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INTERACTIVE STABILITY OF MULTIOBJECTIVE NONLINEAR PROGRAMMING PROBLEMS WITH FUZZY PARAMETERS

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Abstract

In this paper, we introduce an interactive method for treating multiobjective nonlinear programming problems with fuzzy parameters in the objective functions. The decision maker (DM) must make two types of preference statements at each iterate generated by the method. First, the local tradeoff rates between the objective functions. Second, the preference selection among several vectors of objective functions for each considered feasible solution. The stability of solutions which are obtained by using this method is also presented.

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1. Introduction

Osman [7, 8] introduced the notions of the solvability set, stability set of the first kind and stability set of the second kind. Tanaka and Asai [12] formulated multiobjective linear programming problems with fuzzy parameters. Orlovski [6] formulated general multiobjective nonlinear programming problems with fuzzy parameters. Sakawa and Yano [11] introduced the concept of α-multiobjective nonlinear programming and α-Pareto optimality. Osman and El-Banna [9] introduced the qualitative analysis of the stability set of the first kind for fuzzy parametric multiobjective nonlinear programming problems. Kassem [4] introduced the interactive stability of multiobjective nonlinear programming problems with fuzzy parameters in the constraints. Geoffrion et al. [3] introduced the interactive method for multicriterion optimization, this method is called *GDF method*. Recently, Elshafei [2] introduced an interactive stability compromise programming method for solving fuzzy multiobjective integer nonlinear programming problems.

In this paper, we present a solution method for fuzzy multiobjective nonlinear programming problems by using GDF method. Moreover, we determine the stability of the solutions which are obtained from the directional problems generated by this method.

2. Problem Formulation

Let us consider the following fuzzy multiobjective nonlinear programming (FMONLP) problem:

$$\text{(FMONLP)} \quad \begin{array}{l} \max(f_1(x, \tilde{a}_1),\ f_2(x, \tilde{a}_2), \ldots,\ f_k(x, \tilde{a}_k)) \\ \text{subject to } x \in X = \{x \in R^n \mid g_j(x) \le 0,\ j = 1, 2, \ldots, m\}, \end{array}$$

where $f_i(x, \tilde{a}_i)$ is a continuously differentiable and concave function for $i = 1, 2, \ldots, k$, X is nonempty convex and compact, g_j is a continuously differentiable and convex function for $j = 1, 2, \ldots, m$, and $\tilde{a}_i = (\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{iq_i})$ represents a vector of fuzzy parameters in the objective function $f_i(x, \tilde{a}_i)$, these fuzzy parameters are assumed to be characterized as the fuzzy numbers introduced in [1]. A real fuzzy number $\tilde{p}$ is a convex continuous fuzzy subset of the real line whose membership function $\mu_{\tilde{p}}(p)$ is defined by:

(1) a continuous mapping from R to the closed interval $[0, 1]$;

(2) $\mu_{\tilde{p}}(p) = 0$ for all $p \in (-\infty, p_1]$;

(3) strict increase on (p_1, p_2);

(4) $\mu_{\tilde{p}}(p) = 1$ for all $p \in [p_2, p_3]$;

(5) strict decrease on (p_3, p_4);

(6) $\mu_{\tilde{p}}(p) = 0$ for all $p \in [p_4, +\infty)$.

For simplicity of notations we define the following vectors:

$$\tilde{a}_i = (\tilde{a}_{i1}, \tilde{a}_{i2}, ..., \tilde{a}_{iq_i}),$$

$$a = (a_1, a_2, ..., a_k),$$

$$\tilde{a} = (\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_k).$$

Definition 1 (α-level set)**.** The α-*level set* of the numbers $\tilde{a}_i$ $(i = 1, 2, ..., k)$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership functions exceeds the level α:

$$L_\alpha(\tilde{a}) = \{a \mid \mu_{\tilde{a}_i}(a_i) \geq \alpha,\ i = 1, 2, ..., k\}.$$

For a certain degree of α, the problem (FMONLP) can be understood as the following nonfuzzy α-multiobjective nonlinear programming $(\alpha\text{-MONLP})'$ problem:

$$(\alpha\text{-MONLP})' \quad \begin{array}{l} \max(f_1(x, a_1),\ f_2(x, a_2), ...,\ f_k(x, a_k)) \\ \text{subject to } x \in X,\ a \in L_\alpha(\tilde{a}). \end{array}$$

Problem $(\alpha\text{-MONLP})'$ can be rewritten as the following form:

$$\max(f_1(x, a_1),\ f_2(x, a_2), ...,\ f_k(x, a_k))$$

$$(\alpha\text{-MONLP}) \text{ subject to } x \in X,$$

$$A_i \leq a_i \leq B_i, \quad i = 1, 2, ..., k,$$

where A_i and B_i are lower and upper bounds on a_i for $i = 1, 2, ..., k$.

Definition 2 (α-Pareto optimal solution)**.** $x^* \in X$ is said to be an α-*Pareto optimal solution* to the (α-MONLP), if and only if there does not exist another $x \in X$, $a \in L_\alpha(\tilde{a})$ such that $f_i(x, a_i) \geq f_i(x^*, a_i^*)$, $i = 1, 2, \ldots, k$, with strictly inequality holding for at least one i, where the corresponding values of parameters a_i^* are called α-*level optimal parameters*.

For some (unknown) implicit utility function, we have the following problem:

$$(\alpha M) \quad \begin{aligned} &\max U[f_1(x, a_1), f_2(x, a_2), \ldots, f_k(x, a_k)] \\ &\text{subject to } x \in X, \\ &A_i \leq a_i \leq B_i, \; i = 1, 2, \ldots, k, \end{aligned}$$

where $U(\cdot)$ is concave and continuously differentiable.

3. Tradeoff Weights

In this approach, given a point $(x^l, a^l) \in X \times L_\alpha(\tilde{a})$, the DM is asked to estimate the local tradeoff rates w_i^l for $i = 1, 2, \ldots, k$, where

$$w_i^l = \frac{\partial U / \partial f_i(x, a_i)}{\partial U / \partial f_1(x, a_1)} \quad \text{evaluated at } (x, a) = (x^l, a^l). \tag{1}$$

The method employs the local tradeoff rates (1) in a different directional subproblem at each iteration. This subproblem is defined as follows:

$$\begin{aligned} &\max \sum_{i=1}^{k} w_i^l \nabla_{(x,a)} f_i(x^l, a_i^l) \cdot Z \\ &\text{subject to } Z \in X \times L_\alpha(\tilde{a}). \end{aligned} \tag{2}$$

Without loss of generality, the problem (2) can be reformulated to the equivalent form

$$\begin{aligned} &\max \sum_{i=1}^{n} t_i^l x_i + \sum_{i=1}^{k} t_{n+i}^l a_i \\ &\text{subject to } x \in X, \\ &A_i \leq a_i \leq B_i, \; i = 1, 2, \ldots, k. \end{aligned} \tag{3}$$

4. Stability Set of the First Kind

Here, we assume that the problem (FMONLP) is stable, therefore the problem (α-MONLP) is also stable [10]. The stability of problem (2) and problem (3) follows directly from the stability of the (α-MONLP) problem.

Definition 3 (Stability set of the first kind)**.** Given a certain $\bar{t} \in R^{n+k}$ with a corresponding optimal solution $(\bar{x}, \bar{a})$, then the *stability set of the first kind* of problem (3) corresponding to $(\bar{x}, \bar{a})$, denoted by $S(\bar{x}, \bar{a})$, is defined by

$$S(\bar{x}, \bar{a}) = \{t \in R^{n+k} \mid (\bar{x}, \bar{a}) \text{ is an optimal solution of problem (3)}\}.$$

Let a certain $\bar{t} \in R^{n+k}$ with a corresponding optimal solution $(\bar{x}, \bar{a})$ be given. Then from the stability of problem (3), there exist $t \in R^{n+k}$, $\nu \in R^m$, $\lambda \in R^k$ and $\mu \in R^k$ such that the Kuhn-Tucker conditions of problem (3) take the form:

$$\begin{aligned}
& t_r - \sum_{j=1}^{m} \nu_j \frac{\partial g_j}{\partial x_r}(\bar{x}) = 0, && r = 1, 2, \dots, n, \\
& t_{n+i} - \lambda_i + \mu_i = 0, && i = 1, 2, \dots, k, \\
& g_j(\bar{x}) \le 0, && j = 1, 2, \dots, m, \\
& \bar{a}_i - B_i \le 0, && i = 1, 2, \dots, k, \\
& A_i - \bar{a}_i \le 0, && i = 1, 2, \dots, k, \\
& \nu_j g_j(\bar{x}) = 0, && j = 1, 2, \dots, m, \\
& \lambda_i(\bar{a}_i - B_i) = 0, && i = 1, 2, \dots, k, \\
& \mu_i(A_i - \bar{a}_i) = 0, && i = 1, 2, \dots, k, \\
& \nu_j = 0, && \text{for } j \in J \subset \{1, 2, \dots, m\}, \\
& \nu_j \ge 0, && \text{for } j \notin J, \\
& \lambda_i = 0, \ \mu_i = 0 && \text{for } i \in I \subset \{1, 2, \dots, k\}, \\
& \lambda_i \ge 0, \ \mu_i \ge 0, && \text{for } i \notin I.
\end{aligned}$$

Consider the system of equations

$$\begin{aligned} &t_r - \sum_{j \notin J} \nu_j \frac{\partial g_j}{\partial x_r}(\bar{x}) = 0, \qquad r = 1, 2, \ldots, n, \\ &t_{n+i} - \lambda_i + \mu_i = 0, \qquad\qquad i \notin I. \end{aligned} \tag{$*$}$$

It is clear that

$$S(\bar{x}, \bar{a}) = \{t \in R^{n+k} \,|\, (t, \nu, \lambda, \mu) \text{ solves the system } (*)\}.$$

Theorem. *The set* $S(\bar{x}, \bar{a})$ *is convex.*

Proof. The proof is similar to the one in [7].

5. A Solution Algorithm

The steps of the algorithm can be summarized as follows:

Step 1. Set a certain degree α $(0 \leq \alpha \leq 1)$.

Step 2. Determine the α-level set of the fuzzy numbers.

Step 3. Convert the FMONLP in the form of α-MONLP and select an initial feasible point (x^l, a^l). Set $l = 0$.

Step 4. The DM specifies precise values of local tradeoff rates w_i^l at (x^l, a^l) for $i = 1, 2, \ldots, k$.

Step 5. Solve problem (3) to get optimal solution (x^{l+1}, a^{l+1}).

Step 6. Set $d^{l+1} = (x^{l+1}, a^{l+1}) - (x^l, a^l)$. Present to DM the vector

$$f_1((x^l, a^l) + \rho d^{l+1}), \ldots, f_k((x^l, a^l) + \rho d^{l+1}),$$

for various values of $0 \leq \rho \leq 1$, the preference selection defines ρ^l.

Step 7. If $\rho^l = 0$, go to Step 10.

Step 8. Determine the stability set of the first kind $S(x^{l+1}, a^{l+1})$.

Step 9. Put $(x^{l+1}, a^{l+1}) = (x^l, a^l) + \rho^l d^{l+1}$, set $l = l + 1$ and go to Step 4.

Step 10. Terminate with (x^l, a^l) as α-Pareto optimal solution to the α-MONLP problem.

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