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CONNECTIVE STABILITY OF SINGULAR LINEAR TIME-INVARIANT LARGE-SCALE DYNAMICAL SYSTEMS WITH SELF-INTERACTION

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Abstract

The purpose of this paper is to discuss the connective stability of singular time-invariant large-scale dynamical systems with self-interaction by means of singular Lyapunov equations and vector Lyapunov functions. A stable domain of connective parameters is given in the paper which is simple in form. An example to illustrate usage and efficiency of the method is provided at the end.

1. Introduction

With the development of modern control theory and the permeation into other application area, one kind of systems with extensive form has appeared as follows:

$$I\dot{X}(t) = f(X(t), t),$$

where I is an $n \times n$ matrix, it is usually singular. $X(t) \in R^n$ is an n-vector, f is an n-vector function. This kind of system is generally called as the *singular system*. It appeared largely in many areas such as the economy management, the electronic network, robot, bioengineering, aerospace industry and navigation and so forth, and

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possesses more wide application background [7]. The concept of the singular system was first proposed in the 1970s, within short twenty years, it has become an independent branch of the modern control theory. Especially in later decades, more and more scholars participated in the study of the singular system theory, and many important results have been obtained as in [1, 2, 6, 7]. Singular system models exit in many areas of social production activities. Known Dynamic Leontief Input Output Models, Hopfield Neural Network Model, multi-robot Subject Coordinate Work Dynamic Model and so on are all singular systems. It is far-reaching practical significance to study the singular system theory.

The purpose of this paper is to discuss the connective stability of singular time-invariant large-scale dynamical systems with self-interaction by means of singular Lyapunov equation and vector Lyapunov function. A stable domain of connective parameters is given in the paper which is simple in form. At last, we give an example to show the usage of the method and its efficiency.

2. Basic Conception

We consider singular linear time-invariant large-scale dynamical system described by equations of the form:

$$I_i \dot{X}_i(t) = A_i X_i(t) + \sum_{j=1}^{r} e_{ij} A_{ij} X_j(t) \quad (i = 1, 2, \ldots, r), \tag{1}$$

where I_i and A_i are all $n_i \times n_i$ constant matrices, A_{ij} is an $n_i \times n_j$ constant matrix. $X_i \in R^{n_i}$ $(i, j = 1, 2, \ldots, r)$. A_{ij} represents action of the subsystem X_j to X_i $(i \neq j)$, A_{ii} represents self-interaction of the subsystem X_i. Denote $n = \sum_{i=1}^{r} n_i$, $X = (X_1^T, X_2^T, \ldots, X_r^T)^T \in R^n$, $I = \text{Block-diag}(I_1, I_2, \ldots, I_r)$. We assume that the matrix I is a singular matrix, and there is one (and only one) solution $x = x(t; t_0, x_0)$ of the equations (1) for any initial time t_0 and any initial state $x_0 = x(t_0)$.

A matrix $E = (e_{ij})_{r \times r}$ is called an *interconnection matrix* associated with $\overline{E} = (\bar{e}_{ij})_{r \times r}$ (denoted as $E \in \overline{E}$), if $\bar{e}_{ij} = 0$ imply $e_{ij} = 0$, and if $\bar{e}_{ij} = 1$ imply

$e_{ij} = 0$ or $e_{ij} = 1,$ where element $\bar{e}_{ij}$ of the fundamental interconnection matrix of the system (1) is defined as

$$\bar{e}_{ij} = \begin{cases} 1 & X_j \text{ can act on } X_i, \\ 0 & X_j \text{ cannot act on } X_i. \end{cases}$$

The isolated subsystems of the system (1) are

$$I_i \dot{X}_i(t) = A_i X_i(t) \quad (i = 1, 2, ..., r). \tag{2}$$

The system (2) is called as *regular* if there exists a constant $s_0,$ such that

$$\det(s_0 I_i - A_i) \neq 0.$$

The system (2) is called as *impulsive-free* if for any $s \in C$ (C is the domain of complex number), it always holds that

$$\deg \det(sI_i - A_i) = \operatorname{rank}(I_i).$$

The system (2) is called to be *compatible* if it is regular, impulsive-free and is asymptotically stable [2].

3. Main Results

We assume that for every isolated subsystem (2), there exists a positive definite matrix $B_i,$ such that it satisfies the following singular Lyapunov equation:

$$A_i^T B_i I_i + I_i^T B_i A_i = -I_i^T I_i, \tag{3}$$

we take singular Lyapunov function as follows:

$$V_i[I_i X_i(t)] = [I_i X_i(t)]^T B_i [I_i X_i(t)], \tag{4}$$

then

$$\lambda_m(B_i) \| I_i X_i(t) \|^2 \leq V_i[I_i X_i(t)] \leq \lambda_M(B_i) \| I_i X_i(t) \|^2, \tag{5}$$

where $\| \bullet \|$ represents Euclidean norm, $\lambda_m(B)$ and $\lambda_M(B)$ represent respectively, minimum and maximum eigenvalues of the positive definite matrix B, and the total derivative of $V_i[I_i X_i(t)]$ with respect to t along solutions of the system (2) is

$$\dot{V}_i[I_i X_i(t)]_{(2)} = -[I_i X_i(t)]^T [I_i X_i(t)]. \tag{6}$$

Lemma [2]**.** *For any pair of* $X, Y \in R^n$, *a positive definite matrix B and a constant number* $\mu > 0$, *the following inequality holds*:

$$X^T BY + Y^T BX \leq \mu X^T BX + \mu^{-1} Y^T BY. \tag{7}$$

Definition [6]**.** $\overline{E}$ is a fundamental interconnection matrix of the system (1). The system (1) is called as *uniformly connectively asymptotically stable*, if the equilibrium state of the system (1) is always uniformly asymptotically stable for any $E \in \overline{E}$.

Theorem 1. *The equilibrium state of the system* (1) *is uniformly connectively asymptotically stable if the following conditions are satisfied*:

(i) *The isolated subsystem* (2) *is compatible, that is, the every isolated subsystem* (2) *is regular, impulsive-free and there exists a positive definite matrix* B_i *such that* (5) *and* (6) *hold*.

(ii) *For singular large-scale dynamical system* (1), *there exist positive numbers* δ_{ij}, *such that*

$$\| A_{ij} X_j(t) \| \leq \delta_{ij} \| I_j X_j(t) \| \quad (i, j = 1, 2, ..., r). \tag{8}$$

(iii) *All successively principal minors* C_k $(k = 1, 2, ..., r)$ *of the* $r \times r$ *matrix* $C = (c_{ij})$ *satisfy*

$$(-1)^k C_k = (-1)^k \begin{vmatrix} c_{11} & \cdots & c_{ik} \\ \cdots & \cdots & \cdots \\ c_{k1} & \cdots & c_{kk} \end{vmatrix} > 0, \tag{9}$$

where

$$c_{ij} = \begin{cases} \dfrac{[-1 + \lambda_M(B_i)(1 + \bar{e}_{ii}\delta_{ii}^2)]}{2\lambda_M(B_i)}, & i = j, \\ p_{ij} + \dfrac{(r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2}{\lambda_m(B_j)}, & i \neq j, \end{cases} \tag{10}$$

where

$$p_{ij} = \frac{2(n - n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1 + \lambda_M(B_i)(1 + \bar{e}_{ii}\delta_{ii}^2)]\lambda_m(B_j)}, \quad i \neq j.$$

Proof. Let $Y_i = \| I_i X_i(t) \|$ $(i = 1, 2, ..., r)$, for any $E = (e_{ij}) \in \overline{E}$, taking vector singular Lyapunov function as follows:

$$V[IX(t)] = \{V_1[I_1X_1(t)], V_2[I_2X_2(t)], ..., V_r[I_rX_r(t)]\}^T,$$

the total derivative of $V_i[I_iX_i(t)]$ with respect to t along solutions of the system (1) is

$$\begin{aligned}
\dot{V}_i[I_iX_i(t)]_{(1)} &= [I_i\dot{X}_i(t)]^T B_i[I_iX_i(t)] + [I_iX_i(t)]^T B_i[I_i\dot{X}_i(t)] \\
&= \left[A_iX_i(t) + \sum_{j=1}^{r} e_{ij}A_{ij}X_j(t)\right]^T B_i[I_iX_i(t)] \\
&\quad + [I_iX_i(t)]^T B_i\left[A_iX_i(t) + \sum_{j=1}^{r} e_{ij}A_{ij}X_j(t)\right] \\
&= [X_i(t)]^T (A_i^T B_iI_i + I_i^T B_iA_i)X_i(t) \\
&\quad + 2\left[\sum_{j=1}^{r} e_{ij}A_{ij}X_j(t)\right]^T B_i[I_iX_i(t)],
\end{aligned}$$

by (7), we get

$$\begin{aligned}
&\dot{V}_i[I_iX_i(t)]_{(1)} \\
&\leq \dot{V}_i[I_iX_i(t)]_{(2)} + [I_iX_i(t)]^T B_i[I_iX_i(t)] \\
&\quad + \left[\sum_{j=1}^{r} e_{ij}A_{ij}X_j(t)\right]^T B_i\left[\sum_{j=1}^{r} e_{ij}A_{ij}X_j(t)\right]
\end{aligned}$$

$$\leq -\| I_i X_i(t) \|^2 + \lambda_M(B_i) \| I_i X_i(t) \|^2 + \lambda_M(B_i) \left\| \sum_{j=1}^{r} e_{ij} A_{ij} X_j(t) \right\|^2$$

$$\leq [-1 + \lambda_M(B_i)] Y_i^2 + \lambda_M(B_i) \left(\sum_{j=1}^{r} \bar{e}_{ij} \delta_{ij} Y_j \right)^2$$

$$\leq [-1 + \lambda_M(B_i)] Y_i^2$$

$$+ \lambda_M(B_i) \left[\bar{e}_{ii} \delta_{ii}^2 Y_i^2 + 2 \bar{e}_{ii} \delta_{ii} Y_i \sum_{j=1,\, j \neq i}^{r} \bar{e}_{ij} \delta_{ij} Y_j + \left(\sum_{j=1,\, j \neq i}^{r} \bar{e}_{ij} \delta_{ij} Y_j \right)^2 \right]$$

$$\leq \sum_{j=1,\, j \neq i}^{r} \left\{ \frac{n_j}{n - n_i} [-1 + \lambda_M(B_i)(1 + \bar{e}_{ii} \delta_{ii}^2)] Y_i^2 + 2 \bar{e}_{ii} \delta_{ii} \lambda_M(B_i) \delta_{ij} Y_i Y_j \right\}$$

$$+ \lambda_M(B_i) \left(\sum_{j=1,\, j \neq i}^{r} \bar{e}_{ij} \delta_{ij} Y_j \right)^2 .$$

By the hypothesis (iii) of Theorem 1, from $[-1 + \lambda_M(B_i)(1 + \bar{e}_{ii} \delta_{ii}^2)] < 0$, and the following inequality:

$$-\alpha x^2 + xy \leq -\frac{\alpha}{2} x^2 + \frac{1}{2\alpha} y^2,$$

where $x, y \in R$, $\alpha \in R^+$, we get

$$\dot{V}_i [I_i X_i(t)]_{(1)}$$

$$\leq \sum_{j=1,\, j \neq i}^{r} \left\{ \frac{n_j [-1 + \lambda_M(B_i)(1 + \bar{e}_{ii} \delta_{ii}^2)]}{2(n - n_i)} Y_i^2 + \frac{2(n - n_i) \bar{e}_{ij} \delta_{ii}^2 \delta_{ij}^2 \lambda_M^2(B_i)}{n_j [-1 + \lambda_M(B_i)(1 + \bar{e}_{ii} \delta_{ii}^2)]} Y_j^2 \right\}$$

$$+ (r - 1) \lambda_M(B_i) \sum_{j=1,\, j \neq i}^{r} \bar{e}_{ij} \delta_{ij}^2 Y_j^2$$

$$\leq \frac{[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}{2}Y_i^2$$

$$+\sum_{j=1,\, j\neq i}^{r}\left\{\frac{2(n-n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}+(r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2\right\}Y_j^2$$

by (5), we have

$$\dot{V}_i[I_iX_i(t)]_{(1)} \leq \frac{[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}{2\lambda_M(B_i)}V_i[I_iX_i(t)]$$

$$+\sum_{j=1,\, j\neq i}^{r}\left\{\frac{2(n-n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]\lambda_m(B_j)}+\frac{(r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2}{\lambda_m(B_j)}\right\}V_j[I_jX_j(t)],$$

that is,

$$\dot{V}_i[I_iX_i(t)]_{(1)} \leq \sum_{j=1}^{r}c_{ij}V_j[I_jX_j(t)], \quad (i=1,2,\ldots,r),$$

the total derivative of $V[IX(t)]$ with respect to t along solutions of the system (1) holds the following inequality:

$$\dot{V}[IX(t)]_{(1)} \leq CV[IX(t)].$$

Since matrix C satisfies the hypothesis (iii) of Theorem 1, all eigenvalues of matrix C have negative real parts. It follows from the Comparison principle [3] that

$$V[IX(t)] \to 0 \quad (t \to +\infty).$$

Therefore

$$V_i[I_iX_i(t)] \to 0 \quad (t \to +\infty) \quad (i=1,2,\ldots,r)$$

and

$$I_iX_i(t) \to 0 \quad (t \to +\infty) \quad (i=1,2,\ldots,r). \tag{11}$$

Since every isolated subsystem (2) is regular and impulsive-free, so there exist nonsingular matrices P_i, Q_i, such that

$$P_i^{-1} I_i Q_i = \begin{pmatrix} I_i^{(1)} & 0 \\ 0 & 0 \end{pmatrix}, \tag{12}$$

$$P_i^{-1} A_i Q_i = \begin{pmatrix} A_i^{(1)} & 0 \\ 0 & I_i^{(2)} \end{pmatrix} \tag{13}$$

denote

$$Q_i^{-1} X_i(t) = \overline{X}_i(t) = \begin{pmatrix} \overline{x}_{i1}(t) \\ \overline{x}_{i2}(t) \end{pmatrix} \quad (i = 1, 2, ..., r),$$

where $I_i^{(1)}$ and $I_i^{(2)}$ are, respectively, $r_i \times r_i$ and $(n_i - r_i) \times (n_i - r_i)$ unit matrices, $\overline{x}_{i1}(t) \in R^{r_i}$, $\overline{x}_{i2}(t) \in R^{n_i - r_i}$, $A_i^{(1)}$ is an $r_i \times r_i$ constant matrix.

By (11) and

$$P_i^{-1} I_i X_i(t) = \begin{pmatrix} I_i^{(1)} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \overline{x}_{i1}(t) \\ \overline{x}_{i2}(t) \end{pmatrix} = \begin{pmatrix} \overline{x}_{i1}(t) \\ 0 \end{pmatrix},$$

we get

$$\overline{x}_{i1}(t) \to 0 \quad (t \to +\infty) \quad (i = 1, 2, ..., r),$$

by (8) and (11), we have $A_{ij} X_j(t) \to 0 \quad (t \to +\infty) \quad (i, j = 1, 2, ..., r)$. Since the system (1) can be written as

$$\begin{pmatrix} I_i^{(1)} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\overline{x}}_{i1}(t) \\ \dot{\overline{x}}_{i2}(t) \end{pmatrix} = \begin{pmatrix} A_i^{(1)} & 0 \\ 0 & I_i^{(2)} \end{pmatrix} \begin{pmatrix} \overline{x}_{i1}(t) \\ \overline{x}_{i2}(t) \end{pmatrix} + \sum_{j=1}^{r} e_{ij} \begin{pmatrix} A_{ij}^{(1)} & A_{ij}^{(2)} \\ A_{ij}^{(3)} & A_{ij}^{(4)} \end{pmatrix} \begin{pmatrix} \overline{x}_{j1}(t) \\ \overline{x}_{j2}(t) \end{pmatrix},$$

where $P_i^{-1} I_i Q_i = \begin{pmatrix} A_{ij}^{(1)} & A_{ij}^{(2)} \\ A_{ij}^{(3)} & A_{ij}^{(4)} \end{pmatrix} (i, j = 1, 2, ..., r)$, we get

$$\overline{x}_{i2}(t) = -\sum_{j=1}^{r} e_{ij} (A_{ij}^{(3)} \overline{x}_{j1}(t) + A_{ij}^{(4)} \overline{x}_{j2}(t)),$$

thus

$$\bar{x}_{i2}(t) \to 0 \quad (t \to +\infty) \quad (i = 1, 2, ..., r),$$

we have

$$\bar{X}_i(t) \to 0 \quad (t \to +\infty) \quad (i = 1, 2, ..., r),$$

$$X_i(t) \to 0 \quad (t \to +\infty) \quad (i = 1, 2, ..., r),$$

finally, we have $X(t) \to 0 \ (t \to +\infty)$, that is, the equilibrium $X = 0$ of the system (1) is uniformly connectively asymptotically stable.

Theorem 2. *Assume that*:

(a) *There exist a positive definite matrix* B_i *and a constant number* $\mu_i > 0$ *such that the total derivative of* (4) *with respect to t along the solution of the system* (2) *satisfies*

$$\dot{V}_i[I_i X_i(t)]_{(2)} = \mu_i [I_i X_i(t)]^T [I_i X_i(t)]. \tag{14}$$

(b) *For singular large-scale system* (1), *there exist positive constant numbers* δ_{ij}, *such that*

$$\| A_{ij} X_j(t) \| \leq \delta_{ij} \| I_j X_j(t) \| \quad (i, j = 1, 2, ..., r). \tag{15}$$

(c) *All successively principal minors* C_k *of the* $r \times r$ *matrix* $C = (c_{ij})$ *satisfy* $C_k > 0 \quad (k = 1, 2, ..., r)$, *then the equilibrium* $X = 0$ *of the system* (1) *is connectively unstable, where*

$$c_{ij} = \begin{cases} \dfrac{[\mu_i - \lambda_M(B_i)(1 + \bar{e}_{ii}\delta_{ii}^2)]}{2\lambda_M(B_i)}, & i = j, \\ -p_{ij} - \dfrac{(r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2}{\lambda_m(B_j)}, & i \neq j, \end{cases} \tag{16}$$

where

$$p_{ij} = \frac{2(n - n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1 + \lambda_M(B_i)(1 + \bar{e}_{ii}\delta_{ii}^2)]\lambda_m(B_j)}, \quad i \neq j.$$

Proof. Let $Y_i = \| I_i X_i(t) \|$ $(i = 1, 2, ..., r)$, for any $E = (e_{ij}) \in \overline{E}$, taking vector singular Lyapunov function as follows:

$$V[IX(t)] = \{V_1[I_1V_1(t),\ I_2V_2(t),\ ...,\ I_rV_r(t)]\},$$

the total derivative of $V_i[I_i X_i(t)]$ with respect to t along solutions of the system (1) is

$$\dot{V}_i[I_i X_i(t)]_{(1)}$$

$$= [I_i \dot{X}_i(t)]^T B_i[I_i X_i(t)] + [I_i X_i(t)]^T B_i[I_i \dot{X}_i(t)]$$

$$= \left[A_i X_i(t) + \sum_{j=1}^{r} e_{ij} A_{ij} X_j(t)\right]^T B_i[I_i X_i(t)]$$

$$+ [I_i X_i(t)]^T B_i \left[A_i X_i(t) + \sum_{j=1}^{r} e_{ij} A_{ij} X_j(t)\right]$$

$$= [X_i(t)]^T (A_i^T B_i I_i + I_i^T B_i A_i) X_i(t) + 2\left[\sum_{j=1}^{r} e_{ij} A_{ij} X_j(t)\right]^T B_i[I_i X_i(t)]$$

by (7), we get

$$\geq \dot{V}_i[I_i X_i(t)]_{(2)} - [I_i X_i(t)]^T B_i[I_i X_i(t)] - \left[\sum_{j=1}^{r} e_{ij} A_{ij} X_j(t)\right]^T B_i \left[\sum_{j=1}^{r} e_{ij} A_{ij} X_j(t)\right]$$

$$\geq \mu_i \| I_i X_i(t) \|^2 - \lambda_M(B_i) \| I_i X_i(t) \|^2 - \lambda_M(B_i) \left\| \sum_{j=1}^{r} e_{ij} A_{ij} X_j(t) \right\|^2$$

$$\geq [\mu_i - \lambda_M(B_i)] Y_i^2 - \lambda_M(B_i) \left(\sum_{j=1}^{r} \overline{e}_{ij} \delta_{ij} Y_j \right)^2$$

$$\geq [\mu_i - \lambda_M(B_i)]Y_i^2$$

$$- \lambda_M(B_i)\left[\bar{e}_{ii}\delta_{ii}^2 Y_i^2 + 2\bar{e}_{ii}\delta_{ii}Y_i \sum_{j=1,\ j\neq i}^{r} \bar{e}_{ij}\delta_{ij}Y_j + \left(\sum_{j=1,\ j\neq i}^{r} \bar{e}_{ij}\delta_{ij}Y_j\right)^2\right]$$

$$\geq \sum_{j=1,\ j\neq i}^{r} \left\{\frac{n_j}{n-n_i}[\mu_i - \lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]Y_i^2 - 2\bar{e}_{ii}\delta_{ii}\lambda_M(B_i)\delta_{ij}Y_iY_j\right\}$$

$$- \lambda_M(B_i)\left(\sum_{j=1,\ j\neq i}^{r} \bar{e}_{ij}\delta_{ij}Y_j\right)^2.$$

By the hypothesis (c) of Theorem 2, from $[\mu_i - \lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)] > 0$, and the following inequality:

$$\alpha x^2 - xy \geq \frac{\alpha}{2}x^2 - \frac{1}{2\alpha}y^2,$$

where $x, y \in R$, $\alpha \in R^+$, we get

$$\dot{V}_i[I_iX_i(t)]_{(1)}$$

$$\geq \sum_{j=1,\ j\neq i}^{r} \left\{\frac{n_j[\mu_i - \lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}{2(n-n_i)}Y_i^2 - \frac{2(n-n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}Y_j^2\right\}$$

$$- (r-1)\lambda_M(B_i)\sum_{j=1,\ j\neq i}^{r} \bar{e}_{ij}\delta_{ij}^2 Y_j^2$$

$$\geq \frac{[\mu_i - \lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}{2}Y_i^2$$

$$- \sum_{j=1,\ j\neq i}^{r} \left\{\frac{2(n-n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]} + (r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2\right\}Y_j^2,$$

by (5), we have

$$\dot{V}_i[I_i X_i(t)]_{(1)} \geq \frac{[\mu_i - \lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]}{2\lambda_M(B_i)} V_i[I_i X_i(t)]$$

$$- \sum_{j=1,\ j\neq i}^{r} \left\{ \frac{2(n-n_i)\bar{e}_{ij}\delta_{ii}^2\delta_{ij}^2\lambda_M^2(B_i)}{n_j[-1+\lambda_M(B_i)(1+\bar{e}_{ii}\delta_{ii}^2)]\lambda_m(B_j)} + \frac{(r-1)\lambda_M(B_i)\bar{e}_{ij}\delta_{ij}^2}{\lambda_m(B_j)} \right\} V_j[I_j X_j(t)],$$

that is,

$$\dot{V}_i[I_i X_i(t)]_{(1)} \geq \sum_{j=1}^{r} c_{ij} V_j[I_j X_j(t)] \quad (i = 1, 2, \ldots, r),$$

we have

$$\dot{V}[IX(t)]_{(1)} \geq CV[IX(t)]$$

by the condition (c), it follows from the Comparison principle that the equilibrium point $X = 0$ of the system (1) is connectively unstable.

4. Example

Consider the following singular linear large-scale system which includes two subsystems with order 5:

$$\begin{cases} I_1\dot{X}_1(t) = A_1X_1(t) + e_{11}A_{11}X_1(t) + e_{12}A_{12}X_2(t), \\ I_2\dot{X}_2(t) = A_2X_2(t) + e_{21}A_{21}X_1(t) + e_{22}A_{22}X_2(t), \end{cases} \tag{17}$$

where

$$I_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad I_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$A_{11} = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}, \qquad A_{22} = \begin{pmatrix} \frac{1}{5} & \frac{1}{6} & 0 \\ \frac{1}{5} & -\frac{1}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_{12} = \begin{pmatrix} -\frac{1}{6} & -\frac{1}{5} & 0 \\ -\frac{1}{6} & \frac{1}{5} & 0 \end{pmatrix}, \qquad A_{21} = \begin{pmatrix} \frac{1}{6} & \frac{1}{5} \\ -\frac{1}{5} & \frac{1}{6} \\ 0 & 0 \end{pmatrix},$$

we get positive definite matrices

$$B_1 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \qquad B_2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix},$$

taking $\overline{E} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Let $\delta_{ij} = \delta \ (i, \ j = 1, 2)$. Then for any $E \in \overline{E}$, we have the following matrix:

$$C = \begin{pmatrix} \frac{1}{2}(\delta^2 - 1) & \delta^2\left(1 + \frac{4\delta^2}{\delta^2 - 1}\right) \\ \frac{4}{3}\delta^2\left(1 + \frac{2\delta^2}{\delta^2 - \frac{1}{3}}\right) & \frac{1}{2}(\delta^2 - \frac{1}{2}) \end{pmatrix},$$

we get $\frac{1}{2}(\delta^2 - 1) = -\frac{1}{18} < 0$ and $|C| = \frac{7}{81} > 0$ if $\delta = \frac{1}{3}$, matrix C satisfies the condition (iii) of Theorem 1, therefore, the equilibrium point $X = 0$ of the system (17) is uniformly connectively asymptotically stable.

5. Conclusion

The item e_{ij} in the paper is taken to be 0 or 1, in practice, the conclusions above are adapted to the condition $0 \le e_{ij}(t) \le 1$, that is to say, for any moment t, under the different intensities of connectivity among the subsystems, the conclusion still holds.

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