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EFFECT OF MAGNETIC FIELD ON A TRANSIENT MIXED CONVECTION FLOW THROUGH A POROUS MEDIUM BOUNDED BY A SUDDENLY FIXED INFINITE VERTICAL PLATE

N. AHMED and H. K. SARMAH

Department of Mathematics
Gauhati University
Guwahati 781 014, Assam, India
e-mail: nazib_ahmed@rediffmail.com
nsarmah@hotmail.com

Abstract

An exact solution to the problem of an unsteady free and forced convective MHD flow of an incompressible viscous electrically conducting fluid through a porous medium bounded by an infinite vertical hot plate impulsively held fixed in a uniform stream is presented. A uniform magnetic field is assumed to be applied transversely to the direction of the flow. The governing equations are solved by Laplace transform technique. The expressions for the velocity field and skin friction at the plate are obtained and demonstrated graphically for the various values of the parameters involved in the problem. The expressions for the coefficient of rate of heat transfer, temperature field and current density are also derived in non-dimensional form. The effects of the Hartmann number, the Prandtl number, the porosity parameter and the other parameters involved on the velocity field and skin friction at the plate are discussed. It is observed that the velocity field, skin friction and current density are significantly affected by the applied magnetic field.

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1. Introduction

The problems of transient free convection flows past infinite vertical plates were studied by many researchers because of their applications in the cooling process. Some of them were Siegel [17], Gebhart [6], Chung and Anderson [5], Schetz and Eichhorn [16], and Goldstein and Briggs [7]. Siegel [17] first observed that the initial behaviour of temperature and velocity fields for a semi-infinite plate is the same as for a doubly infinite vertical plate where the temperature field is given by the solution of one-dimensional heat conduction equation. Goldstein and Eckert [8] later confirmed the above theoretical result through experiments. Lahurikar *et al*. [13] studied the unsteady forced and free convective flow past an infinite vertical plate through a porous medium of Brinkman model [2]. In recent years, study on MHD unsteady convection problems has attracted numerous authors in view of the application of such problems in Geophysics, Astrophysics and in missile technology. At high temperature gas is ionized and it becomes a good conductor. The ionized gas or plasma interacts with the applied magnetic field and significantly alters the flow and heat transfer characteristics. Jha [9] analysed the effects of magnetic field and permeability of the porous medium on unsteady forced and free convection flow past an infinite vertical porous plate in presence of temperature-dependent heat source. A study of unsteady laminar hydromagnetic flow and heat transfer in porous channel with temperature-dependent properties was presented by Chamkha [3]. Kalita and Borkakati [11] studied the transient free convection MHD flow through a porous medium between two vertical plates. They analysed systematically the flow of a viscous incompressible and electrically conducting fluid in presence of a transverse magnetic field through a porous medium whose effective viscosity is larger than the viscosity of the fluid. The effect of the local acceleration term on the MHD transient free convection flow over a vertical plate was studied by Aldoss *et al*. [1]. Prasad *et al*. [14] investigated the interaction of free convection with thermal radiation of viscous incompressible MHD unsteady flow past an impulsively started vertical plate with uniform heat and mass flux. A study of unsteady MHD free convection flow through a porous vertical flat plate immersed in a porous medium in presence of magnetic field with radiation was carried out by Samad and Rahman [15]. Joaquin [10] presented a numerical analysis of an unsteady free convective MHD flow of a dissipative fluid along a vertical plate subject to a constant heat flux. An exact solution to the problem of the MHD flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free convection and mass transfer was obtained by Chaudhary and Jain [4]. Khan *et al*. [12] analysed the MHD transient flows in a channel of rectangular cross section with porous medium.

As the present authors are aware up till now no attempt has been made to study the effect of the transverse magnetic field on a transient free and forced convective flow through a porous medium bounded by an infinite vertical plate impulsively held fixed in free stream. Such an attempt has been made in the present work, because of the importance of such problems in industry as well as in aerodynamics. We present the results for two fluids viz air $(P = .7)$ and water $(P = 7)$.

2. Basic Equations

The equations governing the motion of an incompressible viscous electrically conducting fluid through a porous medium in presence of a magnetic field are:

the equation of continuity:

$$\operatorname{div} \vec{q} = 0, \tag{2.1}$$

the Gauss's law of magnetism:

$$\operatorname{div} \vec{B} = 0, \tag{2.2}$$

the modified Navier-Stokes equation:

$$\rho\left[\frac{\partial \vec{q}}{\partial \bar{t}} + (\vec{q} \cdot \vec{\nabla})\vec{q}\right] = -\vec{\nabla}p + \vec{J} \times \vec{B} + \mu\nabla^2\vec{q} - \frac{\mu\vec{q}}{\overline{K}} + \rho\vec{g}, \tag{2.3}$$

the energy equation:

$$\rho C_p\left[\frac{\partial \overline{T}}{\partial \bar{t}} + (\vec{q} \cdot \vec{\nabla})\overline{T}\right] = k\nabla^2\overline{T} + \phi + \frac{\vec{J}^2}{\sigma}, \tag{2.4}$$

the Ohms' law:

$$\vec{J} = \sigma[\vec{E} + \vec{q} \times \vec{B}], \tag{2.5}$$

where

$\vec{q}$ is the velocity vector,

$\vec{g}$ is the acceleration due to gravity,

$\bar{t}$ is the time,

$\vec{B}$ is the magnetic induction vector,

$\vec{E}$ is the electric field (here assumed to be zero),

σ is the electrical conductivity,

k is the thermal conductivity,

C_p is the specific heat at constant pressure,

p is the pressure,

μ is the coefficient of the viscosity,

$\overline{T}$ is the temperature,

$\vec{J}$ is the electric current density,

$\overline{K}$ is the permeability parameter,

ρ is the density of fluid,

ϕ is the viscous dissipation per unit volume,

$\vec{J} \times \vec{B}$ is the Lorentz force per unit volume,

$\frac{\vec{J}^2}{\sigma}$ is the Joulean heat per unit volume and the other symbols have their usual meanings.

We now consider an unsteady two-dimensional boundary layer flow of an incompressible viscous electrically conducting fluid through a porous medium of Brinkman model [2] occupying a semi-infinite region of the space bounded by an infinite vertical hot plate suddenly held fixed in a uniform stream in presence of a transverse magnetic field of strength B_0 by making the following assumptions:

(i) All the fluid properties except the density in the buoyancy force term are constant.

(ii) The viscous dissipation of energy is negligible.

(iii) The Joulean heat is negligible.

(iv) The flow is parallel to the plate.

(v) The magnetic Reynolds number is small so that the induced magnetic field can be neglected.

Initially the plate and the surrounding fluid are at the same temperature T'_∞ and the plate is moving parallel to itself with velocity U_0. At time $t' > 0$, the plate is suddenly made stationary and the plate temperature is raised to T'_w.

We introduce a coordinate system $(\bar{x}, \bar{y}, \bar{z})$ with X-axis vertically upwards along the plate, Y-axis normal to the plate into the fluid region and Z-axis along the width of the plate. Let the plate be long enough in X-direction for the flow to be parallel. Let $\vec{q} = (u, 0, 0)$ be the fluid velocity and $\vec{B} = (0, \bar{b}_y, 0)$ be the magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid. Since the plate is infinite in length in X-direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$.

With the foregoing assumptions, Boussinesq approximation and usual boundary layer approximations, the equations (2.1), (2.2), (2.3), (2.4) and (2.5) can be reduced to the following:

Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} = 0 \tag{2.6}$$

which is satisfied by the velocity field $\bar{u} = \bar{u}(\bar{y}, \bar{t})$.

Gauss's law of magnetism:

$$\frac{\partial \bar{b}_y}{\partial \bar{y}} = 0 \tag{2.7}$$

which holds for $\bar{b}_y =$ a constant $= B_0$, the strength of the applied magnetic field.

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = g\beta(\bar{T} - \bar{T}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\sigma B_0^2}{\rho}(U_0 - \bar{u}) + \frac{\nu}{\bar{K}}(U_0 - \bar{u}). \tag{2.8}$$

Energy equation:

$$\rho C_p \frac{\partial \bar{T}}{\partial \bar{t}} = k \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}. \tag{2.9}$$

The initial and boundary conditions of the problem are:

$$\bar{t} \le 0: \quad \bar{u} = U_0, \quad \bar{T} = \bar{T}_\infty, \qquad \forall y, \tag{2.10}$$

$$\left.\begin{aligned} \bar{t} > 0: \quad \bar{u} = 0, \quad & \bar{T} = \bar{T}_w, \quad \text{at} \quad \bar{y} = 0 \\ \bar{u} = U_0, \quad & \bar{T} = \bar{T}_\infty, \quad \text{at} \quad \bar{y} \to \infty \end{aligned}\right\}, \tag{2.11}$$

where

$\bar{u}$ is the velocity of the fluid near the plate,

U_0 is the free stream velocity,

g is the acceleration due to gravity,

$\bar{T}_\infty$ is the temperature in the free stream,

$\bar{T}_w$ is the plate temperature,

β is the coefficient of volume expansion,

ν is the kinematic viscosity,

σ is the electrical conductivity.

We now introduce the following non-dimensional quantities:

$$G = \frac{\nu g \beta (\bar{T}_w - \bar{T}_\infty)}{U_0^3} \text{ (Grashof number)},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho U_0^2 G} \text{ (Hartmann number)},$$

$$P = \frac{\mu C_p}{k} \text{ (Prandtl number)},$$

$$K = \frac{\bar{K} U_0^2 G}{\nu^2} \text{ (Permeability parameter)},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \quad y = \frac{\bar{y} U_0 \sqrt{G}}{\nu}, \quad u = \frac{\bar{u}}{U_0}, \quad t = \frac{\bar{t} U_0^2 G}{\nu},$$

$$w = 1 - u.$$

The non-dimensional forms of the equations (2.9), (2.10) and (2.11); the initial condition (2.12) and the boundary condition (2.13) are as follows:

$$\frac{\partial w}{\partial t} = -\theta + \frac{\partial^2 w}{\partial y^2} - \xi w, \tag{2.12}$$

$$P \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}, \tag{2.13}$$

$$t \le 0: \quad w = 0, \quad \theta = 0, \qquad \forall y, \tag{2.14}$$

$$\left.\begin{array}{lllll} \bar{t} > 0: & w = 1, & \theta = 1, & \text{at} & y = 0 \\ & w = 0, & \theta = 0, & \text{at} & \bar{y} \to \infty \end{array}\right\}, \tag{2.15}$$

where $\xi = M + \dfrac{1}{K}$.

3. Solution of the Equations

The equations (2.12) and (2.13) are solved (by Laplace transform technique) subject to the initial and boundary conditions (2.14) and (2.15), respectively, and the solutions are as follows:

$$\theta = erfc(\eta \sqrt{P}), \tag{3.1}$$

$$u = 1 - \left(1 + \frac{1}{\xi}\right) F_1 + \frac{1}{\xi} F_2 + \frac{1}{\xi} F_3 - \frac{1}{\xi} F_4 \quad \text{for} \quad P \ne 1 \tag{3.2}$$

$$= 1 - \left(1 + \frac{1}{\xi}\right) F_1 + \frac{1}{\xi} erfc(\eta) \quad \text{for} \quad P = 1, \tag{3.3}$$

where

$$\eta = \frac{y}{2\sqrt{t}},$$

$$F_1 = \frac{1}{2}[e^{2\eta\sqrt{\xi t}} erfc(\eta + \sqrt{\xi t}) + e^{-2\eta\sqrt{\xi t}} erfc(\eta - \sqrt{\xi t})],$$

$$F_2 = \frac{e^{\lambda t}}{2}[e^{2\eta\sqrt{(\xi+\lambda)t}} erfc\{\eta + \sqrt{(\xi+\lambda)t}\} + e^{-2\eta\sqrt{(\xi+\lambda)t}} erfc\{\eta - \sqrt{(\xi+\lambda)t}\}],$$

$$F_3 = erfc(P\sqrt{\eta}),$$

$$F_4 = \frac{e^{\lambda t}}{2}\left[e^{2\eta\sqrt{\lambda t}} erfc\left(\eta\sqrt{P} + \sqrt{\frac{\lambda t}{P}}\right) + e^{-2\eta\sqrt{\lambda t}} erfc\left(\eta\sqrt{P} - \sqrt{\frac{\lambda t}{P}}\right)\right].$$

4. Coefficient of Skin Friction, Rate of Heat Transfer and Current Density

The non-dimensional skin friction at the plate in the direction of free stream is given by

$$\tau = -\frac{\mu \frac{\partial u}{\partial \bar{y}}}{\rho U_0^2 \sqrt{G}}\Bigg]_{\bar{y}=0} = -\frac{\partial u}{\partial y}\Bigg]_{y=0} = -\frac{1}{2\sqrt{t}}\frac{\partial u}{\partial \eta}\Bigg]_{\eta=0}$$

$$= -\frac{1}{2\sqrt{t}}\left[\left(1+\frac{1}{\xi}\right)\frac{\partial F_1}{\partial \eta} - \frac{1}{\xi}\frac{\partial F_2}{\partial \eta} - \frac{1}{\xi}\frac{\partial F_3}{\partial \eta} + \frac{1}{\xi}\frac{\partial F_4}{\partial \eta}\right]_{\eta=0}. \tag{4.1}$$

The coefficient of heat transfer from plate to the fluid in terms of Nusselt number at the plate is given by

$$\mathrm{Nu} = -\frac{\nu}{U_0\sqrt{G}(\bar{T}_w - \bar{T}_\infty)}\frac{d\bar{T}}{d\bar{y}}\Bigg]_{\bar{y}=0}$$

$$= -\frac{d\theta}{dy}\Bigg]_{y=0} = -\frac{1}{2}\sqrt{\frac{P}{t}}\, erfc'(\eta\sqrt{P})\Bigg]_{\eta=0}. \tag{4.2}$$

The non-dimensional current density is given by

$$\bar{J} = \frac{J}{\sigma U_0 B_0} = u. \tag{4.3}$$

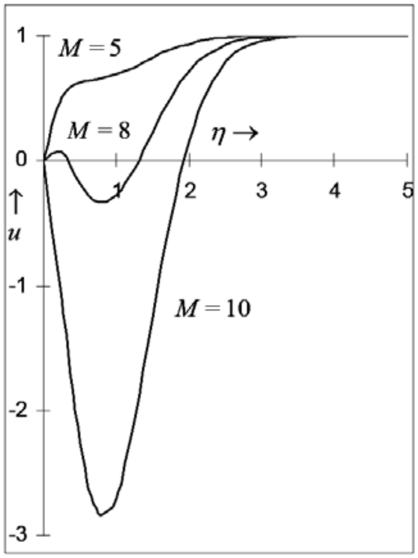

Figure 1. The velocity field u versus η for $P = 0.7$, $K = 0.5$, $t = 0.5$.

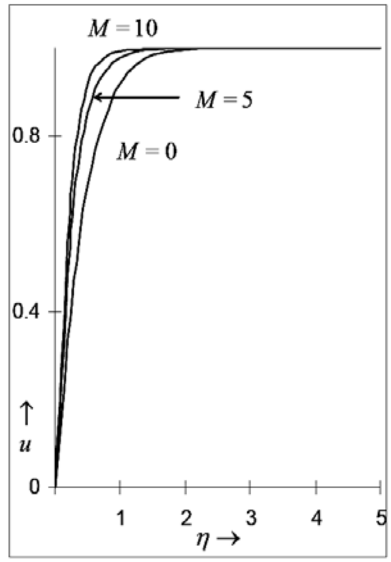

Figure 2. The velocity field u versus η for $P = 7$, $K = 0.5$, $t = 0.5$.

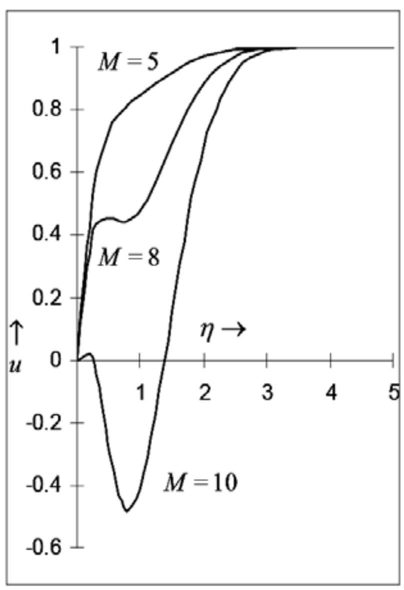

Figure 3. The velocity field u versus η for $P = 0.7$, $K = 5$, $t = 0.5$.

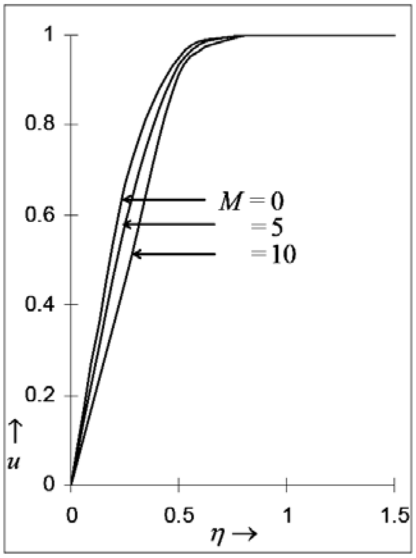

Figure 4. The velocity field u versus η for $P = 7,\ K = .02,\ t = 0.5.$

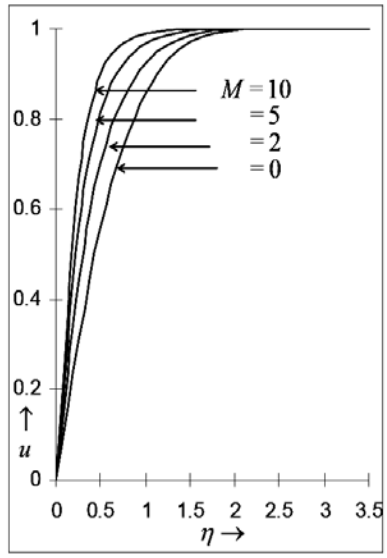

Figure 5. The velocity field u versus η for $P = 7,\ K = 5,\ t = 0.5.$

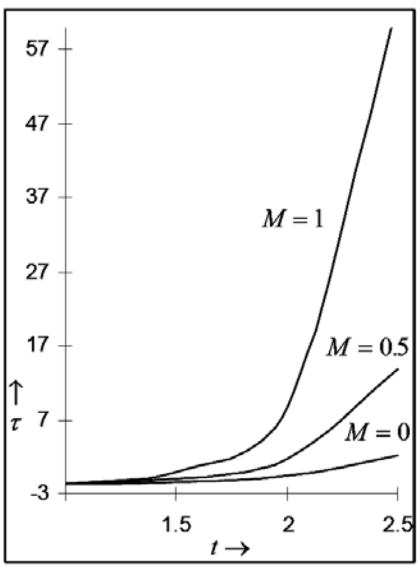

Figure 6. Skin friction τ at the plate $y = 0$ versus time t for $P = 0.7,\ K = 0.5.$

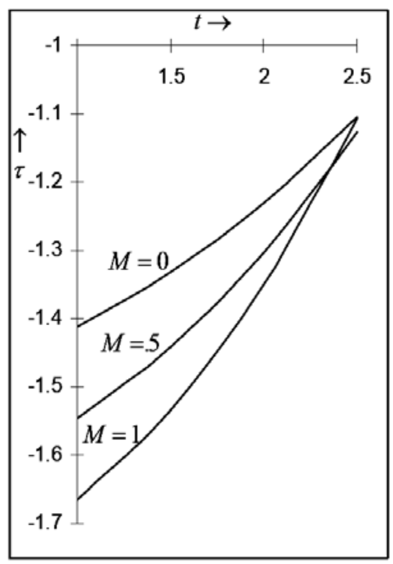

Figure 7. Skin friction τ at the plate $y = 0$ versus time t for $P = 7,\ K = 0.5.$

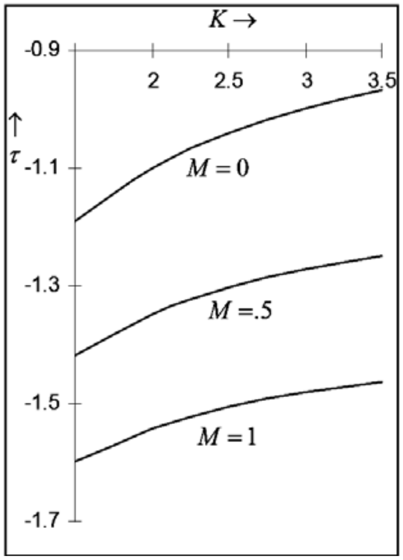

Figure 8. Skin friction τ at the plate $y = 0$ versus K for $P = 0.7$, $t = 0.5$.

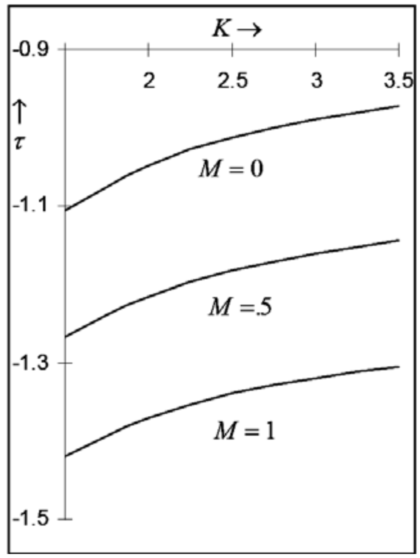

Figure 9. Skin friction τ at the plate $y = 0$ versus K for $P = 7$, $t = 0.5$.

5. Discussion of the Results

In order to get physical insight into the problem, we have carried out the numerical calculations for the velocity field and skin friction at the plate and the values are demonstrated in graphs. The Prandtl number P is taken to be equal to 0.7 and 7 which correspond to air and water, respectively, and the values of the porosity parameter K are chosen as 0.02, 0.5 and 5. The values of M, η, t, H, P_m and y are selected arbitrarily. The velocity profiles under the influence of magnetic field are displayed in Figures 1-5. We observe from Figures 2, 4 and 5 that in case of water $(P = 7)$, the fluid velocity quickly increases up to some layer of the fluid adjacent to the plate and after this fluid layer the fluid velocity asymptotically tends to 1 (free stream velocity). There is an indication from these figures that in case of moderate and high porosity of the medium, an increase in the strength of the applied magnetic field causes the velocity to increase (Figure 2).

In case of air (Figures 1 and 3), it is seen that for the large magnetic field, the fluid flows vertically downwards near the plate up to a layer in the immediate neighbourhood of the plate and after that it begins to flow in the upward vertical direction. That is, there occurs a surface of separation in the boundary layer flow.

The same figures also indicate that when the strength of the applied magnetic field is moderate or small, the velocity of air behaves like water. The influence of the applied magnetic field in the flow seems unaffected due to porosity in case of air.

Figures 6, 7, 8 and 9 depict the change of behaviour of the skin friction τ at the plate due to variation of the Hartmann number M. It is inferred from Figure 6 that in case of air, τ increases as M and t. The same figure also indicates that the effect of M on τ is very pronounced for large t and its effect is negligible for small t. In case of water (Figure 7), it is observed that the application of the magnetic field causes τ to decrease. Figure 7 also shows that τ is significantly affected by M for small t and as t increases the magnetic field ceases to affect τ. Figures 8 and 9 indicate that τ decreases as M increases whereas it increases for the increasing values of K.

It is observed that the rate of coefficient of heat transfer is not affected by the applied field. From equation (4.3), it is inferred that the behaviour of current density is exactly same as the convective velocity.

6. Conclusions

(a) The fluid velocity, skin friction and current density at the plate are significantly affected by the applied magnetic field.

(b) The effect of the magnetic field is not pronounced on the temperature field and on the rate of heat transfer coefficient.

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