Far East Journal of Applied Mathematics
Volume 37, Number 1, 2009, Pages 23-32
Published Online: December 3, 2009
This paper is available online at http://www.pphmj.com

EFFECTS OF VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY ON FLOW OF A MICROPOLAR FLUID BOUNDED BY STRETCHING SHEET

B. BORGOHAIN and G. C. HAZARIKA

Department of Mathematics
Tinsukia College
Tinsukia -786125, India
e-mail: bhairab.borgohain@yahoo.in

Department of Mathematics
Dibrugarh University
Dibrugarh-786004, India
e-mail: gchazarika@gmail.com

Abstract

The effect of variable viscosity and thermal conductivity of two-dimensional boundary layer flow of a micropolar fluid driven by a porous stretching sheet is investigated. The micropolar model due to Eringen is used to describe the working fluid. The partial differential equations governing the motion, angular momentum and energy are reduced to ordinary differential equations using similar transformations and then solved numerically using Ranga-Kutta shooting technique. The effects of the different parameters on velocity distribution, microrotation distribution and temperature distribution have been studied numerically. The results are presented graphically for velocity distribution, temperature distribution and microrotation distributions for various values of non-dimensional parameters. It is found that the effects of the parameters representing variable property of viscosity and thermal conductivity are significant.

2000 Mathematics Subject Classification: 74.

Keywords and phrases: viscosity, thermal conductivity, micropolar fluid, porous stretching sheet, shooting method.

Received July 9, 2009

1. Introduction

Micropolar fluids are those, which contain micro-constituents which can undergo rotation, the presence of which can affect the hydrodynamics of the flow so that it can be distinctly non-Newtonian. The theory of micropolar fluids is originally formulated by Eringen [4, 5]. In essence, the theory introduces new material parameters, an additional independent vector field, the microrotation and new constitutive equations, which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents provided initial motivation for the development of the theory, but subsequent studies have successfully applied the model to a wide range of applications including blood flow, lubricants, porous media, turbulent shear flows and flowing capillaries and micro channels by Lukaszewicz [12]. The two-dimensional boundary layer flow introduced by a stretching sheet in an ambient quiescent fluid was first studied by Crane [2] who obtained a very closed form of exponential solution. Hady [6] studied the solution of heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection. Hassanien and Gorla [7] studied the heat transfer to micropolar fluid from a non-isothermal stretching sheet with suction and blowing. The steady isothermal flow of a micropolar fluid driven by a continuous porous surface is analysed by numerical methods. The two-dimensional boundary layer flow caused by a moving plate or a stretching sheet is of interest in manufacturing of sheeting material through an extrusion process. Most of the existing analytical studies for this problem are based on the constant physical properties. Pop et al. [13] studied the effect of variable viscosity on laminar boundary layer flow and heat transfer due to a continuous moving plate. Heruska et al. [8] studied the micropolar fluid flow past a porous stretching sheet.

In this paper, an attempt has been made to investigate the effect of viscosity and thermal conductivity on flow of a micropolar fluid bounded by a stretching sheet. Mathematical formulation of the problem under consideration is presented and similarity transformations are applied to reduce the system of partial differential equations and their boundary conditions describing this problem, into a boundary value problem of ordinary differential equations. This system of ordinary differential equations is solved numerically by shooting method. The effects of different parameters are studied numerically. The variation of the velocity, microrotation and temperature distribution has been illustrated. Attia [1] studied the stagnation point flow and heat transfer of a micropolar fluid in a porous medium. Wilson [16] studied the micropolar boundary layer flow near a stagnation point with the aid of integral method. Kafoussias and Williams [9] investigated the thermal-diffusion and diffusion-thermo effects on mixed free forced convection and mass transfer boundary layer flow with temperature dependent viscosity. Though the viscosity and thermal conductivity are assumed as constant properties but in actual these are temperature dependent (Schlichting [15] and Eckert [3]). Therefore, in this paper, we consider the effect of variable viscosity and variable thermal conductivity on steady incompressible laminar flow of a micropolar fluid bounded by a stretching sheet by Kelson et al. [10] for constants properties of viscosity and thermal conductivity.

2. Governing Equations

The equation of motion for incompressible viscous micropolar fluid is given by

$$\rho\left\{\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}\right\} = -\nabla p + \nabla(\mu\nabla \cdot \vec{V}) + \kappa\nabla^2\vec{V} + \kappa(\nabla \times \vec{N}) + \vec{F}, \tag{1}$$

where ρ is the mass density of the fluid, p is the pressure, μ is the viscosity, $\vec{N}$ is the angular velocity, κ is the material constant and t denotes time. $\vec{F}$ is the body force per unit volume due to flow through porous media given by

$$\vec{F} = \frac{\nu}{\lambda^*}\vec{V}, \tag{2}$$

where v is the kinematic viscosity of the fluid and λ^* is the coefficient of permeability of the porous media.

The equation of angular momentum for incompressible viscous micropolar fluid is given by

$$\rho j\left\{\frac{\partial \vec{N}}{\partial t}(\vec{V} \cdot \nabla)\vec{N}\right\} = -2\kappa\vec{N} + \kappa(\nabla \times \vec{V}) - \gamma\{\nabla \times (\nabla \times \vec{N})\}, \tag{3}$$

where j is the micro-inertia per unit mass and γ is the material constant. The equation of heat transfer is given by

$$\rho C_p\left\{\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla)T\right\} = \nabla \cdot (\lambda\nabla T) + (\mu + \kappa)\phi, \tag{4}$$

where C_p is specific heat at constant pressure, T is the temperature of the fluid, λ is the coefficient of thermal conductivity of the fluid, ϕ is the viscous dissipation function and is given by

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2. \quad (5)$$

3. Formulation of the Problem

We consider two-dimensional equations governing the isothermal, steady, laminar, incompressible micropolar fluid in a quiescent medium by Ramachandran et al. [14]. Let u and v be the velocity components in x and y directions, respectively. N is the microrotation component and T is the temperature. The basic boundary layer equations for a steady two-dimensional flow of micropolar fluid are as follows:

Mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (6)$$

Momentum equation:

$$\rho\left(\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + k\frac{\partial^2 u}{\partial y^2} + k\frac{\partial N}{\partial y}. \quad (7)$$

Angular momentum equation:

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = -2\kappa N - k\frac{\partial u}{\partial y} + \gamma\frac{\partial^2 N}{\partial y^2}. \quad (8)$$

Energy equation:

$$\rho C_p\left(u\frac{\partial T}{\partial x} + \nu\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + (\mu + k)\left(\frac{\partial u}{\partial y}\right)^2. \quad (9)$$

In the above equations, μ and γ are, respectively, the viscosity and microrotation (or spin-gradient) viscosity, j is the micro-inertia density, ρ is the density of the fluid, κ is the microrotation coupling coefficient (or coefficient of gyro viscosity or vortex viscosity), p is the pressure, C_p is the specific heat at constant pressure and λ is the thermal conductivity.

The appropriate physical boundary conditions are

$$y = 0;\ \ u = u_w,\ \ v = v_w,\ \ N = 0,\ \ T = T_\infty,$$

$$y \to 0;\ \ v \to 0,\ \ N \to 0,\ \ T \to T_\infty. \quad (10)$$

The governing equations subject to the boundary conditions can be expressed in a simpler form by introducing the following transformations

$$\psi = axF(\eta), \;\; u = \frac{\partial \psi}{\partial y} = abxF'(\eta), \;\; v = -\frac{\partial \psi}{\partial x} = -aF(\eta), \;\; \eta = by, \;\; N = cxG(\eta) \quad (11)$$

and

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w = T_\infty + x^2. \quad (12)$$

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [11]) as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \alpha(T - T_\infty)], \;\; \frac{1}{\mu} = a(T - T_r), \;\; a = \frac{\alpha}{\mu_\infty} \;\text{ and }\; T_r = T_\infty - \frac{1}{\alpha}, \quad (13)$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general, $a > 0$ for liquids and $a < 0$ for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T = T_\infty$. Similarly, consider the variation of thermal conductivity as

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}[1 + \xi(T - T_\infty)], \;\; \frac{1}{\lambda} = b(T - T_k), \;\; b = \frac{\xi}{\lambda_\infty} \;\text{ and }\; T_k = T_\infty - \frac{1}{\xi}, \quad (14)$$

where b and T_k are constants and their values depend on the reference state and thermal property of the fluid, ξ is constant based on thermal property and λ_∞ is the viscosity at $T = T_\infty$.

Using equations of velocity components of u and v equation (6) is automatically satisfied. Substituting the expressions (11)-(14) in equations (7)-(9), we have

$$F''' = \frac{\theta_r - \theta}{\theta_r + N_1(\theta_r - \theta)}[F'^2 - FF'' - N_1 G'] - \frac{1}{\theta_r + N_1(\theta_r - \theta)} \cdot \frac{\theta_r}{(\theta_r - \theta)} F''\theta', \quad (15)$$

$$G'' = \frac{N_3}{N_2}(F'G - FG') + \frac{N_1}{N_2}(2G + F''), \quad (16)$$

$$\theta'' = -P_r F\theta' \frac{\theta_k - \theta}{\theta_k} - \frac{\theta'^2}{\theta_k - \theta} - N_4 F'' \frac{\theta_k - \theta}{\theta_k}. \quad (17)$$

By choosing $a^2 = \nu,\ b^2 = c^2 = \dfrac{1}{\nu}$ and introducing the physical parameters

$$N_1 = \frac{k}{\rho\nu}, \quad N_2 = \frac{\gamma}{\rho\nu^2}, \quad N_3 = \frac{j}{\nu} \quad \text{and} \quad N_4 = \frac{\mu + k}{\lambda}, \quad P_r = \frac{\rho\nu C_p}{\lambda}$$

and the transformed boundary conditions are

$$F'(0) = 1, \quad F(0) = -\frac{v_w}{a} = -V, \quad G(0) = 0, \quad \theta(0) = 1,$$

$$\lim_{n\to\infty} F' = 0, \quad \lim_{n\to\infty} G = 0, \quad \lim_{n\to\infty} \theta = 0.$$

4. Conclusion

In this study, we have investigated the effect of variable viscosity and thermal conductivity to the boundary layer equations for the micropolar fluid flow over a stretching sheet. We have considered in some detail the influence of the physical parameters on the similarity solutions using Runga-Kutta shooting method. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity distribution, microrotation distribution and temperature distribution.

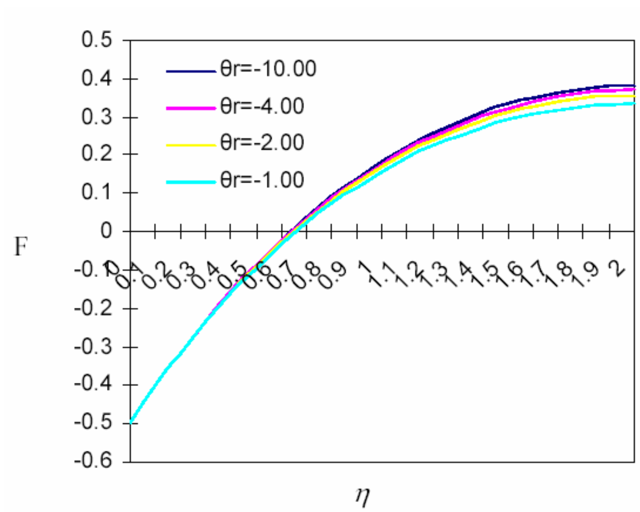

Figure 1. Variation of velocity distribution of F against η for various values of temperature corresponding to the viscosity parameter θ_r taking $P_r = 0.70,\ V = 0.50,\ N_1 = 0.80,\ N_2 = 0.50,\ N_3 = 0.50,\ N_4 = 0.50,\ \theta_k = -10.00.$

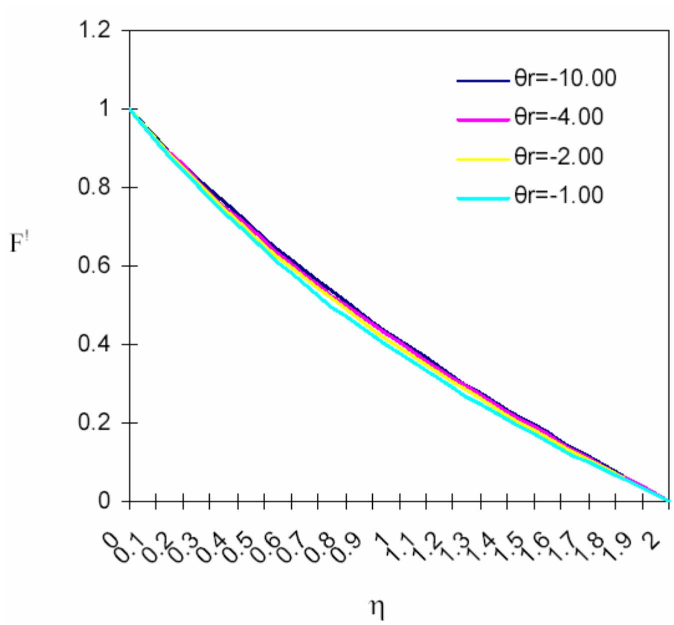

Figure 2. Variation of velocity distribution of F' against η for various values of temperature corresponding to the viscosity parameter θ_r taking $P_r = 0.70$, $V = 0.50$, $N_1 = 0.80$, $N_2 = 0.50$, $N_3 = 0.50$, $N_4 = 0.50$, $\theta_k = -10.00$.

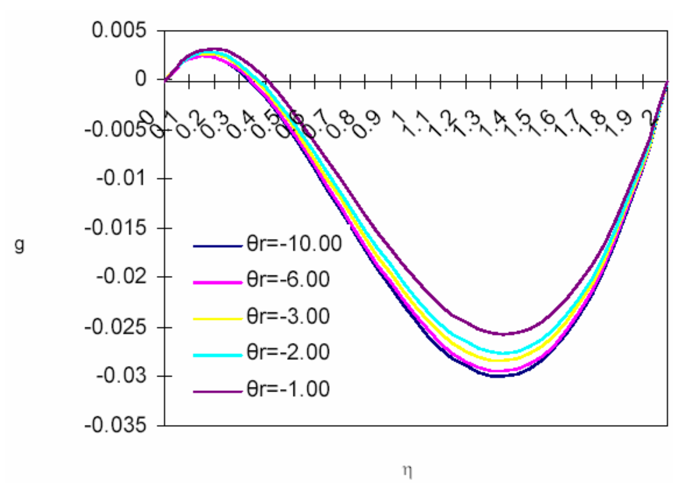

Figure 3. Variation of microrotation distribution of g against η for various values of temperature corresponding to the viscosity parameter θ_r taking $P_r = 0.70$, $V = 0.50$, $N_1 = 0.80$, $N_2 = 0.50$, $N_3 = 0.50$, $N_4 = 0.50$, $\theta_k = -10.00$.

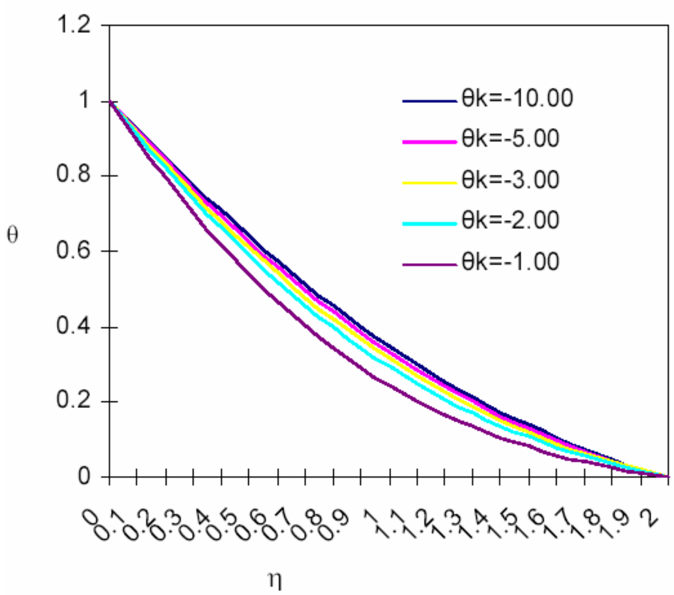

Figure 4. Variation of temperature distribution (θ) against η for the various values of temperature corresponding to thermal conductivity parameter (θ_k) taking $P_r = 0.70$, $V = 0.50$, $N_1 = 0.80$, $N_2 = 0.50$, $N_3 = 0.50$, $N_4 = 0.50$, $\theta_r = -10.00$.

Results and Discussion

The equations (6)-(9) together with the boundary conditions (10) are solved for various values of the parameters involved in the equations using algorithms based on the shooting method. Results are presented for velocity distribution, microrotation distribution and temperature distribution with the variation of different parameters.

Initially solution was taken for constant values of taking $P_r = 0.70$, $V = 0.50$, $N_1 = 0.80$, $N_2 = 0.50$, $N_3 = 0.50$, $N_4 = 0.50$, $\theta_r = -10.00$, $\theta_k = -10$ with the viscosity parameter θ_r ranging from –2 to –1 at the certain values of $\theta_k = -10$. Similarly, the solutions have been found with varying the thermal conductivity parameter θ_k ranging from –12 to –1 at the certain values of $\theta_r = -10$ keeping other values remaining same. The variation in velocity distribution, microrotation distribution and temperature distribution is illustrated in Figures 1 to 4. From the equation (11), it is found that the velocity ‘u’ is dependent on $F'(\eta)$. Figures 1 and

2 represent the variation in velocity (u) distribution with the variation of viscosity parameter θ_r. From Figure 3, it is seen that the variation in microrotation distribution with the variation of θ_r. From Figure 4, it is found that the variation in temperature distribution with variation of temperature corresponding to thermal conductivity parameter (θ_k).

From Figure 1, it is clear that velocity decreases as θ_r increases, i.e., $-\theta_r$ decreases. From Figure 2, it is seen that velocity distribution decreases with the increase of θ_r. From Figure 3, it is seen that microrotation distribution increases as θ_r increases. From Figure 4, it is found that temperature distribution decreases with the increase of θ_k.

References

[1] A. H. Attia, Stagnation point flow and heat transfer of a micropolar fluid in a porous medium, Turkish J. Phys. 30 (2006), 57-65.

[2] L. J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21 (1970), 645-647.

[3] E. R. G. Eckert, Heat and Mass Transfer, McGraw Hill, New York, 1972.

[4] A. C. Eringen, Theory of thermo-micro fluids, J. Math. Anal. Appl. 38 (1964), 480-496.

[5] A. C. Eringen, Theory of micropolar fluids. J. Math. Mech. 16 (1966), 1-18.

[6] F. M. Hady, On the solution of heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection, Int. J. Num. Math. Heat Fluid Flow 6 (1996), 99-104.

[7] I. A. Hassanien and R. S. R. Gorla, Heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing, Acta Mech. 84 (1990), 191-199.

[8] M. W. Heruska, L. T. Watson and K. K. Sankara, Micropolar flow past a porous stretching sheet, Comput. & Fluids 14(2) (1986), 117-129.

[9] N. G. Kafoussias and E. W. Williams, Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity, Int. J. Engng. Sci. 33(9) (1995), 1369-1384.

[10] N. A. Kelson, A. Desseaux and T. W. Farrell, Micropolar flow in a porous channel with high mass transfer, ANZIAM J. 44 (2003), 479-495.

[11] F. C. Lai and F. A. Kulacki, The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, Int. J. Heat Mass Transfer 33(5) (1990), 1028-1031.

[12] G. Lukaszewicz, Micropolar fluids theory and applications, Modeling and Simulation in Science, Engineering and Technology, Birkhäuser Boston, Inc., Boston, MA, 1999.

[13] I. Pop, R. S. R. Gorla and M. Rashidi, The effect of variable viscosity on flow and heat transfer to a continuous moving flat plate, Int. J. Engng. Sci. 30(1) (1992), 1-6.

[14] P. S. Ramachandran, M. N. Mathur and S. K. Ojha, Heat transfer in boundary layer flow of a micropolar fluid past a curved surface with solution and injection, Int. J. Engng. Sci. 17 (1979), 625-639.

[15] H. Schlichting, Boundary Layer Theory, McGraw Hill, New York, 1968.

[16] A. J. Wilson, Boundary layer in micropolar liquids, Proc. Camb. Phil. 67 (1970), 969.