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SOLUTION OF COUPLED NONLINEAR KLEIN-GORDON-SCHRÖDINGER EQUATIONS USING HOMOTOPY-PERTURBATION METHOD

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Abstract

In this paper, we apply the homotopy-perturbation method (HPM) and obtain approximate analytical solutions of the coupled nonlinear Klein-Gordon-Schrödinger equations. The results show that HPM is very effective.

1. Introduction

It is well known that many phenomena in scientific fields can be described by nonlinear partial differential equations. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. There has recently been much attention devoted to the search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models [2].

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Consider the coupled nonlinear Klein-Gordon-Schrödinger equations

$$u_t t - u_{xx} + u - |v|^2 = 0, \quad (1)$$

$$iv_t + v_{xx} + uv = 0, \quad (2)$$

with an initial condition

$$u(x, 0) = 6B^2(\operatorname{sech}(Bx))^2,$$

$$u_t(x, 0) = 12B^2 c(\operatorname{sech}(Bx))^2 \tanh(Bx),$$

$$v(x, 0) = 3B(\operatorname{sech}(Bx))^2 e^{i\alpha x}, \quad (3)$$

where $B \geq 1/2$, c and α are arbitrary constants. Darwish and Fan [4] have been applied an algebraic method to obtain the explicit exact solutions for coupled Klein-Gordon-Schrödinger (KGS) equations. The Jacobi elliptic function expansion method has been proposed to obtain the solitary wave solutions for coupled KGS equations [5]. Bao and Yang [1] have presented efficient, unconditionally stable and accurate numerical methods for approximations of the Klein-Gordon-Schrödinger equations. Recently, Saha Ray [13] implemented the modified decomposition method for solving the coupled Klein-Gordon-Schrödinger equation.

Another powerful analytical method is Homotopy-perturbation method (HPM), application of HPM in nonlinear problems has been presented by many researchers [3, 6-12, 14].

This paper investigates for the first time the applicability and effectiveness of HPM on coupled nonlinear Klein-Gordon-Schrödinger equations, the results prove that HPM is effective and simple than modified decomposition method because we do not need to calculate the Adomian polynomial.

2. Solution Procedure

First write system (1) and (2) in the operator form

$$L_1(u) + N_1(u, v) = 0, \quad (4)$$

$$L_2(v) + N_2(u, v) = 0 \quad (5)$$

subject to the initial conditions (3), where $L_1 = \dfrac{\partial}{\partial t^2}$, $L_2 = \dfrac{\partial}{\partial t}$ and N_1, N_2 are the nonlinear operators. We shall next present the solution approaches for (4) and (5) based on the HPM.

2.1. Solution by HPM

According to HPM, we construct a homotopy for (4) and (5) which satisfies the following relations:

$$L_1(u) - L_1(u_0) + pL_1(u_1) + p[N_1(u, v)] = 0, \tag{6}$$

$$L_2(v) - L_2(v_0) + pL_1(v_1) + p[N_1(u, v)] = 0, \tag{7}$$

where $p \in [0, 1]$ is an embedding parameter and v_1, v_2 are initial approximateness satisfying the given conditions. It is obvious that when the perturbation parameter $p = 0$, equations (6) and (7) become a linear system and when $p = 1$ we get the original nonlinear system.

Let us take the initial approximations as follows:

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \cdots, \tag{8}$$

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots. \tag{9}$$

Substituting equations (8) and (9) into equations (6) and (7) and arranging the coefficients of the same powers of p, we get

$$\frac{\partial^2 u_0}{\partial t^2} = 0,$$

$$u_0(x, 0) = 6B^2(\operatorname{sech}(Bx))^2, \left.\frac{\partial u_0(x, t)}{\partial t}\right|_{t=0} = 12B^2c(\operatorname{sech}(Bx))^2 \tanh(Bx),$$

$$\frac{\partial v_0}{\partial t} = 0, \quad v_0(x, 0) = 3B(\operatorname{sech}(Bx))^2 e^{i\alpha x},$$

$$\frac{\partial^2}{\partial t^2} u_1 - \frac{\partial^2}{\partial x^2} u_0 + u_0 - \overline{v}_0 v_0, \quad u_1(x, 0) = 0, \left.\frac{\partial u_1(x, t)}{\partial t}\right|_{t=0}$$

$$i\frac{\partial}{\partial t}v_1 + \frac{\partial^2}{\partial x^2}v_0 + u_0v_0, \quad v_1(x, 0) = 0,$$

$$\frac{\partial^2}{\partial t^2}u_2 - \frac{\partial^2}{\partial x^2}u_1 - cv_0v_1 - cv_1v_0 + u_1,$$

$$u_2(x, 0) = 0, \quad \left.\frac{\partial u_2(x, t)}{\partial t}\right|_{t=0} = 0$$

$$i\frac{\partial}{\partial t}v_2(x, t) + \frac{\partial^2}{\partial x^2}v_1 + u_0v_1 + u_1v_0, \quad v_2(x, 0) = 0,$$

etc. We solve the above systems of equations for the unknowns u_i and v_i $i = 0, 1, 2, \ldots,$ we obtain

$$u_0(x, t) = 6B^2(\operatorname{sech}(Bx))^2 - 12tB^2c(\operatorname{sech}(Bx))^2\tanh(Bx),$$

$$v_0(x, t) = 3B(\operatorname{sech}(Bx))^2e^{i\alpha x},$$

$$u_1(x, t) = -2\frac{\sinh(Bx)(4B^2(\cosh(Bx))^2 - 12B^2 - (\cosh(Bx))^2)t^3B^2c}{(\cosh(Bx))^5}$$

$$+\frac{1}{2}(36B^4(\operatorname{sech}(Bx))^2(\tanh(Bx))^2 - 12B^4(\operatorname{sech}(Bx))^2$$

$$+ 9B^2(\operatorname{sech}(Bx))^4 - 6B^2(\operatorname{sech}(Bx))^2)t^2,$$

$$v_1(x, t) = -3\frac{Be^{i\alpha x}i\alpha^2t}{(\cosh(Bx))^2} - 12\frac{B^3e^{i\alpha x}t}{(\cosh(Bx))^2i}$$

$$+12\frac{B^2e^{i\alpha x}\alpha\sinh(Bx)t}{(\cosh(Bx))^3} + 18\frac{B^3e^{i\alpha x}t^2c\sinh(Bx)}{(\cosh(Bx))^5i},$$

$$u_2(x, t) = -\frac{8}{5}\frac{B^6t^5c\sinh(Bx)}{(\cosh(Bx))^3} + \frac{4}{5}\frac{t^5B^4c\sinh(Bx)}{(\cosh(Bx))^3} - \frac{1}{102}\frac{B^2t^5\sinh(Bx)c}{(\cosh(Bx))^3}$$

$$-\frac{12}{5}\frac{t^5B^4c\sinh(Bx)}{(\cosh(Bx))^5} - 36\frac{B^6t^5c\sinh(Bx)}{(\cosh(Bx))^7} + 24\frac{B^6t^5c\sinh(Bx)}{(\cosh(Bx))^5}$$

$$+30\frac{B^6t^4}{(\cosh(Bx))^6}-\frac{15}{2}\frac{B^4t^4}{(\cosh(Bx))^6}+9\frac{B^4t^4}{(\cosh(Bx))^4}$$

$$-2\frac{B^4t^4}{(\cosh(Bx))^2}-\frac{3}{8}\frac{B^2t^4}{(\cosh(Bx))^4}+4\frac{B^6t^4}{(\cosh(Bx))^2}$$

$$-30\frac{B^6t^4}{(\cosh(Bx))^4}+\frac{1}{4}\frac{B^2t^4}{(\cosh(Bx))^2}+12\frac{B^3\alpha\sinh(Bx)t^3}{(\cosh(Bx))^5},$$

$$v_2(x,t)=-54\frac{B^5e^{i\alpha x}t^4c^2}{(\cosh(Bx))^8i^2}+6\frac{B^5e^{i\alpha x}t^4\sinh(Bx)c}{(\cosh(Bx))^5i}+54\frac{B^5e^{i\alpha x}t^4c^2}{(\cosh(Bx))^6i^2}$$

$$-\frac{3}{2}\frac{B^3e^{i\alpha x}t^4\sinh(Bx)c}{(\cosh(Bx))^5i}-18\frac{B^5e^{i\alpha x}t^4\sinh(Bx)c}{(\cosh(Bx))^7i}-108\frac{B^4e^{i\alpha x}t^3\alpha c}{(\cosh(Bx))^6i}$$

$$-18\frac{B^3e^{i\alpha x}t^3\alpha^2c\sinh(Bx)}{(\cosh(Bx))^5}-\frac{9}{2}\frac{B^3e^{i\alpha x}t^3}{(\cosh(Bx))^6i}-144\frac{B^5e^{i\alpha x}t^3\sinh(Bx)c}{(\cosh(Bx))^5i^2}$$

$$-12\frac{B^5e^{i\alpha x}t^3}{(\cosh(Bx))^4i}+3\frac{B^3e^{i\alpha x}t^3}{(\cosh(Bx))^4i}+18\frac{B^5e^{i\alpha x}t^3}{(\cosh(Bx))^6i}$$

$$+144\frac{B^5e^{i\alpha x}t^3\sinh(Bx)c}{(\cosh(Bx))^7i^2}+96\frac{B^4e^{i\alpha x}t^3\alpha c}{(\cosh(Bx))^4i}+24\frac{B^5e^{i\alpha x}t^2}{(\cosh(Bx))^2i^2}$$

$$+\frac{3}{2}\frac{Be^{i\alpha x}i^2t^2\alpha^4}{(\cosh(Bx))^2}-36\frac{B^3e^{i\alpha x}t^2\alpha^2}{(\cosh(Bx))^4}-48\frac{B^4e^{i\alpha x}t^2\alpha\sinh(Bx)}{(\cosh(Bx))^3i}$$

$$-12\frac{B^2e^{i\alpha x}it^2\alpha^3\sinh(Bx)}{(\cosh(Bx))^3}+36\frac{B^3e^{i\alpha x}t^2\alpha^2}{(\cosh(Bx))^2}+36\frac{B^4e^{i\alpha x}t^2\alpha\sinh(Bx)}{(\cosh(Bx))^5i}.$$

Therefore, according to HPM the n-term approximations for the solutions of (6) and (7) can be expressed as

$$\phi_n(x,t)=\lim_{p\to 1}u(x,t)=\sum_{k=0}^{n-1}u_0(x,t), \tag{10}$$

$$\varphi_n(x,t)=\lim_{p\to 1}v(x,t)=\sum_{k=0}^{n-1}v_0(x,t), \tag{11}$$

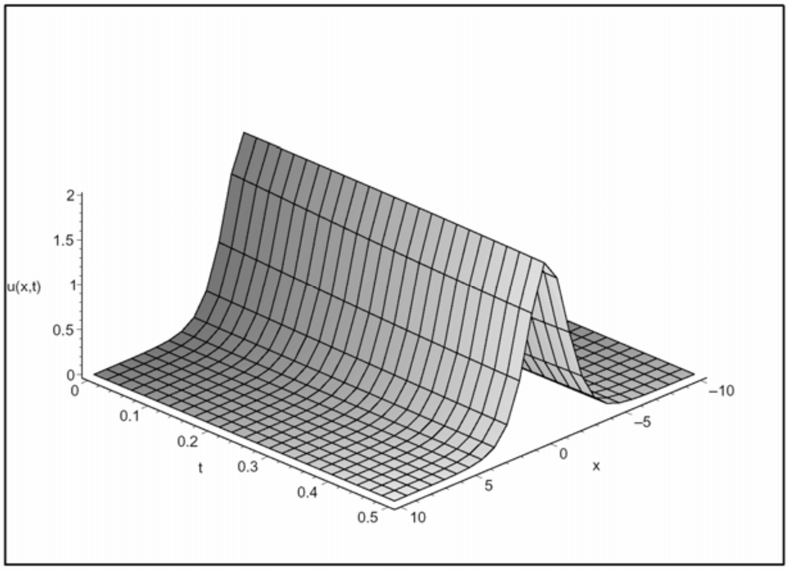
3. Numerical Results and Discussion

In the present numerical experiment, equations (10) and (11) have been used to draw the graphs as shown in Figures (1)-(4), respectively.

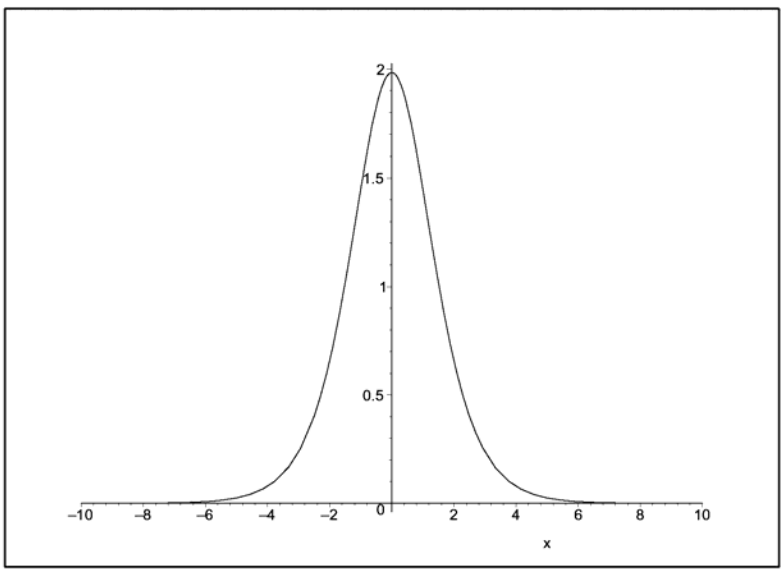
The numerical solutions of the coupled KGS equations (1)-(3) have been shown in Figures (1)-(4) with the help of four-term approximations ϕ_4 and φ_4 for the series solutions of $u(x, t)$ and $v(x, t)$, respectively. In the present numerical computation we have assumed $B = 0.575$, $c = \sqrt{4B^2 - 1}/2$ and $\alpha = -c/2B$. Figures (1)-(4) have been drawn using the Maple software.

4. Conclusions

In this paper, the HPM was used for finding the solutions for the coupled KGS equations with initial conditions. The approximate solutions to the equations have been calculated by using the HPM without any need to a transformation techniques and linearization of the equations. Additionally, it does not need any discretization method to get numerical solutions. This method thus eliminates the difficulties and massive computation work. The HPM is straightforward, without restrictive assumptions and the components of the series solution can be easily computed using any mathematical symbolic package. Moreover, this method does not change the problem into a convenient one for the use of linear theory. It, therefore, provides more realistic series solutions that generally converge very rapidly in real physical problems.

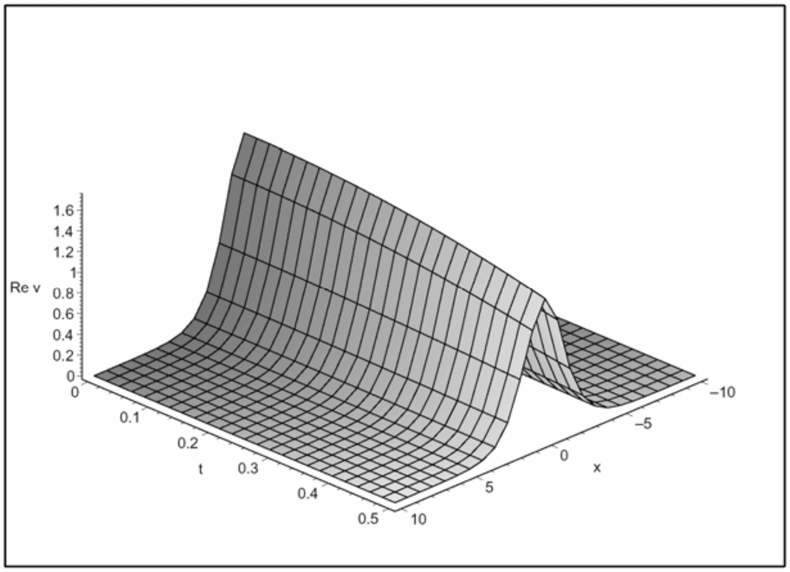


(a)



(b)

Figure 1. (a) The HPM solution for $u(x, t)$, (b) corresponding solution for $u(x, t)$ when $t = 0$.



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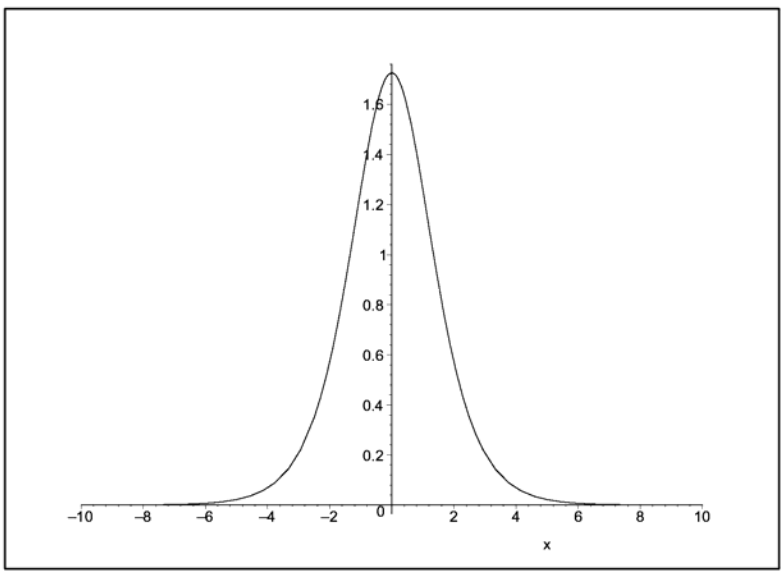
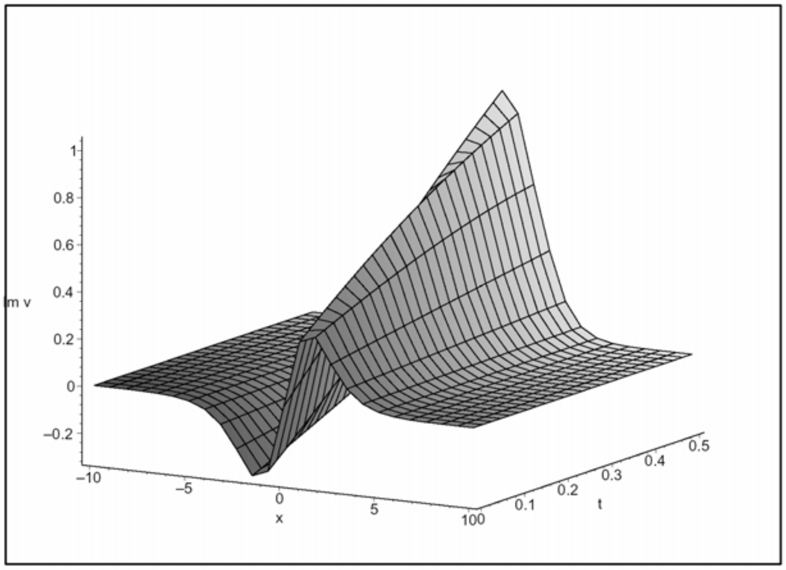


(b)

Figure 2. (a) The HPM solution for $\mathrm{Re}(v(x, t))$, (b) corresponding solution for $\mathrm{Re}(v(x, t))$ when $t = 0$.



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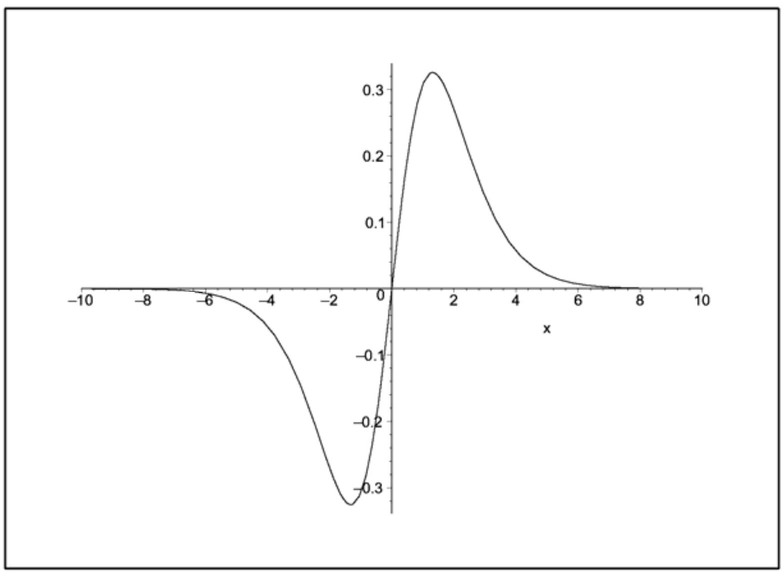
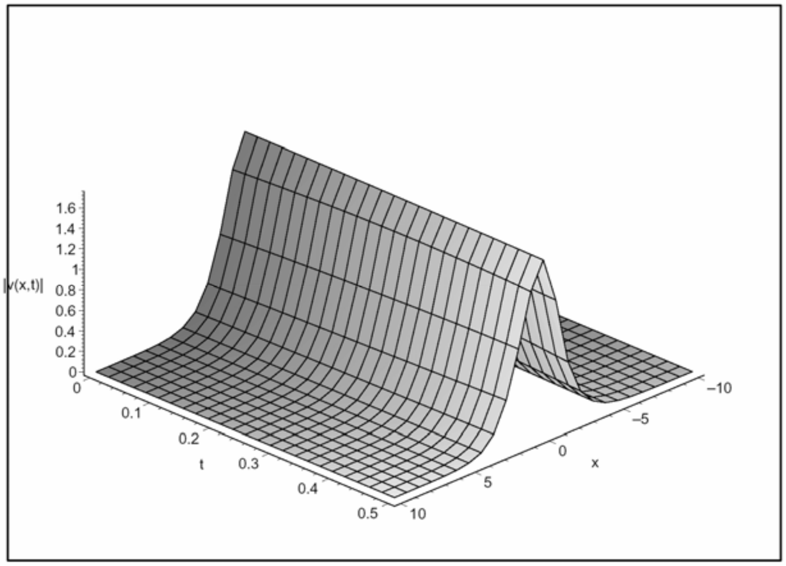
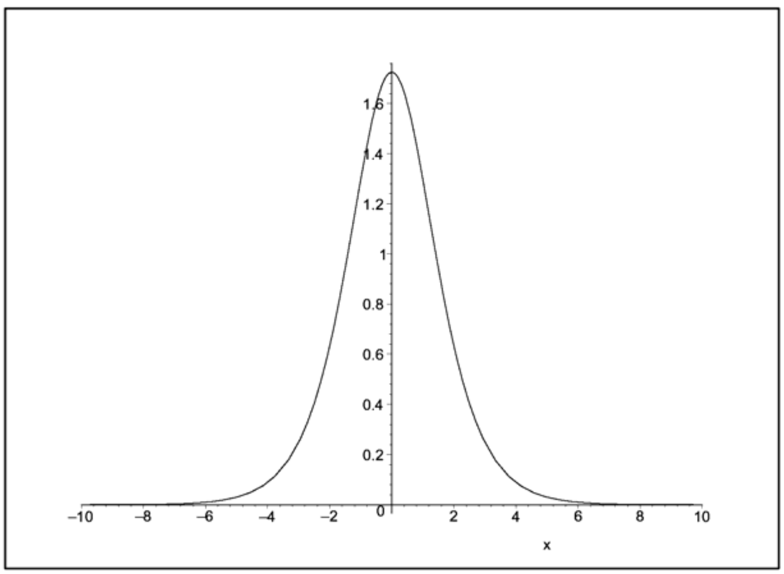


(b)

Figure 3. (a) The HPM solution for $\mathrm{Im}(v(x, t))$, (b) corresponding solution for $\mathrm{Im}(v(x, t))$ when $t = 0$.



(a)



(b)

Figure 4. (a) The HPM solution for $|v(x, t)|$, (b) corresponding solution for $|v(x, t)|$ when $t = 0$.

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