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RANK-ORDER WEIGHTING AS A SURROGATE FOR DIRECT WEIGHTING

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Abstract

This article is concerned with procedures for finding a surrogate for direct weighting method. In this paper, it is tried to determine that among famous rank-order weighting methods, which one can be a surrogate for direct weighting method. Using surrogate weights based on ranking has been proposed as a method for avoiding complexity of weight elicitation. A simulation was conducted to compare using different rank-order weighting methods with the TRUE weights (simulated decision makers' judgments with prior ranking). Two kinds of comparison were done; comparison on weights and comparison on decision scores. From the results, using Rank-Sum (RS) is suggested as a good surrogate for decision makers' weights for the attributes when using direct weighting method to weight the attributes.

I. Introduction

Decision makers often need to choose among several alternatives, that each alternative has some attributes. Actually what happened in selecting an alternative is that the decision makers try to estimate the score of an alternative upon some specific features (attributes). However

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different attributes has different importance in one particular alternative. So what the decision maker should do is to consider these differences of importance somehow in the decision process. Usually they weigh the attributes in terms of the attributes' importance in order to bring this importance to decision process. However understanding the importance and consequently the weights of attributes is not easy. The decision makers should have good experiences to know which attribute is more important than the other attribute in the particular alternative, and after understanding that, s/he may be able to weigh the attribute.

During the past several decades, there has been much research to deal with imprecise information in multiattribute decision analysis. Imprecise information means that decision parameters, such as marginal values (i.e., values of decision alternatives on each attribute) and attribute weights, are known only to the extent that the true values lie within prescribed bounds, while other parameters are known only to satisfy certain ordinal relations. Imprecise information is also referred to as incomplete information or partial information, and assumes that the decision maker may not be willing or able to provide exact estimations of decision parameters [1]. Many procedures are explained in the literature in order to formalize the decision-making process under these conditions.

In most of the researches, first the decision makers determine the importance of attributes using his intuitive and experience or from analyzing the user's questionnaire. Second; after the first step, the weights have to be assigned to the attributes according to their importance. If the attributes' ranks exist, the problem is how to elicit weights from this their ranks.

A. Weighting methods categories

Elicitation of weights in many situations is difficult. To overcome this problem several methods have been proposed. Most of these methods fall in three categories which are Direct Rating Methods, Point Allocation Methods, and Ranking Methods. These approaches are outlined below.

(1) Direct weighting methods: In this procedure, the most important attribute is given an arbitrary weight. Proportionally smaller weights are then given to lower attribute. This procedure is repeated for the next most important criterion until weights are assigned to all criteria (attribute). Finally, the weights are normalized by dividing each weight by the total (sum of weights) [4]. The problem using these methods is its subjectivity using experts or any other subjects' judgments to assign direct weights. The process of weighting is difficult and its subjectivity makes it unrepeatable, and unreliable.

(2) Point allocation method: In this method, weights are estimated by the decision maker on a predetermined scale, for example 0 to 100. In this approach, the more points a criterion receives, the greater its relative importance [4]. For example, the decision maker may be asked to distribute 100 points among the four criteria (attributes) that are important to a specific choice (alternative). It is obvious this method does not need to be normalized as the total sum of 100 (for instance) is already given. This way of weighting is amongst the difficult methods, because it is easier to take for example 100 as the weight for most important attribute, and then relative to this 100, weight consecutive attributes. In this type, the decision maker may be worry of the restriction that the total must be a specified value. The subjectivity problem remains in this method also; means assigning the weight is completely depends on what the expert has in its mind.

(3) Rank-order weighting methods: This category includes those methods that use ranking into 'surrogate' weights that represent an approximation of the "true" weights. The three famous formulas in this category are as below:

(a) Rank-Sum method (RS): In the rank-sum method procedure the weights, w_i (RS), are the individual ranks normalized by dividing by the sum of the ranks. Since adopting the convention that $w_1 \geq w_2 \geq w_3 \geq \cdots \geq w_n$, the most important attribute has a weight of *n*/(sum-of-ranks), and the least important attribute, 1/(sum-of-ranks). In general, weight for the *i*th most important attribute in each formula is: [2]

$$w_i(RS) = \frac{n + 1 - i}{\sum_{j=1}^{n} j} = \frac{2 \times (n + 1 - i)}{n \times (n + 1)}, \; i = 1, \ldots, n.$$

(b) Reciprocal of the Ranks (RR): A similar approach gives relative weights based on the reciprocal of ranks (RR). That is, the non-normalized weights are $1, 1/2, 1/3, \cdots, 1/n$ dividing through by the sum of these terms yields the final weights which meet the restriction of summing to unity [2]

$$w_i(RR) = \frac{\frac{1}{i}}{\sum_{j=1}^{n} \frac{1}{j}}, \; i = 1, \ldots, n.$$

(c) Rank-Order Centroid (ROC): The formula producing the weights is written as below. In general, assuming there are "*n*" attributes, for *i*th most important attribute in the centroid weight is [2]

$$w_i(ROC) = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{j}, \; i = 1, \ldots, n.$$

These methods (RR, RS, ROC) have the advantage of reducing the subjectivity level of weighting to some extent, because when using rank order weighting methods to weigh the attributes, it does not need to assign weights directly, instead it needs to rank the attributes first, which this is done by expert or evaluator; the one who is going to make decision, and then after ranking, using each of the above formulas to elicit weights from the ranks, it is clear that ranking is more easier than weighting [5].

The first and second weighting categories (Direct rating, and point allocation), have the problem of being completely subjective. Furthermore the point allocation method is more difficult than the direct ranking because of having the constraint that the total (total weights of attributes) must be a specified value. Here we assume that the decision makers prefer to use direct weighting method to directly weight the attributes and also we assume that they could agree on the rank of attributes. So each decision maker may weigh the attributes in the way s/he would like. According to some researches, ranking is a necessary first step in most procedures for eliciting more precise weights, and rank-ordering the importance of attributes may be easier than describing other imprecise weights such as bounded weights [1]. We tried to find out which rank-order weighting method can be a good surrogate for decision makers' weights (direct weighting method, with prior ranked attributes). Previously in two researches, ROC has been suggested as a good surrogate for point allocation method [1, 2].

B. Evaluating of weighting methods

In [3], four weighting methods are compared on three efficiency criteria, which are internal consistency, convergent validity, and external validity. Three weighting methods with prior ranking and one weighting method without prior ranking are used to weigh attributes for evaluating nuclear waste repository sites in USA.

Bottomley and Doyle in [6] compared three weighting methods, conducting an experiment and asking people to weight and use the methods in an issue. The above research studies compare the weights obtained from different weighting methods in terms of those three efficiency criteria (Internal Consistency, Convergent Validity, and External Validity).

In another research [2], the efficiency of three rank-order-weighting methods (RR, RS, and ROC) is examined in a simulation study. The researchers simulate experts' weights and compare the results from RR, RS, and ROC with the results from simulated experts. The comparison is done on criteria HitRatio, AverageValueLoss, and Average Proportion of Maximum Value Range Achieved. They simulate the point allocation weighting for the experts (they create a vector at random from the weight space S_n; which S_n is $w_1 \geq w_2 \geq \cdots \geq w_n$, and $\sum_{i=1}^{n} = 1$). Finally, they concluded that ROC method gives better results than RR, and RS. Thus it can be considered as an alternative to expert's judgments (TRUE weights) when weighting attributes in point allocation method.

Also the researchers in [1] examined the efficiency of RR, RS, and ROC in a simulation study. They used a similar simulation set up as what later was used in [2]. They compared the results from rank-order weighting method with the results from simulation. Two measures of efficiency; HitRatio, and Rank Correlation are employed. They simulate the point allocation weighting for the expert. Finally they concluded the same result as in [2]. In [2] and [1], the comparison is not done on weights. They use the weights (from RR, RS, ROC, and simulated experts' judgments) in a multi-attribute decision problem, and employ the HitRatio, Value Loss, and Rank correlation to examine the efficiency of using the weights (from different methods, and expert simulation) in the problem. Actually in recent comparison, the efficiency of weights compared indirectly. It is mentioned that point allocation method is not easy to be used. Direct weighting is easier than point allocation. Consequently it is more probable that experts use the second type (direct weighting) instead. On the other hand, the rank order weighting methods make it easier and less subjective (although still ranking is left subjective but not weighting).

In this research it is intended to investigate the efficiency of rank-order weighting methods against direct weighting (suppose that weighting is done with predefined ranking). In general in any weighting decision problem, using decision makers' judgments to directly assign weights is a problem [2]. The necessity of this research can be mentioned as:

(1) The decision maker may be unavailable, unable, or unwilling to specify sufficiently precise weights; or

(2) There may be no single decision maker, and the group may not be able to directly assign weights.

(3) Direct rating is one of the most used procedures; if it can be replaced by a surrogate it is useful and more reliable.

So the main objective of this research was to find out that among the famous rank order weighting methods (ROC, RR, and RS), which one can be used as the surrogate for direct weighting method (random weight with prior ranked attributes).

The rest of the article is structured as follows: First the description of the research methodology and procedures used in the study are reviewed. This is followed by simulation and comparison sections. Then that the results of the study are discussed. Finally, conclusion is offered.

II. Methodology

To overcome subjectivity of direct weighting (with prior ranking), it is tried to replace it by a less subjective or non-subjective methods. The main steps are as follows:

• Weighting the ranked attributes, using Rank-order Weighting formula.

• Simulating the decision makers' judgments (TRUE weights - simulation of direct weighting).

• Comparison: The three famous rank-weight methods are compared to find which is more efficient in our problem.

A. Developing weighting approach

We assume here that there is "*m*" ranked attributes to weight. The weighting method will be developed to elicit weights for the attributes from their ranks. The important layouts of this section consist of the following steps:

(1) Weighting the attributes: Calculate weight for attributes using rank-order weights: Having the ranks of attributes, and using rank-order weighting formula, the weight of each attribute according to its rank in the group of attributes, can be expressed. Assume that there are:

• "*n*" attribute from $At_1 \cdots At_n$,

• $At_1 > At_2 > \cdots > At_n$; means At_1 is more important than $At_2 \cdots$, and At_{n-1} is more important than At_n. Then it is obvious that the relation between their weights is $w_1 > w_2 > \cdots > w_n$; which w_1 is the weight for At_1.

• So now by using the attributes' ranks and rank-order weighting formulas, one can reach to the vector of weights. The three famous rank order weighting formulas; which are Reciprocal of the Ranks (RR), Rank-Sum (RS), and Rank-Order Centroid (ROC), are mentioned in pervious section.

- $\sum_{i=1}^{n} W_i = 1,\ i = 1, \ldots, n.$

B. Simulation study

Simulation is conducted in order to use its results in the comparison. Here we make a decision matrix in order to use it in making decide about the alternatives. Actually the weights for a number of ranked attributes (m) are calculated using rank-order weighting methods (RR, RS, ROC). For some particular number of alternative (n), and number of attributes (m) the TRUE weights (weights from decision makers' judgments) are simulated. Moreover a value matrix for attributes in the particular alternative is simulated (Called $\text{SVM}_{m \times n}$ matrix). Furthermore the decision matrices are calculated by multiplying weights from RR, RS, ROC and TRUE weights by the $\text{SVM}_{m \times n}$.

In simulation, weighting attributes should be performed in a way reflecting the decision maker's behavior. For example, if ask the decision makers to weigh these attributes, then each decision maker weighs the attributes in a way s/he thinks according to the attributes' ranks. As it is mentioned in the objective, the decision makers' weights are considered subjective, means there is no control how decision makers weight the attributes, so each decision maker may weight the attributes base on their concept from their ranks. Thus in order to reflect the decision makers' behavior and preserves its subjectivity, the weights from decision makers are simulated randomly, for the particular ranked attributes.

The simulation follows the similar simulation setup used by [1, 2], although it is changed and adapted to this research. The simulation setup is as follows:

(1) Step 1. Simulate the decision makers' weights (simulating TRUE attributes' weights based on their ranks) as below:

- A matrix of weights in the rate (0, 100); named the $\text{DMW}_{k \times m}$ matrix, will be generated randomly. The rows of the matrix (k) show the number of simulated decision makers, and the column (m) is the attributes.

• As each weight should be between 0 and 1, and also because the sum of weights for each row should be unit, each row should be normalized.

• At the end because one dimensional matrix from $\mathrm{DMW}_{k\times m}$ is needed, mean of $\mathrm{DMW}_{k\times m}$ as the decision maker's weights simulation matrix will be calculated, which is MDMW_{m} matrix.

(2) Step 2. Compute the weights of attributes using rank-order weighting formulas. There are three sets (matrices) of weights form three rank order weighting methods RR, RS, and ROC using their corresponding formula. Moreover there is another set (matrix) of weights from decision makers' weights simulation (TRUE weights). ROC, RR, and RS formulas will be used to calculate weight for the ranked attributes, and put the weights in matrices ROCW_{m}, RRW_{m}, RSW_{m}.

(3) Step 3. Simulate value matrix for the attributes randomly $(\mathrm{SVM}_{m\times n})$. We assumed the scale of attributes is in range $0-100$. Therefore $\mathrm{SVM}_{m\times n}$ matrix filled with random numbers from uniform distribution $[0, 100]$, this is called *attribute value matrix*. Here we assumed that for each of "n" alternatives, each of "m" attributes have been measured and have filled $\mathrm{SVM}_{m\times n}$ matrix. This matrix $(\mathrm{SVM}_{m\times n})$ is used to calculate final decision score for each alternative.

For example: $\mathrm{ROCW}_{1\times m} \times \mathrm{SVM}_{m\times n} = \mathrm{ROCD}_{1\times n}$.

Totally thirty (48) simulation design are conducted. The simulation design is as follows:

• 8 levels of alternatives $(n = 2, 3, 4, 5, 6, 7, 9, 11)$ and

• 6 different levels of attributes $(m = 2, 3, 4, 5, 6, 7)$

• For each attribute (m), 1000 decision makers' judgments for the weights (called TRUE weights), using random number are simulated.

• For each of the 48 design structures (alternative × attributes) the process of weight assignment and decision calculation is repeated until

10000 trials were obtained. This means total trial of this simulation is 480000 (48 ∗ 10000).

It is required to show that for a number of attributes, results from rank-order weighting methods (RR, RS, and ROC) is closed to results from decision makers (direct weighting); in terms of some criteria (explained in comparison section). So instead of direct weighting and asking the decision maker or evaluator to directly assign weights to the ranked attributes; one can just use formulas to elicit weights from the ranked attributes.

C. Comparison

Results using ROC, RR, and RS, will be compared to determine which weighting formula has better correlation with the results using TRUE weights (simulated decision makers). Two kinds of comparison are conducted; comparison on decision scores and comparison on weight. Some of the comparisons below have been used in previous researches [1, 2]. They are used and adapted to this research.

(1) Comparison on decision score: The first comparison is to compare the decision scores using different rank-order weighting methods (RR, RS, and ROC) with the one using simulated decision maker's weights in terms of three criteria, which are:

(a) HitRatio: The same best alternative (HitRatio) as the decision makers' best alternative. HitRatio is the percent of times that for example ROC method chooses the same best alternative as decision makers' judgments selection [2].

(b) ValueLoss: The different value with true decision (ValueLoss): It is the sum of differences between best decision scores obtained from rank-order weighting method with the best decision score from decision makers' judgments' divided by number of iterations of the program. It is clear that having smaller ValueLoss is more desirable.

(c) Decision-Maker's-Rank-preservation: It is checked that which method preserves ranks of decision makers in terms of decision score.

The first and second criteria (HitRatio and ValueLoss) are used in previous researches [2].

(2) Comparison on weights: The second comparison (comparison on weights) is to compare the attributes' weights obtained from three rank order weighting methods and also the weights obtained from simulated decision makers'; TRUE weights, through the below criteria which are:

(a) Convergent validity: To determine the correspondence between weights elicited through different weighting methods, individual correlations were computed across attributes [3].

(b) External validity: Here it should be examined if weights from the alternative methods correlated positively with decision makers' weights (simulated decision makers' judgments) [3]. The rank correlation between weights from each rank-order waiting methods (RR, RS, and ROC) and the weights from simulated decision makers' judgments (TRUE weights), are examined using Kendall tau *b* correlation.

III. Results

For each decision scores obtained using weights of rank order weighting methods (RR, RS, ROC), Table 1 illustrates the min, max, and average hit ratio and value losses obtained for the 48 simulation design.

A. HitRatio

Table 1 shows the percentage of times that three rank order weighting methods, selected the same best alternative as the decision makers' one (actually simulation of direct weighting). The match for RS scores is shown there. This means that 98.44% of times, using RS weights, produced the same best decision score as the TRUE weights do. It can be seen that ranges for RS is from 99.5 to a 97.1. Having a look at variance in the below table, it is clear that having a small variance for RS (less than 1), it can be stated that RS remains mostly constant and changes very little but it is not true for the other two methods. In terms of average percent of HitRatio, it can be said that the poorest one here is ROC, with the average of 88.31% over 48 design structure. So in terms of hit-ratio the relation among three waiting methods is RS > RR > ROC.

Table 1. Mean and variances of results of all 48 simulation structure

	HitROC	HitRR	HitRS	VLROC	VLRR	VLRS
Average	88.31	88.99	98.44	2.67	2.05	0.33
Min	80.50	76.00	97.10	1.44	0.37	0.18
Max	95.50	98.90	99.50	3.23	3.42	0.61
Variance	17.38	47.44	0.37	0.23	0.94	0.02

To precise this result, a statistic test is done. The results of paired T-Test are summarized in Table 2. In this test, it is checked to see if HitRatio of RS has meaningful different with the Hitrate from the other two methods. For instance to compare the ROC and RS methods, a t-statistic of 15.79 is obtained and we found that RS is significantly perform ROC at the significance level of 0.01. This is also true for RS and RR.

Table 2. Paired t-test results for average HitRatio

	ROC	RR
RS	15.79	8.71

The results show that RS method is significantly superior to other methods at the significant level of 0.01.

B. Value Loss (VL)

The results for VL are illustrated in Table 1. The results show that among rank order weighting methods, RS has the least mean Value loss (0.02 in scale 0-100 or 0.0002 in scale 0 to 1). It can be said that ValueLoss (VL) is small for RS method comparing to other methods. Actually in every simulation structure, results using RS out performs the results by using other weighting methods in terms of ValueLoss. It is obvious that having less ValueLoss is desirable.

To precise this result, a statistic test is done. The results of paired *T*-Test are summarized in Table 3. In this test, it is checked to see if ValueLoss of RS has meaningful different with the ValueLoss from the other two methods. For instance to compare the ROC and RS methods, a *t*-statistic of 26.642 is obtained and we found that RS is significantly perform ROC at the significance level of 0.01. This is also true for RS and RR.

Table 3. Paired *t*-test for the average ValueLoss

	ROC	**RR**
RS	26.64	10.34

The results show that RS method is significantly superior to other methods at the significant level of 0.01.

Another comparison using the HitRatio and ValueLoss that can be fetched, is the best method in terms of HitRatio and VL (Table 4).

Table 4. Number of times (out of 48) each method has the best HitRatio and best ValueLoss

		Rank-Order Weighting Methods		
		ROC	RS	RR
Criteria	Best (Max) HitRatio	-	48	-
	Best (Min) ValueLoss	-	48	-

Among 48 design structure of simulation, always (48 times out of 48) RS has the best HitRatio. It means in each simulation design, RS has the highest score for the HitRatio. Also for ValueLoss, RS in each design has the least VL.

C. Decision makers-ranks-preservation

Actually in order to understand that how much using rank-order weighting methods in calculating decision scores preserve the ranks as same as the one using TRUE weights (weights from decision makers or direct weighting) this criterion is used. The results of decision scores of each simulation structure are shown (Table 5). Moreover to understand it clearly one of them is illustrated in chart.

Table 5. Number of times (out of 48) each rank-order weighting method preserves decision makers' ranks

		Rank-Order Weighting Methods		
		ROC	**RS**	**RR**
Criteria: Rank Preservation	Experts' Rank Preservation	35 (of 48)	48 (of 48)	39 (of 48)
	Percent	72.91%	100%	81.25%

From Table 5 it can be seen that in all cases (100%) the RS method shows better results. This means 100% of times, ranks of alternatives' decision scores obtained using weights of RS method, preserves the one using TRUE weights (weights using simulation decision makers' judgments). In terms of this criterion the relation between these methods is RS > RR > ROC.

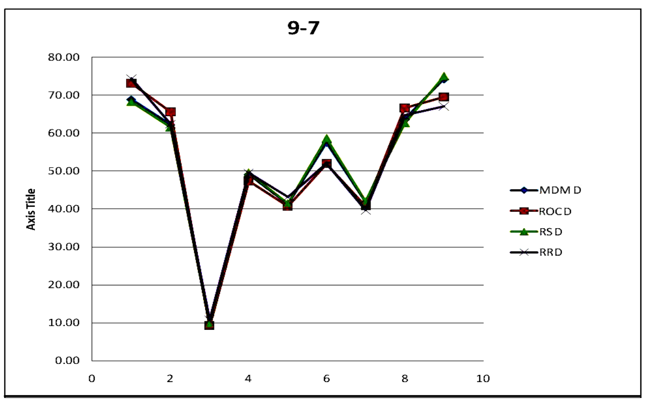

Figure 1. Simulation structure with 9 alternatives, 7 attributes-10000 iterations.

In the above chart (Fig. 1), the final decision scores for seven attributes and for nine alternatives are illustrated, it can be seen that the results from all three different weighting methods (ROCD, RSD, RRD), are to some extent close to the result using simulated decision makers' weights (MDMD). Also from the above chart is understood that using all three weighting methods, the ranks of alternative is the same as using TRUE weights.

D. Convergent validity

As all three methods used same ranked attributes to elicit weights, so each pair of (RR & RS), (RR & ROC), and (RS & ROC) will be tested on this criterion. From the results (Table 6), it can be seen that for each simulation structure, the result of the test is 1. This means that each pair is completely convergent and there is perfect agreement in terms of their ranks between them.

Table 6. Mean convergent validity between each pair of rank-order weighting methods in 48 simulation design structure

	Weighting Methods		
	ROC & RR	**ROC & RS**	**RS & RR**
Mean Convergent Validity	1	1	1

E. External validity

The results of external validity between each method and simulated decision makers' judgments are represented in Table 7. It can be observed that the mean result of each rank-order weighting method and simulated-decision makers' is positive. This is the mean for 48 simulation design.

Table 7 Mean Kendall Tau b correlation between each rank-order weighting method and TRUE weights (simulated decision makers' judgments)

	Weighting Method		
Simulated Experts' (TRUE Weights)	**ROC**	**RR**	**RS**
	1	1	1

So in another word all three methods have perfect correlation with the simulated decision makers' judgments, in terms of their weights.

IV. Conclusion

We have proposed using rank-order weighting method (RS) as a surrogate for direct weighting method (with prior ranking) in weighting attributes. We tried to show that the weights using RS can be used instead of decision makers' weights (weights by direct weighting method with prior ranked attributes); in another study when using non-prior ranked attributes to weight by direct weighting methods, the results again showed that RS can be a surrogate for it also. Using a simulation study, we have compared the performance of rank-order weighting methods with direct weighting method (decision makers' judgments). The simulation result shows that using RS outperforms the other two rank-order weighting methods (RR, and ROC) in terms of selecting the best alternative (HitRatio), ValueLoss, and decision makers'-ranks-preservation.

When a weighing method has a small ValueLoss in every simulation design structure, it means that the particular weighting method can be easily used instead of decision makers' weights, without the fear of losing the value of decision score for an alternative; it is reliable. Also when it has high HitRatio, this means that in terms of choosing the best alternative using surrogate method select the best alternative as the previous method in 98.44% of times (RS for example). Also when having 100% rank-preservation for RS, this means that 100% of times when comparing several alternatives, RS put the alternatives in the same order as the decision makers' do.

In particular, using the RS method appears to be the best performer throughout the simulation. RS shows surprising good results; this may because that actually the procedure acquiring experts is some how more similar to how the RS is being calculated than other two rank-order weighting methods.

Here the performance of using rank-order weighting methods (RS, RR, and ROC) is compared with direct weighting having ranked attributes. In another research we have used non-ranked attributes for decision makers' judgments (instead of using ranked attributes), again the results of RS shows much better match.

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