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FUZZY PAIRWISE ALMOST (r, s)-CONTINUOUS MAPPINGS

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Abstract

In this paper, we introduce the concepts of fuzzy pairwise almost (r, s)-continuous, fuzzy pairwise almost (r, s)-open and fuzzy pairwise almost (r, s)-closed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

1. Introduction

Chang [2] introduced fuzzy topological spaces and other authors continued the investigation of such spaces. Azad [1] introduced the concepts of fuzzy regular open set and fuzzy almost continuous mappings in fuzzy topological spaces. Chattopadhyay et al. [3] introduced another definition of smooth topology as a generalization of fuzzy topology. Kandil [4] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee [5] introduced the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

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In this paper, we introduce the concepts of fuzzy pairwise almost (r, s)-continuous, fuzzy pairwise almost (r, s)-open and fuzzy pairwise almost (r, s)-closed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

2. Preliminaries

In this paper, we denote by I the unit interval $[0, 1]$ of the real line and $I_0 = (0, 1]$. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are the standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$, then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_k \in T$ for all k, then $\vee \mu_k \in T$.

The pair (X, T) is called a *Chang's fuzzy topological space*. Members of T are called T-fuzzy open sets of X and their complements T-fuzzy closed sets of X.

A system (X, T_1, T_2) consisting of a set X with two Chang's fuzzy topologies T_1 and T_2 on X is called a *Kandil's fuzzy bitopological space*.

A *smooth topology* on X is a mapping $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.

(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$.

(3) $\mathcal{T}(\vee \mu_i) \geq \wedge \mathcal{T}(\mu_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*. For $r \in I_0$, we call μ a $\mathcal{T}$-fuzzy r-open set of X if $\mathcal{T}(\mu) \geq r$ and μ a $\mathcal{T}$-fuzzy r-closed set of X if $\mathcal{T}(\mu^c) \geq r$.

A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *smooth bitopological space*. Throughout this paper the indices i, j take values in $\{1, 2\}$ and $i \neq j$.

Let $(X, \mathcal{T})$ be a smooth topological space. Then it is easy to see that for each $r \in I_0$, an r-cut

$$\mathcal{T}_r = \{\mu \in I^X \mid \mathcal{T}(\mu) \geq r\}$$

is a Chang's fuzzy topology on X.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Then the mapping $\mathcal{T}^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1} \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\} \\ 0 & \text{otherwise} \end{cases}$$

becomes a smooth topology.

Hence, we obtain that if $(X, \mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopological space and $r, s \in I_0$, then $(X, (\mathcal{T}_1)_r, (\mathcal{T}_2)_s)$ is a Kandil's fuzzy bitopological space. Also, if (X, T_1, T_2) is a Kandil's fuzzy bitopological space and $r, s \in I_0$, then $(X, (T_1)^r, (T_2)^s)$ is a smooth bitopological space.

Definition 2.1 [5]**.** Let $(X, \mathcal{T})$ be a smooth topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the $\mathcal{T}$*-fuzzy r-closure* is defined by

$$\mathcal{T}\text{-Cl}(\mu, r) = \bigwedge\{\rho \in I^X \mid \mu \leq \rho, \mathcal{T}(\rho^c) \geq r\}$$

and the $\mathcal{T}$*-fuzzy r-interior* is defined by

$$\mathcal{T}\text{-Int}(\mu, r) = \bigvee\{\rho \in I^X \mid \mu \geq \rho, \mathcal{T}(\rho) \geq r\}.$$

Lemma 2.2 [5]. *Let* μ *be a fuzzy set of a smooth topological space* $(X, \mathcal{T})$ *and* $r \in I_0$. *Then we have*

(1) $\mathcal{T}\text{-Cl}(\mu, r)^c = \mathcal{T}\text{-Int}(\mu^c, r)$.

(2) $\mathcal{T}\text{-Int}(\mu, r)^c = \mathcal{T}\text{-Cl}(\mu^c, r)$.

Definition 2.3 [5]. Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiopen* set if there is a $\mathcal{T}_i$-fuzzy r-open set ρ in X such that $\rho \leq \mu \leq \mathcal{T}_j\text{-Cl}(\rho, s)$,

(2) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiclosed* set if there is a $\mathcal{T}_i$-fuzzy r-closed set ρ in X such that $\mathcal{T}_j\text{-Int}(\rho, s) \leq \mu \leq \rho$.

Definition 2.4 [5]. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a smooth bitopological space. For each $r, s \in I_0$ and for each $\mu \in I^X$, the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiclosure* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sCl}(\mu, r, s)$$

$$= \bigwedge\{\rho \in I^X \mid \mu \leq \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-semiclosed}\}$$

and the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiinterior* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sInt}(\mu, r, s)$$

$$= \bigvee\{\rho \in I^X \mid \mu \geq \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-semiopen}\}.$$

Lemma 2.5 [5]. *Let* μ *be a fuzzy set of a smooth bitopological space* $(X, \mathcal{T}_1, \mathcal{T}_2)$ *and* $r, s \in I_0$. *Then the following statements are equivalent*:

(1) μ *is a* $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiopen set*.

(2) μ^c *is a* $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-semiclosed set*.

(3) $\mu \leq \mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s)$.

(4) $\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\mu^c, r), s) \leq \mu^c$.

Definition 2.6 [6]**.** Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-preopen* set if $\mu \le \mathcal{T}_i$-Int$(\mathcal{T}_j$-Cl$(\mu, s), r)$,

(2) a $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-preclosed* set if $\mathcal{T}_i$-Cl$(\mathcal{T}_j$-Int$(\mu, s), r) \le \mu$.

Definition 2.7 [6]**.** Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a smooth bitopological space. For each $r, s \in I_0$ and for each $\mu \in I^X$, the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-preclosure* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pCl}(\mu, r, s)$$

$$= \bigwedge\{\rho \in I^X \mid \mu \le \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-preclosed}\}$$

and the $(\mathcal{T}_i, \mathcal{T}_j)$*-fuzzy* (r, s)*-preinterior* is defined by

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pInt}(\mu, r, s)$$

$$= \bigvee\{\rho \in I^X \mid \mu \ge \rho, \rho \text{ is } (\mathcal{T}_i, \mathcal{T}_j)\text{-fuzzy } (r, s)\text{-preopen}\}.$$

Definition 2.8 [5, 6]**.** Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is said to be

(1) a *fuzzy pairwise* (r, s)*-continuous* mapping if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is a fuzzy r-continuous mapping and the induced mapping $f(X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is a fuzzy s-continuous mapping,

(2) a *fuzzy pairwise* (r, s)*-semicontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy (r, s)-semiopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy (s, r)-semiopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y,

(3) a *fuzzy pairwise* (r, s)*-precontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy (r, s)-preopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy (s, r)-preopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y.

3. Fuzzy Pairwise Almost (r, s)-continuous, Fuzzy Pairwise Almost (r, s)-open and Fuzzy Pairwise Almost (r, s)-closed Mappings

Definition 3.1. Let μ be a fuzzy set in a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular open* if $\mathcal{T}_i$-Int$(\mathcal{T}_j$-Cl$(\mu, s), r) = \mu$,

(2) $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular closed* if $\mathcal{T}_i$-Cl$(\mathcal{T}_j$-Int$(\mu, s), r) = \mu$.

Theorem 3.2. *Let* μ *be a fuzzy set in a smooth bitopological space* $(X, \mathcal{T}_1, \mathcal{T}_2)$ *and* $r, s \in I_0$. *Then* μ *is* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular open if and only if* μ^c *is* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular closed.*

Proof. It follows from Lemma 2.2. □

Theorem 3.3. (1) *The intersection of two* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular open sets is a* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular open set.*

(2) *The union of two* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular closed sets is a* $(\mathcal{T}_i, \mathcal{T}_j)$-*fuzzy* (r, s)-*regular closed set.*

Proof. (1) Let μ and ρ be any $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open sets in a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then μ and ρ are $\mathcal{T}_i$-fuzzy r-open sets and hence $\mathcal{T}_i(\mu \wedge \rho) \geq \mathcal{T}_i(\mu) \wedge \mathcal{T}_i(\rho) \geq r$. Thus $\mu \wedge \rho$ is a $\mathcal{T}_i$-fuzzy r-open set. Since $\mu \wedge \rho \leq \mathcal{T}_j$-Cl$(\mu \wedge \rho, s)$,

$$\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu \wedge \rho, s), r) \geq \mathcal{T}_i\text{-Int}(\mu \wedge \rho, r) = \mu \wedge \rho.$$

Now, $\mu \wedge \rho \leq \mu$ and $\mu \wedge \rho \leq \rho$ imply

$$\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu \wedge \rho, s), r) \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu, s), r) = \mu$$

and

$$\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu \wedge \rho, s), r) \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r) = \rho.$$

Thus $\mathcal{T}_i$-Int$(\mathcal{T}_j$-Cl$(\mu \wedge \rho, s), r) \leq \mu \wedge \rho$. Therefore

$$\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu \wedge \rho, s), r) = \mu \wedge \rho$$

and hence $\mu \wedge \rho$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set.

(2) Let μ and ρ be any $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular closed sets in a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then μ and ρ are $\mathcal{T}_i$-fuzzy r-closed sets and hence μ^c and ρ^c are $\mathcal{T}_i$-fuzzy r-open sets. So, $\mathcal{T}_i((\mu \vee \rho)^c) = \mathcal{T}_i(\mu^c \wedge \rho^c) \geq \mathcal{T}_i(\mu^c) \wedge \mathcal{T}_i(\rho^c) \geq r$ and hence $\mu \vee \rho$ is a $\mathcal{T}_i$-fuzzy r-closed set. Since $\mathcal{T}_j$-Int$(\mu \vee \rho, s) \leq \mu \vee \rho$,

$$\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu \vee \rho, s), r) \leq \mathcal{T}_i\text{-Cl}(\mu \vee \rho, r) = \mu \vee \rho.$$

Now, $\mu \vee \rho \geq \mu$ and $\mu \vee \rho \geq \rho$ imply

$$\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu \vee \rho, s), r) \geq \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu, s), r) = \mu$$

and

$$\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu \vee \rho, s), r) \geq \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\rho, s), r) = \rho.$$

Thus $\mathcal{T}_i$-Cl$(\mathcal{T}_j$-Int$(\mu \vee \rho, s), r) \geq \mu \vee \rho$. Therefore

$$\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu \vee \rho, s), r) = \mu \vee \rho$$

and hence $\mu \vee \rho$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular closed set. □

Theorem 3.4. (1) *The $\mathcal{T}_i$-fuzzy r-interior of $\mathcal{T}_j$-fuzzy s-closed set is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set.*

(2) *The $\mathcal{T}_i$-fuzzy r-closure of $\mathcal{T}_j$-fuzzy s-open set is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular closed set.*

Proof. (1) Let μ be a $\mathcal{T}_j$-fuzzy s-closed set in a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then clearly $\mathcal{T}_j$-Cl$(\mathcal{T}_i$-Int$(\mu, r), s) \geq \mathcal{T}_i$-Int$(\mu, r)$ implies that

$$\begin{aligned}\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r) &\geq \mathcal{T}_i\text{-Int}(\mathcal{T}_i\text{-Int}(\mu, r), r)\\ &= \mathcal{T}_i\text{-Int}(\mu, r).\end{aligned}$$

Since μ is a $\mathcal{T}_j$-fuzzy s-closed set, $\mu = \mathcal{T}_j$-Cl(μ, s). Also since $\mathcal{T}_i$-Int(μ, r) $\leq \mu$, $\mathcal{T}_j$-Cl$(\mathcal{T}_i$-Int$(\mu, r), s) \leq \mathcal{T}_j$-Cl$(\mu, s) = \mu$. Thus

$$\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r) \leq \mathcal{T}_i\text{-Int}(\mu, r).$$

Therefore

$$\mathcal{T}_i\text{-Int}(\mu, r) = \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r)$$

and hence $\mathcal{T}_i\text{-Int}(\mu, r)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set.

(2) Let ρ be a $\mathcal{T}_j$-fuzzy s-open set in a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then clearly $\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\rho, r), s) \leq \mathcal{T}_i\text{-Cl}(\rho, r)$ implies that

$$\begin{aligned} \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\rho, r), s), r) &\leq \mathcal{T}_i\text{-Cl}(\mathcal{T}_i\text{-Cl}(\rho, r), r) \\ &= \mathcal{T}_i\text{-Cl}(\rho, r). \end{aligned}$$

Since ρ is a $\mathcal{T}_j$-fuzzy s-open set, $\rho = \mathcal{T}_j\text{-Int}(\rho, s)$. Also since $\rho \leq \mathcal{T}_i\text{-Cl}(\rho, r)$, $\rho = \mathcal{T}_j\text{-Int}(\rho, s) \leq \mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\rho, r), s)$. Thus

$$\mathcal{T}_i\text{-Cl}(\rho, r) \leq \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\rho, r), s), r).$$

Therefore

$$\mathcal{T}_i\text{-Cl}(\rho, r) = \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\rho, r), s), r)$$

and hence $\mathcal{T}_i\text{-Cl}(\rho, r)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular closed set. □

Definition 3.5. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to a smooth bitopological space Y and $r, s \in I_0$. Then f is called

(1) a *fuzzy pairwise almost* (r, s)*-continuous* mapping if $f^{-1}(\mu)$ is a $\mathcal{T}_i$-fuzzy r-open set of X for each $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-regular open set μ of Y,

(2) a *fuzzy pairwise almost* (r, s)*-open* mapping if $f(\rho)$ is a $\mathcal{U}_i$-fuzzy r-open set of Y for each $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set ρ of X,

(3) a *fuzzy pairwise almost* (r, s)*-closed* mapping if $f(\rho)$ is a $\mathcal{U}_i$-fuzzy r-closed set of Y for each $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular closed set ρ of X.

Theorem 3.6. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping and* $r, s \in I_0$. *Then the following statements are equivalent*:

(1) *f is a fuzzy pairwise almost* (r, s)*-continuous mapping.*

(2) $f^{-1}(\mu) \leq \mathcal{T}_i\text{-Int}(f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)), r)$ *for each* $\mathcal{U}_i$*-fuzzy r-open set* μ *of Y.*

(3) $\mathcal{T}_i\text{-Cl}(f^{-1}(\mathcal{U}_i\text{-Cl}(\mathcal{U}_j\text{-Int}(\mu, s), r)), r) \leq f^{-1}(\mu)$ *for each* $\mathcal{U}_i$*-fuzzy r-closed set* μ *of Y.*

Proof. (1) $\Rightarrow$ (2) Let f be a fuzzy pairwise almost (r, s)-continuous mapping and μ be a $\mathcal{U}_i$-fuzzy r-open set of Y. Since μ is $\mathcal{U}_i$-fuzzy r-open and $\mu \leq \mathcal{U}_j\text{-Cl}(\mu, s)$,

$$\mu = \mathcal{U}_i\text{-Int}(\mu, r) \leq \mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r).$$

By Theorem 3.4(1), $\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)$ is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-regular open set of Y. Since f is a fuzzy pairwise almost (r, s)-continuous mapping, $f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r))$ is a $\mathcal{T}_i$-fuzzy r-open set of X. Hence

$$\begin{aligned} f^{-1}(\mu) &\leq f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)) \\ &= \mathcal{T}_i\text{-Int}(f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)), r). \end{aligned}$$

(2) $\Rightarrow$ (3) Let μ be a $\mathcal{U}_i$-fuzzy r-closed set of Y. Then μ^c is a $\mathcal{U}_i$-fuzzy r-open set of Y. By (2),

$$f^{-1}(\mu^c) \leq \mathcal{T}_i\text{-Int}(f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu^c, s), r)), r).$$

Hence

$$\begin{aligned} f^{-1}(\mu) = (f^{-1}(\mu^c))^c &\geq (\mathcal{T}_i\text{-Int}(f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu^c, s), r)), r))^c \\ &= \mathcal{T}_i\text{-Cl}(f^{-1}(\mathcal{U}_i\text{-Cl}(\mathcal{U}_j\text{-Int}(\mu, s), r)), r). \end{aligned}$$

(3) $\Rightarrow$ (1) Let μ be a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-regular open set of Y. Then $\mu = \mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)$. Since μ^c is a $(\mathcal{U}_i, \mathcal{U}_j)$-fuzzy (r, s)-regular closed set of Y, μ^c is a $\mathcal{U}_i$-fuzzy r-closed set of Y. By (3),

$$\mathcal{T}_i\text{-Cl}(f^{-1}(\mathcal{U}_i\text{-Cl}(\mathcal{U}_j\text{-Int}(\mu^c, s), r)), r) \leq f^{-1}(\mu^c).$$

Hence

$$\begin{aligned} f^{-1}(\mu) = (f^{-1}(\mu^c))^c &\leq (\mathcal{T}_i\text{-Cl}(f^{-1}(\mathcal{U}_i\text{-Cl}(\mathcal{U}_j\text{-Int}(\mu^c, s), r)), r))^c \\ &= \mathcal{T}_i\text{-Int}(f^{-1}(\mathcal{U}_i\text{-Int}(\mathcal{U}_j\text{-Cl}(\mu, s), r)), r) \\ &= \mathcal{T}_i\text{-Int}(f^{-1}(\mu), r) \\ &\leq f^{-1}(\mu). \end{aligned}$$

Thus $f^{-1}(\mu) = \mathcal{T}_i\text{-Int}(f^{-1}(\mu), r)$ and hence $f^{-1}(\mu)$ is a $\mathcal{T}_i$-fuzzy r-open set of X. Therefore f is a fuzzy pairwise almost (r, s)-continuous mapping. □

Theorem 3.7. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping and* $r, s \in I_0$. *Then f is a fuzzy pairwise almost* (r, s)*-open mapping if and only if* $f(\mathcal{T}_i\text{-Int}(\rho, r)) \leq \mathcal{U}_i\text{-Int}(f(\rho), r)$ *for each* $(\mathcal{T}_j, \mathcal{T}_i)$*-fuzzy* (s, r)*-semiclosed set* ρ *of X.*

Proof. Let f be a fuzzy pairwise almost (r, s)-open mapping and ρ be a $(\mathcal{T}_j, \mathcal{T}_i)$-fuzzy (s, r)-semiclosed set of X. Then

$$\mathcal{T}_i\text{-Int}(\rho, r) \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r) \leq \rho.$$

Note that $\mathcal{T}_j\text{-Cl}(\rho, s)$ is a $\mathcal{T}_j$-fuzzy s-closed set of X. By Theorem 3.4(1), $\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r)$ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set of X. Since f is a fuzzy pairwise almost (r, s)-open mapping, $f(\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r))$ is a $\mathcal{U}_i$-fuzzy r-open set of Y. Thus we have

$$\begin{aligned} f(\mathcal{T}_i\text{-Int}(\rho, r)) &\leq f(\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r)) \\ &= \mathcal{U}_i\text{-Int}(f(\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r)), r) \\ &\leq \mathcal{U}_i\text{-Int}(f(\rho), r). \end{aligned}$$

Conversely, let ρ be a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-regular open set of X. Then ρ is $\mathcal{T}_i$-fuzzy r-open and hence $\mathcal{T}_i\text{-Int}(\rho, r) = \rho$. Since $\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r) = \rho$, ρ is a $(\mathcal{T}_j, \mathcal{T}_i)$-fuzzy (s, r)-semiclosed set of X. So,

$$f(\rho) = f(\mathcal{T}_i\text{-Int}(\rho, r)) \leq \mathcal{U}_i\text{-Int}(f(\rho), r) \leq f(\rho).$$

Thus $f(\rho) = \mathcal{U}_i\text{-Int}(f(\rho),\ r)$ and hence $f(\rho)$ is a U_i-fuzzy r-open set of Y. Therefore f is a pairwise almost $(r,\ s)$-continuous mapping. □

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