_k^-, $k = 1, 2, \ldots, K$.

Step 3. Solving the programming (11), the α level optimal solutions of original problem (7) are obtained. If existing k_0, satisfy $Z_{k_0}^+ = Z_{k_0}^-$, then we only solve the following parametric programming

$$\begin{aligned}
&\max\ \beta \\
&\text{s.t.}\quad \mu_k \geq \beta,\ k = 1, 2, \ldots, K,\ k \neq k_0, \\
&\qquad (\tilde{a}_i)_\alpha^L \leq \sum_{j=1}^{n} x_{ij} \leq (\tilde{a}_i)_\alpha^R,\ \ i = 1, 2, \ldots, m, \\
&\qquad (\tilde{b}_j)_\alpha^L \leq \sum_{i=1}^{m} x_{ij} \leq (\tilde{b}_j)_\alpha^R,\ \ j = 1, 2, \ldots, n, \\
&\qquad x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.
\end{aligned} \tag{12}$$

We obtain the α level optimal solutions x^* of (7).

Example 2.

$$\begin{aligned}
&\min\ \tilde{Z}_1 = \tilde{3}x_{11} + \tilde{4}x_{12} + \tilde{5}x_{13} + \tilde{4}x_{21} + \tilde{6}x_{22} + \tilde{5}x_{23} \\
&\min\ \tilde{Z}_2 = \tilde{2}x_{11} + \tilde{3}x_{12} + \tilde{4}x_{13} + \tilde{1}x_{21} + \tilde{9}x_{22} + \tilde{6}x_{23} \\
&\min\ \tilde{Z}_3 = \tilde{4}x_{11} + \tilde{2}x_{12} + \tilde{10}x_{13} + \tilde{3}x_{21} + \tilde{8}x_{22} + \tilde{1}x_{23} \\
&\text{s.t.}\ \sum_{j=1}^{3} x_{1j} \leq \tilde{11}, \\
&\qquad \sum_{j=1}^{3} x_{2j} \leq \tilde{15},
\end{aligned}$$

$$\sum_{j=1}^{3} x_{2j} \le \tilde{7},$$

$$\sum_{j=1}^{3} x_{2j} \le 1\tilde{0},$$

$$\sum_{j=1}^{3} x_{2j} \le \tilde{9},$$

$$x_{ij} \ge 0,\ i = 1, 2,\ j = 1, 2, 3, \tag{13}$$

where trapezoidal fuzzy numbers as follows:

$\tilde{1} = [0.2, 0.8, 1, 0.2]$, $\tilde{2} = [1, 1.5, 2, 0.5]$, $\tilde{3} = [1, 2, 3, 1]$, $\tilde{4} = [1, 4, 5, 1.5]$,

$\tilde{5} = [1, 4.2, 5, 1.2]$, $\tilde{6} = [1, 5.5, 6, 1]$, $\tilde{7} = [1, 6, 7, 1.3]$, $\tilde{8} = [1, 7, 8, 1]$,

$\tilde{9} = [1, 8.5, 9, 1]$, $1\tilde{0} = [1, 9, 10, 1.5]$, $1\tilde{1} = [1, 10, 11, 1.6]$, $1\tilde{5} = [1, 13.5, 15, 1.4]$.

By Algorithm 2, the α level optimal solutions of (7) are given by Table 1.

Table 1. The α level optimal solutions

α	Z_1^+	Z_2^+	Z_3^+	Z_1^-	Z_2^-	Z_3^-	β	Z_1^*	Z_2^*	Z_3^*
0.1	64.29	59.97	34.63	123.79	90.56	51.15	0.9393	67.90	60.25	35.63
0.2	67.12	62.73	37.32	122.53	89.88	50.77	0.9348	70.73	63.01	38.19
0.3	69.99	65.53	40.07	121.22	89.16	50.36	0.9295	73.60	65.81	40.79
0.4	72.90	68.37	42.88	119.85	88.40	49.91	0.9229	76.52	68.65	43.42
0.5	75.85	71.25	45.75	118.43	87.60	49.43	0.9004	80.09	72.04	46.12
0.6	78.84	74.17	48.68	116.95	86.76	48.91	0.8048	85.22	76.63	48.72
0.7	81.87	77.13	51.67	115.42	85.88	51.67	0.6468	88.96	80.22	51.47
0.8	84.94	80.13	54.72	113.83	84.96	54.72	0.2671	92.48	83.67	54.72
0.9	84.94	80.13	54.72	113.83	84.96	54.72	0.2671	92.48	83.67	54.72
1.0	84.94	80.13	54.72	113.83	84.96	54.72	0.2671	92.48	83.67	54.72

But applying the algorithm of [3, 4], the programming (7) can be turned into a parametric programming, the obtained α level solutions are given by Table 2.

From Table 2, we know that the algorithm of [3, 4] is no longer effective for Example 2.

Table 2. The α level optimal solutions

α	Z_1^+	Z_2^+	Z_3^+	Z_1^-	Z_2^-	Z_3^-	β	Z_1^*	Z_2^*	Z_3^*
0.1	0	0	0	104.17	133.70	162.70	1	0	0	0
0.2	0	0	0	105.92	135.08	163.64	1	0	0	0
0.3	0	0	0	107.61	136.40	164.52	0.99	0	0	0
0.4	0	0	0	109.25	137.67	165.34	1	0	0	0
0.5	0	0	0	110.82	138.87	166.10	1	0	0	0
0.6	0	0	0	112.34	140.02	166.80	1	0	0	0
0.7	0	0	0	113.80	141.10	167.44	1	0	0	0
0.8	0	0	0	115.19	142.13	168.02	1	0	0	0
0.9	0	0	0	116.52	143.10	168.54	1	0	0	0
1.0	0	0	0	117.80	144.00	169.00	0.99	0	0	0

4. Concluding Remarks

In this paper, algorithms of multi-objective interval number programming and multi-objective fuzzy number programming for transportation problem are presented, respectively. Comparing with other method, the proposed algorithm of multi-objective transportation problem fuzzy number programming is illustrated its effectiveness. Specially, the model of interval number programming for multi-objective transportation problem is formulated, an optimal satisfactory degree is obtained by introducing the concept of satisfactory degree function.

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