Advances in Fuzzy Sets and Systems
Volume 4, Number 2, 2009, Pages 189-202
Published Online: July 8, 2009
This paper is available online at http://www.pphmj.com

STATE TIME-DELAY FEEDBACK ROBUST H_∞ CONTROL FOR UNCERTAIN FUZZY DISCRETE SYSTEMS WITH TIME-DELAY

HAN-LIANG HUANG and FU-GUI SHI

Department of Mathematics and Information Science
Zhangzhou Normal University
Zhangzhou 363000, P. R. China
e-mail: h_l_huang1980@eyou.com

Department of Mathematics
Beijing Institute of Technology
Beijing 100081, P. R. China
e-mail: f.g.shi@263.net

Abstract

In this paper, we consider the problem of state time-delay feedback robust H_∞ control for uncertain fuzzy discrete systems with time-delay. Takagi-Sugeno (T-S) fuzzy model is used to describe this kind of systems. Based on Lyapunov approach, a sufficient condition for the existence of robust H_∞ controller is obtained. The state time-delay feedback controller gains are derived by solving some LMIs. A numerical example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

As it is well known, time-delay and system uncertainty are commonly encountered and are often the sources of instability [11, 14]. The

2000 Mathematics Subject Classification: 93C42.

Keywords and phrases: robust control, linear matrix inequality (LMI), time-delay, T-S fuzzy systems, uncertainty.

This work was supported by the National Natural Science Foundation of China (10371079).

Received May 21, 2008

uncertainties may include modeling errors, parameter perturbations, fuzzy approximation errors and external disturbances. In the past few decades, the robust stability analysis problems for uncertain systems with time-delay have gained much research attention ([6, 8, 10] for continuous-time systems and [1, 3, 11] for discrete-time systems). This kind of systems can be found in many real life systems such as electric power systems, large electric networks, rolling mill systems, economic systems, aerospace systems, different types of societal systems and ecological systems. So we consider the nonlinear time-delay systems with parameter uncertainties in this paper.

In the past two decades, the fuzzy logic theory [13] has been proven to be a practical approach to deal with the analysis and synthesis problems for nonlinear systems. There exist two different types of fuzzy controllers: the Mamdani type [2] and the Takagi-Sugeno (T-S) type [5]. The T-S fuzzy model can represent nonlinear systems using fuzzy rules with consequent part as local linear subsystems. This kind of model can provide an effective representation of complex nonlinear systems. When the nonlinear plant is represented by a so-called T-S type fuzzy model, local dynamics in different state-space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these fuzzy models. The control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme.

The linear matrix inequality (LMI) theory is a new and fast growing mathematical tool for optimization problems [4, 7]. Many control problems can be transferred into a feasible problem of an LMI system or into a convex optimization problem which has the LMI restriction. In [7], Wang and Tanaka proposed an LMI approach to design the T-S fuzzy controllers for nonlinear systems described by the T-S fuzzy models.

In this paper, our purpose is to present an LMI-based controller design method for uncertain discrete systems with time-delay described by the T-S fuzzy models. A sufficient condition for the existence of the robust H_∞ controller is derived by Lyapunov functional approach. The state time-delay feedback gains can be obtained by solving some LMIs. A numerical example will be given to show the effectiveness of our method.

The paper is organized as follows: In Section 2, the T-S fuzzy model is presented to model an uncertain discrete system with time-delay and an assumption which the uncertainties should satisfy is introduced. In Section 3, the existence condition of a state time-delay feedback robust H_∞ controller is derived in an LMI form based on Lyapunov approach. In Section 4, a numerical example is given to demonstrate the results. Finally a conclusion is given in Section 5.

Notation. For a symmetric matrix X, the notation $X > 0$ means that the matrix X is positive definite. I is an identity matrix of appropriate dimension. X^T denotes the transpose of the matrix X. For any nonsingular matrix X, X^{-1} denotes the inverse of the matrix X. R^n denotes the n-dimensional Euclidean space. $R^{m\times n}$ is the set of all $m \times n$ matrices. $L_2[0, \infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $L_2[0, \infty)$ norm. $*$ denotes the transposed element in the symmetric position of a matrix.

2. System and Problem Description

Consider the following parameter uncertain discrete system with time-delay described by Takagi-Sugeno fuzzy model, the ith rule of the model is

Plant Rule i:

If

$$z_1(t) \text{ is } M_{i1},\ z_2(t) \text{ is } M_{i2},\ \ldots,\ z_g(t) \text{ is } M_{ig},$$

then

$$\begin{cases} x(t+1) = (A_{i1} + \Delta A_{i1}(t))x(t) + (A_{i2} + \Delta A_{i2}(t))x(t-d) \\ \qquad\qquad + (B_i + \Delta \mathrm{B}_i(t))u(t) + B_{\omega i}\omega(t), \\ \tilde{z}(t) = (C_{i1} + \Delta C_{i1}(t))x(t) + (C_{i2} + \Delta C_{i2}(t))x(t-d) \\ \qquad + (D_i + \Delta D_i(t))u(t), \\ x(t) = \varphi(t), \quad t \in [-d, 0], \end{cases} \tag{1}$$

where $i = 1, 2, ..., n$. n is the number of rules, $z_1(t), z_2(t), ..., z_g(t)$ are the premise variables and M_{ij} $(i = 1, 2, ..., n; j = 1, 2, ..., g)$ is the fuzzy set, $x(t) \in R^l$ is the state vector, $u(t) \in R^m$ is the input vector, $\omega(t)$ is the disturbance which belongs to $L_2[0, \infty)$, $\tilde{z}(t) \in R^p$ is the controlled output. $A_{i1}, A_{i2}, B_i, B_{\omega i}, C_{i1}, C_{i2}$ and D_i $(i = 1, 2, ..., n)$ are constant matrices of appropriate dimensions, $\varphi(t)$ is the initial condition of system (1), $d > 0$ is the upper bound of time-delay, $\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t), \Delta C_{i1}(t), \Delta C_{i2}(t)$ and $\Delta D_i(t)$ $(i = 1, 2, ..., n)$ are realvalued unknown matrices representing time-varying parameter uncertainties of (1) and satisfying the following assumption:

Assumption 1.

$$[\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)] = U_i F_i(t)[E_{i1}, E_{i2}, E_i], \tag{2}$$

$$[\Delta C_{i1}(t), \Delta C_{i2}(t), \Delta D_i(t)] = H_i V_i(t)[G_{i1}, G_{i2}, G_i], \tag{3}$$

where U_i, E_{i1}, E_{i2}, E_i, H_i, G_{i1}, G_{i2} and G_i $(i = 1, 2, ..., n)$ are known real constant matrices of appropriate dimensions. $F_i(t)$ and $V_i(t)$ $(i = 1, 2, ..., n)$ are unknown real time-varying matrices with Lebesgue measurable elements satisfying

$$F_i(t)^T F_i(t) \le I, \quad V_i(t)^T V_i(t) \le I, \quad i = 1, 2, ..., n. \tag{4}$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\rho_i(z(t))$, i.e.,

$$\mu_i(z(t)) = \frac{\rho_i(z(t))}{\sum_{i=1}^{n} \rho_i(z(t))},$$

where $z(t) = [z_1(t), z_2(t), ..., z_g(t)]$, $\rho_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$. $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$\rho_i(z(t)) \ge 0, \quad i = 1, 2, ..., n, \quad \sum_{i=1}^{n} \rho_i(z(t)) > 0, \quad \forall t \ge 0.$$

Then it can be seen that

$$\mu_i(z(t)) \ge 0, \quad i = 1, 2, ..., n, \quad \sum_{i=1}^{n} \mu_i(z(t)) = 1, \quad \forall t \ge 0. \tag{5}$$

By using the center-average defuzzifier product inference and singleton fuzzifier, the T-S fuzzy model (1) can be expressed by the following global model:

$$\begin{cases} x(t+1) = \sum_{i=1}^{n} \mu_i(z(t))(A_{i1} + \Delta A_{i1}(t))x(t) + (A_{i2} + \Delta A_{i2}(t))x(t-d) \\ \qquad\qquad + (B_i + \Delta B_i(t))u(t) + B_{\omega i}\omega(t), \\ \tilde{z}(t) = \sum_{i=1}^{n} \mu_i(z(t))(C_{i1} + \Delta C_{i1}(t))x(t) + (C_{i2} + \Delta C_{i2}(t))x(t-d) \\ \qquad\qquad + (D_i + \Delta D_i(t))u(t), \\ x(t) = \varphi(t), \quad t \in [-d, 0]. \end{cases} \tag{6}$$

In this paper, state time-delay feedback T-S fuzzy-model-based H_∞ controller will be designed for the robust stabilization of T-S fuzzy discrete system (6). The ith controller rule is

$$\begin{aligned} R^i &: \text{If } z_1(t) \text{ is } M_{i1},\ z_2(t) \text{ is } M_{i2},\ \ldots,\ z_g(t) \text{ is } M_{ig}, \\ &\quad \text{then } u(t) = K_{i1}x(t) + K_{i2}x(t-d), \end{aligned} \tag{7}$$

where K_{i1} and K_{i2} $(i = 1, 2, \ldots, n)$ are the controller gains of (7) to be determined. The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t))[K_{i1}x(t) + K_{i2}x(t-d)]. \tag{8}$$

Combining (6) and (8), the closed-loop fuzzy system can be obtained as

$$\begin{cases} x(t+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))[(\tilde{A}_{i1} + \tilde{B}_i K_{j1})x(t) \\ \qquad\qquad + (\tilde{A}_{i2} + \tilde{B}_i K_{j2})x(t-d) + B_{\omega i}\omega(t)], \\ \tilde{z}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))[(\tilde{C}_{i1} + \tilde{D}_i K_{j1})x(t) \\ \qquad\qquad + (\tilde{C}_{i2} + \tilde{D}_i K_{j2})x(t-d)], \\ x(t) = \varphi(t), \quad t \in [-d, 0], \end{cases} \tag{9}$$

where for $i = 1, 2, \ldots, n,$

$$\widetilde{A}_{i1} \triangleq A_{i1} + \Delta A_{i1}(t), \quad \widetilde{A}_{i2} \triangleq A_{i2} + \Delta A_{i2}(t), \quad \widetilde{B}_i \triangleq B_i + \Delta B_i(t),$$

$$\widetilde{C}_{i1} \triangleq C_{i1} + \Delta C_{i1}(t), \quad \widetilde{C}_{i2} \triangleq C_{i2} + \Delta C_{i2}(t), \quad \widetilde{D}_i \triangleq D_i + \Delta D_i(t).$$

Definition 1. For a prescribed scalar $\gamma > 0$, define the performance index as

$$J(\omega) \triangleq \sum_{t=0}^{N-1} [\widetilde{z}^T(t)\widetilde{z}(t) - \gamma^2 \omega^T(t)\omega(t)], \quad \forall N > 0. \tag{10}$$

Remark 1. The purpose of this paper is to design a robust H_∞ controller (8) for the T-S discrete system (6) such that for all admissible uncertainties satisfying (2), (3), (4) and for a prescribed scalar $\gamma > 0$,

(a) the closed-loop fuzzy discrete system (9) is asymptotically stable when $\omega(t) = 0$;

(b) the closed-loop fuzzy discrete system (9) satisfies $\| \widetilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$ for all nonzero $\omega(t) \in L_2[0, \infty]$ under the zero initial condition.

3. Main Results

Two important lemmas should be introduced because they are the key to prove the main theorem.

Lemma 1 [9]**.** *For any two matrices X and Y, we have*

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y,$$

where $X \in R^{m \times n}$, $Y \in R^{m \times n}$, *and* ε *is any positive constant.*

Lemma 2 [12]**.** *Giving the matrices Y, U and E of appropriate dimensions and* $Y = Y^T$, *then for any matrix F satisfying* $F^T F \leq I$,

$$Y + UFE + E^T F^T U^T < 0$$

holds if and only if there exists a constant $\varepsilon > 0$ *satisfying*

$$Y + \varepsilon U U^T + \varepsilon^{-1} E^T E < 0.$$

In the following based on the Lyapunov approach, we will present a new method to design the robust H_∞ controller for uncertain discrete system with time-delay.

Theorem 1. *For a prescribed scalar* $\gamma > 0$, *discrete system* (9) *is stable and satisfies* $\| \tilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$ *for all nonzero* $\omega(t) \in L_2[0, \infty]$ *under the zero initial condition if there exist matrices* $P > 0$, $S > 0$, K_{j1} *and* K_{j2} $(j = 1, 2, \ldots, n)$ *of appropriate dimensions and positive constants* ε_{ij}, η_{ij} $(i, j = 1, 2, \ldots, n)$ *such that the following matrix inequalities simultaneously hold*:

$$\begin{bmatrix} \Xi_{11}^{ii} & * & * \\ \Xi_{21}^{ii} & \Xi_{22}^{ii} & * \\ \Xi_{31}^{ii} & 0 & \Xi_{33}^{ii} \end{bmatrix} < 0, \quad 1 \le i \le n, \tag{11}$$

$$\begin{bmatrix} \Xi_{11}^{ij} + \Xi_{11}^{ji} & * & * & * & * \\ \Xi_{21}^{ij} & \Xi_{22}^{ij} & * & * & * \\ \Xi_{21}^{ji} & 0 & \Xi_{22}^{ji} & * & * \\ \Xi_{31}^{ij} & 0 & 0 & \Xi_{33}^{ij} & * \\ \Xi_{31}^{ji} & 0 & 0 & 0 & \Xi_{33}^{ji} \end{bmatrix} < 0, \quad 1 \le i < j \le n, \tag{12}$$

where

$$\Xi_{11}^{ij} = \begin{bmatrix} -P + S & * & * & * & * \\ 0 & -S & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{i1} + B_i K_{j1} & A_{i2} + B_i K_{j2} & B_{\omega i} & -P^{-1} + 2\varepsilon_{ij} U_i U_i^T & * \\ C_{i1} + D_i K_{j1} & C_{i2} + D_i K_{j2} & 0 & 0 & -I + 2\eta_{ij} H_i H_i^T \end{bmatrix},$$

$$\Xi_{21}^{ij} = \begin{bmatrix} E_{i1} & E_{i2} & 0 & 0 & 0 \\ E_i K_{j1} & E_i K_{j2} & 0 & 0 & 0 \end{bmatrix}, \quad \Xi_{22}^{ij} = \operatorname{diag}\{-\varepsilon_{ij} I - \varepsilon_{ij} I\}, \quad \varepsilon_{ij} > 0,$$

$$\Xi_{31}^{ij} = \begin{bmatrix} G_{i1} & G_{i2} & 0 & 0 & 0 \\ G_i K_{j1} & G_i K_{j2} & 0 & 0 & 0 \end{bmatrix}, \quad \Xi_{33}^{ij} = \operatorname{diag}\{-\eta_{ij} I - \eta_{ij} I\}, \quad \eta_{ij} > 0.$$

Proof. Choose the Lyapunov functional as

$$V(x(t)) = x^T(t)Px(t) + \sum_{m=1}^{d} x^T(t-m)Sx(t-m), \tag{13}$$

where $P > 0$ and $S > 0$. From (9), we have

$$\begin{aligned}
&\Delta V(x(t))\\
&= V(x(t+1)) - V(x(t))\\
&= x^T(t+1)Px(t+1) - x^T(t)Px(t) + x^T(t)Sx(t) - x^T(t-d)Sx(t-d)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i(z(t))\mu_j(z(t))\mu_k(z(t))\mu_l(z(t))\{x^T(t)(-P+S)x(t)\\
&\quad + \alpha_{ij}(t)^T P\alpha_{kl}(t) - x^T(t-d)Sx(t-d)\}\\
&= \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i(z(t))\mu_j(z(t))\mu_k(z(t))\mu_l(z(t))\{[\alpha_{ij}(t)\\
&\quad + \alpha_{ji}(t)]^T P[\alpha_{kl}(t) + \alpha_{lk}(t)] - x^T(t)(-4P+4S)x(t) - x^T(t-d)(4S)x(t-d)\}\\
&\le \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))\{[\alpha_{ij}(t) + \alpha_{ji}(t)]^T P[\alpha_{ij}(t) + \alpha_{ji}(t)]\\
&\quad - x^T(t)(-4P+4S)x(t) - x^T(t-d)(4S)x(t-d)\}\\
&= \sum_{i=1}^{n} \mu_i^2(z(t))\{\alpha_{ii}(t)^T P\alpha_{ii}(t) + x^T(t)(-P+S)x(t) - x^T(t-d)Sx(t-d)\}\\
&\quad + \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j>i}^{n} \mu_i(z(t))\mu_j(z(t))\{[\alpha_{ij}(t) + \alpha_{ji}(t)]^T P[\alpha_{ij}(t) + \alpha_{ji}(t)]\\
&\quad - x^T(t)(-4P+4S)x(t) - x^T(t-d)(4S)x(t-d)\}\\
&= \sum_{i=1}^{n} \mu_i^2(z(t))\left\{\xi^T(t)\left(\begin{bmatrix} -P+S & * & * \\ 0 & -S & * \\ 0 & 0 & 0 \end{bmatrix} + \Omega_{ii}\right)\xi(t)\right\}\\
&\quad + \sum_{i=1}^{n-1}\sum_{j>i}^{n} \mu_i(z(t))\mu_j(z(t))\left\{\xi^T(t)\left(\begin{bmatrix} -2P+2S & * & * \\ 0 & -2S & * \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{2}\Gamma_{ij}\right)\xi(t)\right\},
\end{aligned} \tag{14}$$

where

$$\alpha_{ij}(t) = (\tilde{A}_{i1} + \tilde{B}_i K_{j1})x(t) + (\tilde{A}_{i2} + \tilde{B}_i K_{j2})x(t-d) + B_{\omega i}\omega(t),$$

$$\xi^T(t) = [x^T(t) \quad x^T(t-d) \quad \omega^T(t)],$$

$$\Omega_{ii} = \begin{bmatrix} (\tilde{A}_{i1} + \tilde{B}_i K_{i1})^T \\ (\tilde{A}_{i2} + \tilde{B}_i K_{i2})^T \\ B_{\omega i}^T \end{bmatrix} P[\tilde{A}_{i1} + \tilde{B}_i K_{i1} \;\; \tilde{A}_{i2} + \tilde{B}_i K_{i2} \;\; B_{\omega i}],$$

$$\Gamma_{ij} = \Psi_{ij}^T P \Psi_{ij},$$

$$\Psi_{ij} = [\tilde{A}_{i1} + \tilde{B}_i K_{j1} + \tilde{A}_{j1} + \tilde{B}_j K_{i1} \quad \tilde{A}_{i2} + \tilde{B}_i K_{j2} + \tilde{A}_{j2} + \tilde{B}_j K_{i2} \quad B_{\omega i} + B_{\omega j}].$$

By setting $\omega(t) = 0,$ we can easily verify that $\Delta V(x(t)) < 0$ when (11) and (12) hold.

From (9), we have

$$\begin{aligned}
&\tilde{z}^T(t)\tilde{z}(t) - \gamma^2\omega^T(t)\omega(t) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i(z(t))\mu_j(z(t))\mu_k(z(t))\mu_l(z(t))\{\beta_{ij}^T(t)\beta_{kl}(t) - \gamma^2\omega^T(t)\omega(t)\} \\
&\le \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))\{[\beta_{ij}(t) + \beta_{ji}(t)]^T[\beta_{ij}(t) + \beta_{ji}(t)] - 4\gamma^2\omega^T(t)\omega(t)\} \\
&= \sum_{i=1}^{n} \mu_i^2(z(t))\{\beta_{ii}^T(t)\beta_{ii}(t) - \gamma^2\omega^T(t)\omega(t)\} \\
&\quad + \sum_{i=1}^{n-1}\sum_{j>i}^{n} \mu_i(z(t))\mu_j(z(t))\Big\{\frac{1}{2}[\beta_{ij}(t) + \beta_{ji}(t)]^T[\beta_{ij}(t) \\
&\quad + \beta_{ji}(t)] - 2\gamma^2\omega^T(t)\omega(t)\Big\},
\end{aligned} \tag{15}$$

where

$$\beta_{ij}(t) = (\tilde{C}_{i1} + \tilde{D}_i K_{j1})x(t) + (\tilde{C}_{i2} + \tilde{D}_i K_{j2})x(t-d).$$

Assume that $\varphi(t) = 0$. Then we consider the performance index (10) of system (9)

$$J(\omega) = \sum_{t=0}^{N-1}[\tilde{z}^T(t)\tilde{z}(t) - \gamma^2\omega^T(t)\omega(t) + \Delta V(x(t))] - V(x(N))$$

$$\leq \sum_{t=0}^{N-1}[\tilde{z}^T(t)\tilde{z}(t) - \gamma^2\omega^T(t)\omega(t) + \Delta V(x(t))].$$

Combining (14) and (15), by using the Schur-complements, we can prove that when (11) and (12) hold, $J(\omega) < 0$, i.e., $\| \tilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$. This completes the proof.

Noting that (11) and (12) are not LMIs, we cannot solve them directly by using MATLAB LMI Toolbox, so it is necessary to transfer them into LMIs. The following theorem gives a sufficient condition for the existence of robust H_∞ controller. State time-delay feedback gains can be derived by solving some LMIs.

Theorem 2. *For a prescribed scalar* $\gamma > 0$, *discrete system* (9) *is stable and satisfies* $\| \tilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$ *for all nonzero* $\omega(t) \in L_2[0, \infty]$ *under the zero initial condition if there exist matrices* $X > 0$, $\tilde{S} > 0$, Y_{j1} *and* Y_{j2} $(j = 1, 2, \ldots, n)$ *of appropriate dimensions and positive constants* ε_{ij}, η_{ij} $(i, j = 1, 2, \ldots, n)$ *such that the following LMIs simultaneously hold*:

$$\begin{bmatrix} \Phi_{11}^{ii} & * & * \\ \Phi_{21}^{ii} & \Phi_{22}^{ii} & * \\ \Phi_{31}^{ii} & 0 & \Phi_{33}^{ii} \end{bmatrix} < 0, \quad 1 \leq i \leq n, \tag{16}$$

$$\begin{bmatrix} \Phi_{11}^{ij} + \Phi_{11}^{ji} & * & * & * & * \\ \Phi_{21}^{ij} & \Phi_{22}^{ij} & * & * & * \\ \Phi_{21}^{ji} & 0 & \Phi_{22}^{ji} & * & * \\ \Phi_{31}^{ij} & 0 & 0 & \Phi_{33}^{ij} & * \\ \Phi_{31}^{ji} & 0 & 0 & 0 & \Phi_{33}^{ji} \end{bmatrix} < 0, \quad 1 \leq i < j \leq n, \tag{17}$$

where

$$\Phi_{11}^{ij} = \begin{bmatrix} -X+\tilde{S} & * & * & * & * \\ 0 & -\tilde{S} & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{i1}X + B_iY_{j1} & A_{i2}X + B_iY_{j2} & B_{\omega i} & -X + 2\varepsilon_{ij}U_iU_i^T & * \\ C_{i1}X + D_iY_{j1} & C_{i2}X + D_iY_{j2} & 0 & 0 & -I + 2\eta_{ij}H_iH_i^T \end{bmatrix},$$

$$\Phi_{21}^{ij} = \begin{bmatrix} E_{i1}X & E_{i2}X & 0 & 0 & 0 \\ E_iY_{j1} & E_iY_{j2} & 0 & 0 & 0 \end{bmatrix}, \ \Phi_{22}^{ij} = \Xi_{22}^{ij} = \operatorname{diag}\{-\varepsilon_{ij}I - \varepsilon_{ij}I\}, \ \ \varepsilon_{ij} > 0,$$

$$\Phi_{31}^{ij} = \begin{bmatrix} G_{i1}X & G_{i2}X & 0 & 0 & 0 \\ G_iY_{j1} & G_iY_{j2} & 0 & 0 & 0 \end{bmatrix}, \ \Phi_{33}^{ij} = \Xi_{33}^{ij} = \operatorname{diag}\{-\eta_{ij}I - \eta_{ij}I\}, \ \ \eta_{ij} > 0.$$

Moreover, the state time-delay feedback controller gains of (8) *are given by*

$$K_{j1} = Y_{j1}X^{-1}, \quad K_{j2} = Y_{j2}X^{-1}, \quad j = 1, 2, \ldots, n. \tag{18}$$

Proof. Define $X = P^{-1}$, $\tilde{S} = XSX$, $Y_{j1} = K_{j1}X$ and $Y_{j2} = K_{j2}X$. Then pre- and post-multiplying both sides of (11) with $\operatorname{diag}\{X\ X\ I\ I\ I\ I\ I\ I\}$, pre- and post-multiplying both sides of (12) with $\operatorname{diag}\{X\ X\ I\ I\ I\ I\ I\ I\ I\ I\ I\}$, we can obtain (16) and (17).

4. A Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed robust H_∞ controller design method. Consider an Uncertain T-S Fuzzy Discrete System with Time-Delay as follows:

Plant Rule 1:

If $x_2(t)$ is M_1, then

$$\begin{cases} x(t+1) = (A_{11} + \Delta A_{11})x(t) + (A_{12} + \Delta A_{12})x(t-d) \\ \qquad\qquad + (B_1 + \Delta B_1)u(t) + B_{\omega 1}\omega(t), \\ \tilde{z}(t) = (C_{11} + \Delta C_{11})x(t) + (C_{12} + \Delta C_{12})x(t-d) + (D_1 + \Delta D_1)u(t). \end{cases}$$

Plant Rule 2:

If $x_2(t)$ is M_2, then

$$\begin{cases} x(t+1) = (A_{21} + \Delta A_{21})x(t) + (A_{22} + \Delta A_{22})x(t-d) \\ \qquad\qquad + (B_2 + \Delta B_2)u(t) + B_{\omega 2}\omega(t), \\ \tilde{z}(t) = (C_{21} + \Delta C_{21})x(t) + (C_{22} + \Delta C_{22})x(t-d) + (D_2 + \Delta D_2)u(t), \end{cases}$$

where

$$A_{11} = \begin{bmatrix} -0.01 & 0.1 \\ -0.5 & 1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0.5 & -0.6 \\ 0.6 & 0.5 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -0.1 & -0.5 \\ 0 & 0 \end{bmatrix},\ A_{22} = \begin{bmatrix} -0.01 & -0.01 \\ 0 & 0 \end{bmatrix},$$

$$C_{11} = [0.05 \quad 1],\ C_{12} = [-1 \quad 0.3],\ C_{21} = [0 \quad 0.2],\ C_{22} = [0 \quad 0.2],$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_{\omega 1} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},\ B_{\omega 2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ D_1 = 1,\ D_2 = 0.5,$$

$$U_1 = U_2 = \begin{bmatrix} 0.05 & 0.1 \\ 0.1 & 0 \end{bmatrix},\ E_{11} = E_{21} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},\ E_{12} = E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$H_1 = H_2 = [1 \quad 0],\ G_{11} = G_{21} = G_{12} = G_{22} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_1 = E_2 = G_1 = G_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Choosing $\gamma = 1$, $d = 2$, by using MATLAB LMI Toolbox to solve the LMIs (16) and (17), we can get the positive definite matrices X, $\tilde{S}$ and matrices $Y_{11}, Y_{12}, Y_{21}, Y_{22}$:

$$X = \begin{bmatrix} 0.1981 & 0.0706 \\ 0.0706 & 0.2023 \end{bmatrix}, \quad \tilde{S} = \begin{bmatrix} 0.1404 & 0.0350 \\ 0.0350 & 0.1040 \end{bmatrix},$$

$$Y_{11} = [0.0211 \quad -0.1643], \quad Y_{12} = [-0.1053 \quad -0.1571],$$

$$Y_{21} = [-0.0188 \quad -0.0402], \quad Y_{22} = [-0.0037 \quad -0.0062].$$

By (18), we can get the state time-delay feedback gains as

$$K_{11} = [0.4514 \quad -0.9685], \quad K_{12} = [-0.2913 \quad -0.6741],$$

$$K_{21} = [-0.0277 \quad -0.1889], \quad K_{22} = [-0.0089 \quad -0.0274]. \tag{19}$$

From Theorem 2, we can see that the given T-S fuzzy discrete system is robustly stabilizable under controller (8) with controller gains (19) and satisfies $\| \tilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$ for all nonzero $\omega(t) \in L_2[0, \infty)$ under the zero initial condition.

5. Conclusion

In this paper, we have studied the robust H_∞ control for a class of T-S fuzzy-model-based discrete systems with time-delay and norm-bounded parameter uncertainties. A sufficient condition for the existence of the robust H_∞ controller has been obtained in an LMI form by Lyapunov functional approach. Finally, a numerical example has been illustrated to show the effectiveness of the proposed design method. In future, we will consider the output feedback control for this kind of systems when the states are unmeasurable.

References

[1] Y. Y. Cao and P. M. Frank, Robust H_∞ disturbance attenuation for a class of uncertain discrete-time fuzzy systems, IEEE Trans. Fuzzy Syst. 8 (2000), 406-415.

[2] C. C. Lee, Fuzzy logic in control system: fuzzy logic controller (Parts I and II), IEEE Trans. Syst. Man Cybern. 20 (1990), 408-435.

[3] J. R. Li and Z. L. Xia, Generalized H_2 controller design for uncertain discrete-time-delay piecewise systems, Fuzzy Systems and Mathematics 20 (2006), 298-304.

[4] C. Lin, Q.-G. Wang and T. H. Lee, Delay-dependent LMI conditions for stability and stabilization of T-S fuzzy systems with bounded time-delay, Fuzzy Sets and Systems 157 (2006), 1229-1247.

[5] T. Takagi and M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Trans. Syst. Man Cybern. 15 (1985), 116-132.

[6] E. G. Tian and C. Peng, Delay-dependent stability analysis and synthesis of uncertain T-S fuzzy systems with time-varying delay, Fuzzy Sets and Systems 157 (2006), 544-559.

[7] H. Wang and K. Tanaka, An LMI-based stable fuzzy control of nonlinear systems and its application to control of chaos, Proc. 5th IEEE Internat. Conf. on Fuzzy Systems, New Orleans, Louisiana, 1996, pp. 1433-1438.

[8] R. J. Wang, W. W. Lin and W. J. Wang, Stabilizability of linear quadratic state feedback for uncertain fuzzy time-delay systems, IEEE Trans. Syst. Man Cybern. (Part B) 34 (2004), 1288-1292.

[9] W.-J. Wang, C.-C. Kao and C.-S. Chen, Stabilization, estimation and robustness for large scale time-delay systems, Control Theory Adv. Tech. 7(4) (1991), 569-585.

[10] L. Xie, Output feedback H_∞ control of systems with parameter uncertainty, Internat. J. Control 63 (1996), 741-750.

[11] S. Y. Xu and J. Lam, Robust H_∞ control for uncertain discrete-time-delay fuzzy systems via output feedback controllers, IEEE Trans. Fuzzy Syst. 13 (2005), 82-93.

[12] L. Yu, Robust Control - An LMI Method, Tsinghua University Press, 2002.

[13] L. A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. Syst. Man Cybern. 3 (1973), 28-44.

[14] M. M. Zavarei and M. Jamshidi, Time-delay Systems: Analysis, Optimization and Application, Amsterdam, The Netherlands, North-Holland, 1987.