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A NOVEL FRACTIONAL-ORDER HYPERCHAOTIC SYSTEM AND ITS SYNCHRONIZATION

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Abstract

In this paper, a novel hyperchaotic system with one nonlinear term and its fractional-order system are proposed. Furthermore, synchronization between two fractional-order systems is achieved. The proposed synchronization scheme is simple and theoretically rigorous. Numerical simulations coincide with the theoretical analysis.

1. Introduction

Although fractional calculus is a 300-year-old topic, the basic theory of fractional-order differintegration was founded mainly in the 19th

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century, and developed comprehensively in the last century due to its applications in a wide variety of scientific and technological fields such as thermal, viscoelastic, acoustic, electrochemical, rheological and polymeric disciplines. In recent years, study on the dynamics of fractional-order differential systems has attracted interest of many researchers. It is demonstrated that some fractional-order differential systems behave chaotically or hyperchaotically such as the fractional-order Chua's system, the fractional Rossler system, the fractional modified Duffing system and Chen system [5-8, 11, 17].

Over the last two decades, since the pioneering work by Carroll and Pecora [1, 13], synchronization of chaotic systems has become more and more interesting to researchers in different fields. Synchronization of chaotic system with integer order is understood well. Recently, studies on chaos synchronization for the fractional-order systems are just beginning to attract some attention due to its potential applications in secure communication and control processing [9, 10, 12, 15, 16, 18]. But in these literatures [9, 10, 12, 15, 16, 18], the synchronization among the fractional-order systems is only investigated through numerical simulations. In our work, a novel hyperchaotic system with one nonlinear term is proposed, and its fractional-order system is studied numerically. In this paper, we also investigate the theoretically rigorous synchronization of the novel fractional-order hyperchaotic system.

This paper is organized as follows: In Section 2, a novel hyperchaotic system with one nonlinear term as well as the corresponding fractional-order system is proposed. In Section 3, the synchronization of the novel fractional-order hyperchaotic system is investigated. Finally, in Section 4, conclusions are drawn.

2. A Novel Hyperchaotic System with One Nonlinear Term

Consider the following new 4D dynamical system:

$$\begin{cases} \dot{x} = 0.56x - y, \\ \dot{y} = x - 0.1yz^2, \\ \dot{z} = 4y - z - 6w, \\ \dot{w} = 0.5z + 0.8w, \end{cases} \tag{1}$$

where x, y, z and w are state variables. The new system (1) is symmetric with respect to the origin, since it is invariant for the coordinate transformation $(x, y, z, w) \to (-x, -y, -z, -w)$.

The Lyapunov exponents of system (1) are $\lambda_1 = 0.074726$, $\lambda_2 = 0.021509$, $\lambda_3 = 0$, $\lambda_4 = -0.43386$, respectively. So, system (1) is a hyperchaotic system only with one nonlinear term. The Lyapunov dimension is $D_L = 3 + (\lambda_1 + \lambda_2)/|\lambda_4| = 3.2218$. Figure 1 shows the hyperchaos phase portraits.

The corresponding fractional-order hyperchaotic system is presented as follows:

$$\begin{cases} \dfrac{d^q x}{dt^q} = 0.56x - y, \\ \dfrac{d^q y}{dt^q} = x - 0.1yz^2, \\ \dfrac{d^q z}{dt^q} = 4y - z - 6w, \\ \dfrac{d^q w}{dt^q} = 0.5z + 0.8w, \end{cases} \tag{2}$$

where q is fractional order, $0 < q \leq 1$. The numerical simulation is shown in Figure 2 with $q = 0.95$.

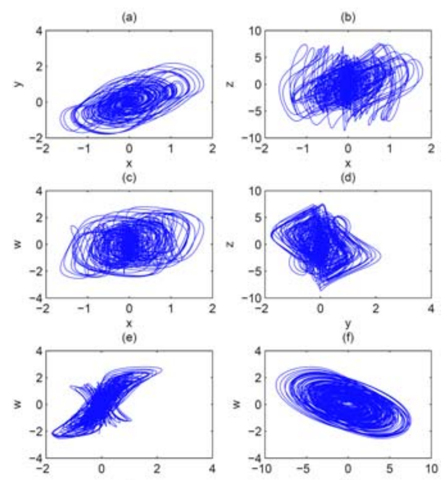

Figure 1. Hyperchaotic phase portraits of system (1): (a) x-y plane; (b) x-z plane; (c) x-w plane; (d) y-z plane; (e) y-w plane; (f) z-w plane.

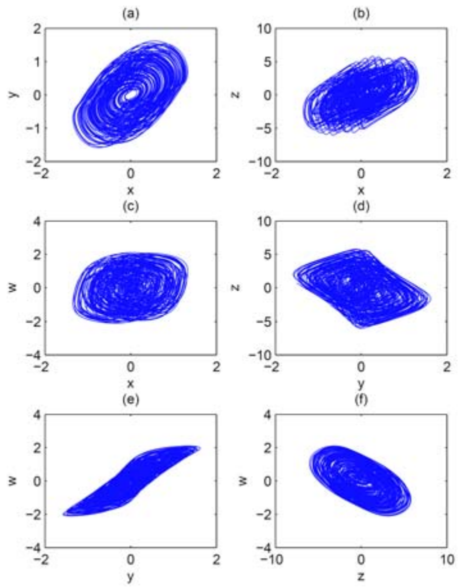

Figure 2. Hyperchaotic phase portraits of the system (2) for $q = 0.95$: (a) x-y plane; (b) x-z plane; (c) x-w plane; (d) y-z plane; (e) y-w plane; (f) z-w plane.

Remark 1. There are two approximation methods for solving fractional differential equation. The first method is an improved version of Adams-Bashforth-Moulton algorithm [3, 4, 11] and is proposed based on the predictor-correctors scheme for this system [2]. The second method is frequency domain approximation. It is well known that the frequency domain approximation of fractional-order operators is a very good candidate to realize the fractional-order controllers. Also, these approximation methods provide a simple procedure to simulate the fractional-order systems numerically. However, the frequency domain approximation in the numerical simulations of fractional systems may result in wrong consequences [14]. For example, this approximation can numerically demonstrate chaos in the non-chaotic fractional-order systems. Unfortunately, this mistake has occurred in the recent literature [14] that found the lowest-order chaotic systems among fractional-order systems. In this paper, we use the first method for solving fractional differential equation (2).

3. Synchronization between Two Fractional-Order Hyperchaotic Systems

In this section, we will focus on the synchronization of two identical fractional-order hyperchaotic systems (2) with different initial values.

Consider another fractional-order hyperchaotic system which is described by

$$\begin{cases} \dfrac{d^q X}{dt^q} = 0.56X - Y, \\ \dfrac{d^q Y}{dt^q} = X - 0.1YZ^2, \\ \dfrac{d^q Z}{dt^q} = 4Y - Z - 6W, \\ \dfrac{d^q W}{dt^q} = 0.5Z + 0.8W. \end{cases} \tag{3}$$

System (2) is regarded as the driving system, then the coupled slave fractional-order hyperchaotic system is given as follows:

$$\begin{cases} \dfrac{d^q X}{dt^q} = 0.56X - Y - k(X - x), \\ \dfrac{d^q Y}{dt^q} = X - 0.1YZ^2 + 0.1Z^2(Y - y) + 0.2yz(Z - z), \\ \dfrac{d^q Z}{dt^q} = 4Y - Z - 6W, \\ \dfrac{d^q W}{dt^q} = 0.5Z + 0.8W, \end{cases} \tag{4}$$

where k is a constant to be designed. To investigate the synchronization of systems (3) and (4), we define the error states $e_1 = X - x$, $e_2 = Y - y$, $e_3 = Z - z$, and $e_4 = W - w$. Then, the corresponding error dynamics system can be obtained by subtracting system (2) from system (4):

$$\begin{pmatrix} \dfrac{d^q e_1}{dt^q} \\ \dfrac{d^q e_2}{dt^q} \\ \dfrac{d^q e_3}{dt^q} \\ \dfrac{d^q e_4}{dt^q} \end{pmatrix} = \begin{pmatrix} 0.56 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & -1 & -6 \\ 0 & 0 & 0.5 & 0.8 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} + \begin{pmatrix} -ke_1 \\ -0.1e_2e_3^2 - 0.1ye_3^2 - 0.2e_2e_3z \\ 0 \\ 0 \end{pmatrix}. \tag{5}$$

If the zero solution of error dynamics system (5) is globally asymptotically stable under a suitable constant k, then the two systems (2) and (4) are realized to synchronization.

From error dynamics system (5), we can obtain that $e_1 = e_2 = e_3 = e_4 = 0$ is an equilibrium point of system (5). Linearizing system (5) at this equilibrium yields the Jacobian matrix

$$J = \begin{pmatrix} 0.56 - k & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & -1 & -6 \\ 0 & 0 & 0.5 & 0.8 \end{pmatrix}$$

and its characteristic equation

$$(r^2 - (k + 0.56)r + 1)(r^2 + 0.2r + 2.2) = 0. \tag{6}$$

Then the eigenvalues of (6) are

$$r_{1,2} = 0.5(k + 0.56) \pm 0.5\sqrt{(k + 0.56)^2 - 4}, \quad r_{3,4} = -0.1 \pm i\sqrt{2.19}.$$

When $k < -0.56$, the eigenvalues of matrix J all have negative real part. Therefore, the equilibrium point $e_1 = e_2 = e_3 = e_4 = 0$ of error dynamics system (5) is globally asymptotically stable, which implies that the systems (2) and (4) are realized to synchronization.

When $k \geq -0.56$ and $-2 < k + 0.56 < 2, \arctan\left|\sqrt{4 - (k + 0.56)^2}\,/k + 0.56\right| > 0.5\pi q$, i.e., $-0.56 \leq k < -0.4031$, then the eigenvalues of matrix J

satisfy $|r_j| > 0.5\pi q$ $(j = 1, 2, 3, 4)$, which implies that the equilibrium point $e_1 = e_2 = e_3 = e_4 = 0$ of error dynamics system (6) is globally asymptotically stable, which implies that the synchronization between (2) and (4) can be achieved.

The above results manifest the fractional-order hyperchaotic systems (2) and (4) can be synchronized under suitable constant k. The simulation result is shown in Figure 3 with $q = 0.95$ and $k = -1$ and the initial states of the drive system and the response system are taken as $x(0) = -1$, $y(0) = 1.5$, $z(0) = -1$, $w(0) = -2$ and $X(0) = 0$, $Y(0) = 1$, $Z(0) = 1$, $W(0) = -1$, respectively.

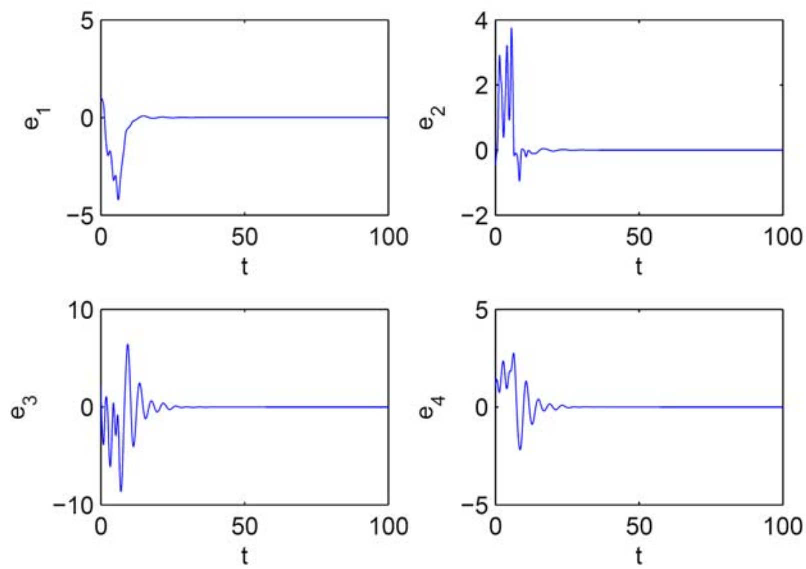

Figure 3. The synchronization error functions of four state variables versus time t.

4. Conclusion

In this paper, a novel hyperchaotic system with one nonlinear term and its fractional-order system were proposed, and its dynamical behaviors were studied. Moreover, synchronization between two such fractional-order hyperchaotic systems has been achieved via feedback control. The proposed synchronization approach is simple and theoretically rigorous. Finally, numerical simulation was given to verify the effectiveness of the proposed synchronization scheme.

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