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A MULTI-OBJECTIVE FUZZY ASSIGNMENT PROBLEM

S. MURUGANANDAM[*], R. NALLASWAMY[†] and K. ABDUL RAZAK

Department of Mathematics
M. A. M. College of Engineering
Trichy – 621 105, Tamilnadu, India
e-mail: asmsacet@yahoo.co.in
karnaz76@yahoo.com

[†]Department of Mathematics
National Institute of Technology
Trichy – 620 015, Tamilnadu, India
e-mail: nalla@nitt.edu

Abstract

This paper is to extend the traditional single-objective minimizing assignment problem to the multi-objective minimizing assignment problem associated with cost, time and quality. Due to the uncertainty of the real life, it is assumed that the elements of the cost matrix, time matrix and quality matrix are fuzzy variables. In order to obtain an assignment plan, a graded mean integration representation is discussed. The results of the numerical example show that an excellent compromise solution can be reached effectively.

1. Introduction

A Management is designed to control organization resources on a

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[*]Corresponding author

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given set of activities, within time, within cost, and within quality. The limited resources must be utilized efficiently such that the optimal available resources can be assigned to the most needed tasks so as to maximize the profit and minimize the cost. Traditional assignment problems deal primarily with N tasks that must assign to N workers, where each worker has the competence to do all tasks.

An assignment problem can be viewed as balanced transportation problem in which all supplies and demands equal to 1, and the number of rows and columns in the matrix are identical. Hence, the transportation simplex method [10] can be used to solve the assignment problem. However, it is often inefficient and not recommended due to the high degree of degeneracy in the problem. Another technique called Hungarian method is commonly employed to solve the minimizing assignment problems. The single objective assignment problems are usually divided into two categories: (1) cost minimizing assignment problems [2, 9] and (2) time minimizing assignment problems [6, 11]. The cost minimizing assignment problems focus on the optimal assignment that minimizes the total project cost. Such problems have been generally discussed and well developed in many operations research text books and relevant papers [1, 2, 8, 10, 12]. The time minimizing assignment problems are formulated, searching for the shortest duration when the total project time is of vital concern. Unfortunately, it can only minimize time or cost, however, cost and time are frequently correlated. Geetha and Nair [7] first provide a solution for an assignment problem that minimizes both cost and time. The total project duration is converted to the supervisory cost and defined as the minimum completion time required to complete all the jobs. Therefore, the decision is to minimize the total project cost and the duration of the time-consuming job.

In many problems, assigning jobs are only based on cost and time. The quality is another major factor in making decisions. A fuzzy approach is applied to minimize the cost, time and quality assignment problem in the next section.

2. Fuzzy Assignment Problem

In this section, we give the description of the assignment problem.

Assume that there are n tasks and m workers. We want to find an assignment plan satisfying the following requirements:

1. There are three goals, that is, trying to minimize the total cost, minimize the total consumed time and maintaining the quality level, after all the tasks are completed.

2. All the tasks must be completed, each task is completed by only one worker, and a worker can undertake more than one task.

3. It is allowed not to assign any task to some workers.

4. In order to balance the amount of work between the workers, it is necessary to stipulate the number of workers who have been assigned to tasks.

5. In the process of decision-making, the ability of working for each worker is considered. We assume that the number of tasks which are assigned to each worker should be in a certain range.

6. If worker i has ability to undertake some tasks j and we think that he will probably produce much less profit or consume very long time, in such conditions, worker i will be deprived of the opportunity to undertake these tasks.

When worker i finishes task j, we denote the costs, the consumed time and the quality by c_{ij}, t_{ij} and q_{ij}, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, respectively. The amount of cost needed and time required to complete a task with certain level of quality are highly interrelated with the difficulty of the task and the capability of the worker. Moreover, the level of quality does not necessarily relate to the time taken and the cost needed because a task can be completed on time consuming reasonable budget, while quality is unacceptable. The quality level 1 is the best and level 9 is the worst. Since we make decision before the tasks are completed, it is difficult for us to know the concrete values of c_{ij}, t_{ij} and q_{ij}. In order to obtain a decision, we assume that c_{ij}, t_{ij} and q_{ij} are fuzzy variables. Furthermore, the membership functions of these fuzzy variables can be obtained by means of experience evaluation.

In this paper, we suppose that the membership functions of c_{ij}, t_{ij} and q_{ij} $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ have been given. Then the cost matrix, the consumed time matrix and the quality matrix individually of the cost, time and quality assignment problem are shown below:

Tasks

		1	2	$\cdots$	n
	1	c_{11} t_{11} q_{11}	c_{12} t_{12} q_{12}	$\cdots$ $\cdots$ $\cdots$	c_{1n} t_{1n} q_{1n}
Workers	2	c_{21} t_{21} q_{21}	c_{22} t_{22} q_{22}	$\cdots$ $\cdots$ $\cdots$	c_{2n} t_{2n} q_{2n}
	$\vdots$	$\vdots$	$\vdots$		$\vdots$
	m	c_{m1} t_{m1} q_{m1}	c_{m2} t_{m2} q_{m2}	$\cdots$ $\cdots$ $\cdots$	c_{mn} t_{mn} q_{mn}

Therefore,

$$C = (c_{ij})_{m\times n} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{pmatrix},$$

$$T = (t_{ij})_{m\times n} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{pmatrix},$$

$$Q = (q_{ij})_{m\times n} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{pmatrix}.$$

In order to model the above mentioned multi-objective fuzzy assignment problem, the total cost, the total consumed time and the total maintained quality level are to be optimized individually, then the problem can be stated as equation:

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij},$$

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} T_{ij} x_{ij},$$

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} Q_{ij} x_{ij},$$

$$x_{ij} = \begin{cases} 1, & \text{if task } j \text{ is assigned to worker } i, \\ 0, & \text{otherwise,} \end{cases}$$

where $i = 1, 2, \ldots, m, \;\; j = 1, 2, \ldots, n.$

Since the elements of the cost matrix, the consumed time matrix and maintained quality matrix are all fuzzy variables, it follows that the total cost, the total consumed time and the total maintained quality are fuzzy variables. As we know, we cannot maximize or minimize fuzzy objective function directly. Thus we use a special method to solve this problem in the next section.

3. Proposed Method

In order to solve uncertain programming model, the most important step is defuzzification. The literature presents different defuzzification methods [13] and clearly each of them has its own features that are suitable for slightly different kinds of problems.

3.1. Representation of generalized fuzzy number

In general, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

1. μ_A is a continuous mapping from R to the closed interval $[0, 1]$.

2. $\mu_A(x) = 0, -\infty < x \leq a$.

3. $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$.

4. $\mu_A(x) = w, b \leq x \leq c$.

5. $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$.

6. $\mu_A(x) = 0, d \leq x \leq \infty$.

We denote this type of generalized fuzzy number as $A = (a, b, c, d : w)_{LR}$.

3.2. Trapezoidal fuzzy number

A fuzzy number A is a trapezoidal fuzzy number denoted by (a, b, c, d) as Figure 1 and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \dfrac{x-d}{c-d}, & c \leq x \leq d, \\ 0, & \text{otherwise.} \end{cases}$$

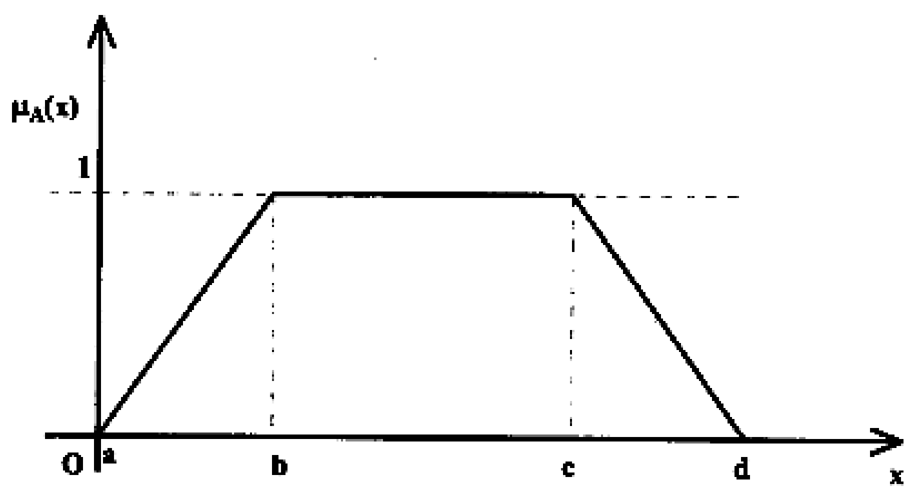

Figure 1. Membership function of trapezoidal fuzzy number.

3.3. Graded mean integration representation method

Chen and Hsieh [3-5] propose graded mean integration representation for representing generalized fuzzy number.

Suppose L^{-1} and R^{-1} are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalized fuzzy number $A = (a, b, c, d : w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h level is

$$P(A) = \frac{\int_0^w h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right]dh}{\int_0^w hdh},$$

where h is between 0 and w, $0 < w \leq 1$.

Generalized trapezoidal fuzzy number is denoted as (a, b, c, d), Chen and Hsieh [3-5] already found the general formulae of the representation of generalized trapezoidal fuzzy number as follows:

Suppose $A = (a, b, c, d)$ is a trapezoidal fuzzy number, then the defuzzified value of $P(A)$ by graded mean integration representation is

$$P(A) = \frac{\int_0^1 h\left[\frac{a + d + (b - a + c - d)h}{2}\right]dh}{\int_0^1 hdh}$$

$$= \frac{a + 2b + 2c + d}{6}.$$

4. Solution Algorithm

First, we defuzzify the fuzzy number into crisp number then follow the steps:

Step 1. For the original cost matrix, time matrix and quality matrix, identify each row's minimum and subtract it from all the entries of the row.

Step 2. For the matrix resulting from Step 1, identify each column's minimum and subtract it from all the entries of the column.

Step 3. Identify the optimal solution as the feasible assignment associated with the zero elements of the matrix obtained in Step 2.

Step 4. If no feasible assignment can be secured from Steps 1 and 2, then

(i) draw the minimum number of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries,

(ii) select the smallest uncovered element, subtract it from every uncovered element, and then add it to every element at the intersection of two lines,

(iii) if no feasible assignment can be found among the resulting zero entries, repeat Step 4. Otherwise, go to Step 3, to determine the optimal assignment.

5. Numerical Example

The cost, time and quality assignment problem whose elements are trapezoidal fuzzy number is given below:

		Tasks			
		1	2	3	
	1	(7, 9, 11, 13)	(11, 13, 15, 17)	(12, 16, 20, 24)	c_{ij}
		(7, 10, 13, 15)	(5, 9, 11, 13)	(11, 15, 17, 20)	t_{ij}
		(1, 3, 5, 7)	(1, 5, 7, 9)	(3, 5, 7, 9)	q_{ij}
Workers	2	(15, 18, 21, 23)	(12, 15, 18, 22)	(9, 11, 13, 14)	
		(9, 11, 15, 17)	(8, 12, 20, 22)	(15, 18, 21, 23)	
		(1, 5, 7, 9)	(1, 3, 5, 7)	(3, 5, 7, 9)	
	3	(12, 16, 22, 25)	(10, 13, 14, 16)	(12, 15, 18, 24)	
		(9, 11, 15, 18)	(12, 16, 21, 25)	(5, 7, 10, 13)	
		(3, 5, 7, 9)	(1, 5, 7, 9)	(1, 3, 5, 7)	

To determine the optimal compromise assignment of the problem, we obtain cost matrix, time matrix and quality matrix individually.

$$C = \begin{pmatrix} (7, 9, 11, 13) & (11, 13, 15, 17) & (12, 16, 20, 24) \\ (15, 18, 21, 23) & (12, 15, 18, 22) & (9, 11, 13, 14) \\ (12, 16, 22, 25) & (10, 13, 14, 16) & (12, 15, 18, 24) \end{pmatrix},$$

$$T = \begin{pmatrix} (7, 10, 13, 15) & (5, 9, 11, 13) & (11, 15, 17, 20) \\ (9, 11, 15, 17) & (8, 12, 20, 22) & (15, 18, 21, 23) \\ (9, 11, 15, 18) & (12, 16, 21, 25) & (5, 7, 10, 13) \end{pmatrix},$$

$$Q = \begin{pmatrix} (1, 3, 5, 7) & (1, 5, 7, 9) & (3, 5, 7, 9) \\ (1, 5, 7, 9) & (1, 3, 5, 7) & (3, 5, 7, 9) \\ (3, 5, 7, 9) & (1, 5, 7, 9) & (1, 3, 5, 7) \end{pmatrix}.$$

We use the method of defuzzification of a generalized trapezoidal fuzzy number by its graded mean integration representation, i.e.,

$$P(A) = \frac{a + 2b + 2c + d}{6},$$

$c_{11} = 10$, $c_{12} = 14$, $c_{13} = 18$, $c_{21} = 19$, $c_{22} = 17$, $c_{23} = 12$,

$c_{31} = 19$, $c_{32} = 13$, $c_{33} = 17$,

$t_{11} = 11$, $t_{12} = 10$, $t_{13} = 16$, $t_{21} = 13$, $t_{22} = 17$, $t_{23} = 19$,

$t_{31} = 13$, $t_{32} = 19$, $t_{33} = 7$,

$q_{11} = 4$, $q_{12} = 7$, $q_{13} = 6$, $q_{21} = 7$, $q_{22} = 4$, $q_{23} = 6$,

$q_{31} = 6$, $q_{32} = 7$, $q_{33} = 4$.

Therefore,

$$\text{Cost Matrix } C = \begin{bmatrix} 10 & 14 & 18 \\ 19 & 17 & 12 \\ 19 & 13 & 17 \end{bmatrix},$$

$$\text{Time Matrix } T = \begin{bmatrix} 11 & 10 & 16 \\ 13 & 17 & 19 \\ 13 & 19 & 7 \end{bmatrix},$$

$$\text{Quality Matrix } Q = \begin{bmatrix} 4 & 7 & 6 \\ 7 & 4 & 6 \\ 6 & 7 & 4 \end{bmatrix}.$$

Using solution algorithm, we obtain optimal solutions

$$C = \begin{bmatrix} 10^* & 14 & 18 \\ 19 & 17 & 12^* \\ 19 & 13^* & 17 \end{bmatrix}, \quad \text{therefore, total cost} = 35,$$

$$T = \begin{bmatrix} 11 & 10^* & 16 \\ 13^* & 17 & 19 \\ 13 & 19 & 7^* \end{bmatrix}, \quad \text{therefore, total time} = 30,$$

$$Q = \begin{bmatrix} 4^* & 7 & 6 \\ 7 & 4^* & 6 \\ 6 & 7 & 4^* \end{bmatrix}, \quad \text{therefore, total quality} = 12,$$

*Optimal assignment.

6. Conclusion

In this paper, we mainly studied a fuzzy multi-objective assignment problem. As a result, we presented a graded mean integration representation method for defuzzification in order to obtain the best solution. There are several uncertain environments in the real life, such as random environment, fuzzy environment, random fuzzy environment, fuzzy random environment and so on. This paper only investigated the problem under the fuzzy environment.

References

[1] R. D. Anderson, J. D. Sweeney and A. T. Williams, An Introduction to Management Science: Quantitative Approaches to Decision Making, 9th ed., Thomson, New York, 2000.

[2] R. S. Barr, F. Glover and D. Klingman, The alternating basis algorithm for assignment problems, Math. Program. 13(1) (1977), 1-13.

[3] Shan Huo Chen and Chih Hsun Hsieh, Graded mean integration representation of generalized fuzzy number, Proceedings of Conference of Taiwan Fuzzy System Association, Taiwan, 1998.

[4] Shan Huo Chen and Chih Hsun Hsieh, Graded mean integration representation of generalized fuzzy number, J. Chinese Fuzzy System Association, Taiwan 5(2) (1999), 1-7.

[5] Shan Huo Chen and Chih Hsun Hsieh, Representation, ranking distance and similarity of L-R type fuzzy number and application, Australia J. Intelligent Information Processing Systems, Australia 6(4) (2000), 217-229.

[6] R. S. Garfinkel, An improved algorithm for bottleneck assignment problem, Oper. Res. 18 (1971), 1717-1751.

[7] S. Geetha and K. P. K. Nair, A variation of the assignment problem, European J. Operational Research 68(3) (1993), 422-426.

[8] S. F. Hilier and J. G. Lieberman, Introduction to Operations Research, 7th ed., McGraw-Hill, Boston, 2001.

[9] M. S. Hung and W. O. Rom, Solving the assignment problem by relaxation, Oper. Res. 28(4) (1980), 969-982.

[10] A. Ravindran, Don T. Phillips and James J. Solberg, Operations Research: Principles and Practice, 2nd ed., John Wiley and Sons, 1987.

[11] A. Ravindran and V. Ramaswami, On the bottleneck assignment problem, J. Optim. Theory Appl. 21(4) (1977), 451-458.

[12] H. Taha, Operations Research: An Introduction, 7th ed., Macmillan, Basingstoke, Hampshire, 2005.

[13] H. J. Zimmermann, Fuzzy Set Theory and its Applications, 3rd ed., Kluwer Academic Publishers, Boston, Dordrecht, London, 1996.