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MEAN SQUARE EXPONENTIAL STABILITY OF UNCERTAIN NONLINEAR TIME DELAY NEUTRAL SYSTEMS WITH MARKOVIAN JUMPING PARAMETERS

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Abstract

This paper studies the problem of mean square exponential stability (MSES) for a class of uncertain nonlinear time delay neutral systems with Markovian jumping parameters by the method of constructing Lyapunov-Krasovskii function. The system under consideration is subjected to time delay in the state and bounded nonlinear term. Our attention is to obtain sufficient conditions which ensure the system is MSES and shows that a robust stabilizing state feedback controller can be constructed through solving a couple of linear matrix inequalities.

1. Introduction

A great deal of attention has been devoted to the Markovian linear

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systems [1, 2, 4] and nonlinear systems [3, 9] in recent years due to the fact that Markovian jump systems can model systems with abrupt structural variation from the occurrence of some inner discrete events in the systems such as failures and repairs of machine, etc. Therefore, the study of Markovian jump systems is of both theoretical and practical value. [6] regards affection of the uncertainty, stochastic perturbation and time lag to exponential stable of stochastic linear differential equation by some important inequalities. Mariton [7] obtained a sufficient condition on the mean square stability for linear systems without delay by the stochastic Lyapunov function approach.

However, there are less results associated with stability of neutral system with Markovian jumping parameters. In this correspondence, we study the MSES and design method of robust stochastic stabilizing state feedback controller for a class of uncertain nonlinear time delay neutral systems with Markovian jumping parameters by the stochastic Lyapunov functional approach first developed by Kushner [5].

Notation: The notations of this paper are quite standard. R^n and $R^{n\times m}$, respectively, denote the n dimensional Euclidean space and the set of all $n \times m$ dimensional matrices. The superscript "T" expresses the transpose and the notation $X \geq Y$ (respectively, $X > Y$), where X, Y are symmetric matrices, means that $X - Y$ is symmetric semidefinite (respectively, positive definite). "$\star$" means the symmetric elements about main diagonal, $L^2([-h, 0], R^n)$ is a set of all square integral vector functions from $[-h, 0]$ to R^n, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ stand for the maximum and minimum eigenvalue of the corresponding matrix, respectively, $E[\cdot]$ represses the mathematical expectation operator. Throughout this paper, all of matrices are with appropriate dimensions.

2. System Description

We shall consider the following system:

$$\dot{x}(t) + G\dot{x}(t - h_0) = \hat{A}(r(t))x(t) + \hat{A}_1(r(t))x(t - h_1) + B(r(t))u(t) + \hat{f}(t), \quad (1a)$$

$$x(t) = \eta(t), \quad t \in [-\hat{h}, 0], \quad (1b)$$

where $x(t) \in R^n$ denotes state vector, $u(t) \in R^m$ denotes the control input vector, $r(t)$ is continuous-time markov chain with right continuous taking values in a finite state set of $S = \{1, 2, \ldots, N\}$, with transition probability matrix $\Pi = \{\pi_{ij}\}$ given by

$$\Pr\{r(t+h) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}h + o(h) & i \neq j, \\ 1 + \pi_{ii}h + o(h) & i = j, \end{cases} \tag{2}$$

where

$$h > 0, \ \lim_{h \to 0} \frac{o(h)}{h} = 0, \tag{3}$$

and $\pi_{ij} > 0$, $i \neq j$, $i, j \in S$, is the transition rate from mode i at time t to mode j at time $t + h$, which satisfies

$$\pi_{ii} = -\sum_{j=1,\ j \neq i}^{N} \pi_{ij}, \tag{4}$$

$\hat{A}(r(t))$ and $\hat{A}_1(r(t))$ are time-varying uncertain matrices with

$$\hat{A}(r(t)) = A(r(t)) + E(r(t))F(r(t))H(r(t)),\ \hat{A}_1(r(t))$$
$$= A_1(r(t)) + E(r(t))F(r(t))H_1(r(t)),$$

where $E(r(t))$, $H(r(t))$, $H_1(r(t))$ are known constant matrices with suitable dimensions and depend on Markovian chain $r(t)$, $F(r(t))$ is unknown matrix with Lebesgue measurable elements satisfying

$$F^T(r(t))F(r(t)) \leq I, \quad \forall r(t) \in S,\ t > 0.$$

The nonlinear term $\hat{f}(t)$ in system (1) is continuous differential function, furthermore,

$$\hat{f}^T(t)\hat{f}(t) \leq \alpha_0 x^T(t - h_0)M_{0i}^T M_{0i} x(t - h_0) + \alpha_1 x^T(t - h_1)M_{1i}^T M_{1i} x(t - h_1)$$
$$+ \alpha_2 x^T(t) M_{2i}^T M_{2i} x(t), \tag{5}$$

where $M_{ji} := M_j(r(t))$, $j = 0, 1, 2$, $i \in S$, are known constant matrices.

Constants $h_0 > 0$ and $h_1 > 0$ denote the time delay of system (1), $\hat{h} = \max\{h_0, h_1\}$, $\eta(t) \in L^2([-\hat{h}, 0], R^n)$ is continuous initial vector function.

For notational simplicity, let $r(t) := i$ for each possible $r(t) \in S$.

Lemma 1 (see [8]). *Let M, N and F be matrices with suitable dimensions and $F^T F \leq I$. Then, for any scalar $\nu > 0$, we have*

$$MFN + N^T F^T M^T \leq \nu MM^T + \nu^{-1} N^T N.$$

Definition 1. The equilibrium point 0 of nominal system (1) (i.e., $\hat{A}(r(t)) = A(r(t)), \hat{A}_1(r(t)) = A_1(r(t)), u(t) = 0$, for $r(t) \in S$) is mean square asymptotically stability for all initial $\eta(t)$ and $r(0)$ if

$$\lim_{t \to \infty} E|\, x(t;\, \eta(0),\, r(0))\,|^2 = 0, \tag{6}$$

and is MSES if there exist positive scalars $\alpha > 0$ and $\beta > 0$ such that

$$E|\, x(t;\, \eta(0),\, r(0))\,|^2 \leq \alpha e^{-\beta t} \sup_{-\hat{h} \leq \vartheta \leq 0} E|\, \eta(\vartheta)\,|^2, \tag{7}$$

where $x(t;\, \eta(0),\, r(0))$ expresses the state trajectory of system (1) with initial value $(\eta(0),\, r(0))$.

The attention of this note is to investigate some sufficient conditions which guarantee system (1) with $u(t) = 0$ is MSES and design state feedback controller such that resulting closed-loop system is MSES.

3. Main Results

In this section, we state some main results of our paper. The following theorem presents a sufficient condition under which nominal system of (1), i.e.,

$$\dot{x}(t) + G\dot{x}(t - h_0) = A(r(t))x(t) + A_1(r(t))x(t - h_1) + \hat{f}(t) \tag{8}$$

is MSES.

Theorem 1. *The nominal system* (8) *is MSES if there exist some positive symmetric matrices* P_i, $i \in S$, *semi-positive symmetric matrices* N_0, N_1, *and scalars* $\beta > 0$, $\gamma > 0$, *such that*

$$\Phi = \begin{bmatrix} \overline{\Phi} & \Phi_{12} & \Phi_{13} \\ \star & -\gamma I & 0 \\ \star & \star & -\beta I \end{bmatrix} < 0 \tag{9}$$

holds, where

$$\overline{\Phi} = \begin{bmatrix} \overline{\Phi}_{11} & A_i^T P_i G + \sum_{j=1}^{N} \pi_{ij} P_j G & P_i A_{1i} \\ \star & \overline{\Phi}_{22} & G^T P_i^T A_{1i} \\ \star & \star & \overline{\Phi}_{33} \end{bmatrix},$$

$$\Phi_{12}^T = [P_i \quad 0 \quad 0], \quad \Phi_{13}^T = [0 \quad P_i G \quad 0],$$

$$\overline{\Phi}_{11} = A_i^T P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij} P_j + (\beta + \gamma)\alpha_2 M_{2i}^T M_{2i} + N_0 + N_1,$$

$$\overline{\Phi}_{22} = \sum_{j=1}^{N} \pi_{ij} G^T P_j G - N_0 + (\beta + \gamma)\alpha_0 M_{0i}^T M_{0i},$$

$$\overline{\Phi}_{33} = (\beta + \gamma)\alpha_1 M_{1i}^T M_{1i} - N_1.$$

Proof. Let

$$V(x(t), r(t)) = (x(t) + Gx(t - h_0))^T P_i (x(t) + Gx(t - h_0)) + \int_{t-h_0}^{t} x^T(s) N_0 x(s) ds + \int_{t-h_1}^{t} x^T(s) N_1 x(s) ds,$$

where P_i, $i \in S$, N_0 and N_1 are solutions of (9), define infinitesimal operator as

$$\mathcal{L}V(x(t), r(t)) = \lim_{h \to 0^+} \frac{1}{h} [E\{V(x(t+h), r(t+h)) | x(t), r(t)\} - V(x(t), r(t))] \tag{10}$$

utilizing the operator $\mathcal{L}$ to function $V(x(t), r(t))$ can easily obtain:

$$\mathcal{L}V(x(t), r(t)) \leq \hat{x}^T(t)\hat{\Phi}\hat{x}(t), \tag{11}$$

where $\hat{x}^T(t) = [x^T(t) \quad x^T(t-h_0) \quad x^T(t-h_1)]$,

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & \hat{\Phi}_{13} \\ \star & \hat{\Phi}_{22} & \hat{\Phi}_{23} \\ \star & \star & \hat{\Phi}_{33} \end{bmatrix},$$

$$\hat{\Phi}_{11} = A_i^T P_i + P_i A_i + \sum_{j=1}^N \pi_{ij} P_j + (\beta + \gamma)\alpha_2 M_{2i}^T M_{2i} + \gamma^{-1} P_i^2 + N_0 + N_1,$$

$$\hat{\Phi}_{12} = A_i^T P_i G + \sum_{j=1}^N \pi_{ij} P_j G,$$

$$\hat{\Phi}_{13} = P_i A_{1i},$$

$$\hat{\Phi}_{22} = G^T \left(\sum_{j=1}^N \pi_{ij} P_j + \beta^{-1} P_i^2 \right) G + (\beta + \gamma)\alpha_0 M_{0i}^T M_{0i} - N_0,$$

$$\hat{\Phi}_{23} = G^T P_i^T A_{1i},$$

$$\hat{\Phi}_{33} = (\beta + \gamma)\alpha_1 M_{1i}^T M_{1i} - N_1.$$

In this process, we use the facts that there exist constants $\gamma > 0$, $\beta > 0$ satisfying

$$2\hat{f}^T(t) P_i x(t) \leq \gamma \hat{f}^T(t)\hat{f}(t) + \gamma^{-1} x^T(t) P_i^2 x(t) \tag{12}$$

and

$$2\hat{f}^T(t) P_i G x(t-h_0) \leq \beta \hat{f}^T(t)\hat{f}(t) + \beta^{-1} x^T(t-h_0) G^T P_i^2 G x(t-h_0). \tag{13}$$

Since $\hat{\Phi} = \overline{\Phi} + diag\{\gamma^{-1} P_i^2, \ \beta^{-1} G^T P_i^2 G, \ 0\}$, $\overline{\Phi}$ is the same as in Theorem 1, it is easy to get $\hat{\Phi} < 0$ by Schur complement lemma and (9).

We choose Lyapunov function candidate $W(x(t), r(t)) = e^{\varepsilon t} V(x(t), r(t))$ in order to investigate MSES of system (8), then

$$\mathcal{L}W(x(t), r(t)) = \varepsilon e^{\varepsilon t} V(x(t), r(t)) + e^{\varepsilon t} \mathcal{L}V(x(t), r(t)) \tag{14}$$

integral about time t from 0 to T on both sides of (14) for arbitrary $T > 0$ can obtain:

$$e^{\varepsilon T}V(x(T), r(T)) = V(x(0), r(0)) + \int_0^T \varepsilon e^{\varepsilon t}V(x(t), r(t))dt$$

$$+ \int_0^T e^{\varepsilon t}\mathcal{L}V(x(t), r(t))dt. \quad (15)$$

Next, we estimate each term of right side of equation (15), respectively. By the definition of function $V(x(r(t)), r(t))$, we can derive that

$$V(x(0), r(0)) \leq \{\lambda_{\max}[(I + G)^T P_i(I + G)] + h_0\lambda_{\max}(N_0)$$

$$+ h_1\lambda_{\max}(N_1)\} \sup_{-\hat{h}\leq\vartheta\leq 0}|\eta(\vartheta)|^2 \quad (16)$$

and

$$\varepsilon e^{\varepsilon t}V(x(t), r(t))$$

$$\leq 2\varepsilon e^{\varepsilon t} \cdot \lambda_{\max}(P_i)(x^T(t)x(t) + x^T(t - h_0)G^T Gx(t - h_0))$$

$$+ \varepsilon e^{\varepsilon t}\lambda_{\max}(N_0)\int_{t-h_0}^t x^T(s)x(s)ds + \varepsilon e^{\varepsilon t}\lambda_{\max}(N_1)\int_{t-h_1}^t x^T(s)x(s)ds. \quad (17)$$

In order to estimate the second term of right side of (15), we shall use the following inequalities

$$\int_0^T e^{\varepsilon t}\int_{t-h_0}^t x^T(s)x(s)dsdt$$

$$\leq h_0 e^{\varepsilon h_0}\int_0^T e^{\varepsilon t}|x(t)|^2 dt + h_0^2 e^{\varepsilon h_0} \sup_{-\hat{h}\leq\vartheta\leq 0}|\eta(\vartheta)|^2, \quad (18)$$

$$\int_0^T e^{\varepsilon t}\int_{t-h_1}^t x^T(s)x(s)dsdt$$

$$\leq h_1 e^{\varepsilon h_1}\int_0^T e^{\varepsilon t}|x(t)|^2 dt + h_1^2 e^{\varepsilon h_1} \sup_{-\hat{h}\leq\vartheta\leq 0}|\eta(\vartheta)|^2, \quad (19)$$

$$\int_0^T e^{\varepsilon t}x^T(t - h_0)x(t - h_0)dt \geq e^{\varepsilon h_0}\int_{-h_0}^0 e^{\varepsilon s}x^T(s)x(s)ds, \quad (20)$$

owing to $\hat{\Phi}(t) < 0$, hence

$$\int_0^T e^{\varepsilon t}\hat{x}^T(t)\hat{\Phi}(t)\hat{x}(t)dt \leq -\lambda_{\min}(-\hat{\Phi})e^{\varepsilon h_0}\int_{-h_0}^0 e^{\varepsilon s}x^T(s)x(s)ds$$

$$\leq h_0\lambda_{\min}(-\hat{\Phi})e^{\varepsilon h_0}\sup_{-\hat{h}\leq\vartheta\leq 0}|\eta(\vartheta)|^2, \tag{21}$$

similarly compute as before, we have

$$-\lambda_{\min}(-\hat{\Phi})\int_0^T e^{\varepsilon t}x^T(t-h_1)x(t-h_1)dt$$

$$\leq h_1\lambda_{\min}(-\hat{\Phi})e^{\varepsilon h_1}\sup_{-\hat{h}\leq\vartheta\leq 0}|\eta(\vartheta)|^2, \tag{22}$$

substituting (16)-(22) to (15) yield

$$e^{\varepsilon T}V(x(T), r(T)) \leq \Psi_1 \sup_{-\hat{h}\leq\varphi\leq 0}|\eta(\varphi)|^2 + \Psi_2\int_0^T e^{\varepsilon t}|x(t)|^2 dt, \tag{23}$$

where

$$\Psi_1 = \lambda_{\max}[(I+G)^T P_i(I+G)] + h_0\lambda_{\max}(N_0) + h_1\lambda_{\max}(N_1)$$

$$- h_0\lambda_{\min}(-\hat{\Phi})e^{\varepsilon h_0} - h_1\lambda_{\min}(-\hat{\Phi})e^{\varepsilon h_1} + 2h_0\varepsilon\lambda_{\max}(P_i)\cdot\lambda_{\max}(G^TG)$$

$$\cdot e^{\varepsilon h_0} + \varepsilon h_0^2 e^{\varepsilon h_0}\lambda_{\max}(N_0) + \varepsilon h_1^2 e^{\varepsilon h_1}\lambda_{\max}(N_1),$$

$$\psi_2 = -\lambda_{\min}(-\hat{\Phi}) + 2\varepsilon\lambda_{\max}(P_i) + \varepsilon\lambda_{\max}(N_0)\cdot h_0 e^{\varepsilon h_0}$$

$$+ \varepsilon\lambda_{\max}(N_1)\cdot h_1 e^{\varepsilon h_1} + 2\varepsilon\lambda_{\max}(P_i)\cdot\lambda_{\max}(G^TG)\cdot e^{\varepsilon h_0}.$$

Assume $\varepsilon > 0$ is the unique root of the equation $\Psi_2 = 0$, furthermore,

$$V(x(T), r(T)) \geq \lambda_{\min}(\hat{P}_i)x^T(T)x(T), \tag{24}$$

where

$$\hat{P}_i = \begin{bmatrix} P_i & P_iG \\ \star & G^TP_iG \end{bmatrix},$$

substituting (24) into (23) can get

$$x^T(T)x(T) \leq \lambda_{\min}^{-1}(\hat{P}_i)\Psi_1 e^{-\varepsilon T} \sup_{-\hat{h}\leq\varphi\leq 0} |\eta(\varphi)|^2,$$

by Definition 1 and T is arbitrary, system (8) is MSES.

Inequality (9) is a linear matrix inequality about matrices P_i, N_0, N_1 and can be solved easily by Matlab LMI control Toolbox.

Theorem 2. *The uncertain system*

$$\dot{x}(t) + G\dot{x}(t-h_0) = \hat{A}_i x(t) + \hat{A}_{1i}x(t-h_1) + \hat{f}(t) \tag{25}$$

is MSES if there exist some positive symmetric matrices P_i, *semi-positive symmetric matrices* N_0, N_1, *and scalars* $\sigma_{1i} > 0$ *for all* $i \in S$, *satisfying*

$$\Phi_\Delta = \begin{bmatrix} \Phi_{\Delta 11} & \Phi_{\Delta 12} & P_i A_{1i} + \sigma_{1i} H_i^T H_{1i} & P_i & 0 & P_i E_i \\ \star & \Phi_{\Delta 22} & G^T P_i A_{1i} & 0 & G^T P_i & G^T P_i E_i \\ \star & \star & \Phi_{\Delta 33} & 0 & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 & 0 \\ \star & \star & \star & \star & -\beta I & 0 \\ \star & \star & \star & \star & \star & -\sigma_{1i} I \end{bmatrix} < 0, \tag{26}$$

where

$$\Phi_{\Delta 11} = A_i^T P_i + P_i A_i + \sum_{j=1}^N \pi_{ij} P_j + \alpha_2(\beta+\gamma) M_{2i}^T M_{2i} + N_0 + N_1 + \sigma_{1i} H_i^T H_i,$$

$$\Phi_{\Delta 12} = A_i^T P_i G + \sum_{j=1}^N \pi_{ij} P_j G,$$

$$\Phi_{\Delta 22} = \sum_{j=1}^N \pi_{ij} G^T P_j G - N_0 + \alpha_0(\beta+\gamma) M_{0i}^T M_{0i},$$

$$\Phi_{\Delta 33} = \alpha_1(\beta+\gamma) M_{1i}^T M_{1i} - N_1 + \sigma_{1i} H_{1i}^T H_{1i}.$$

Proof. By Theorem 1, the uncertain system (25) is MSES if (9) holds after displaced A_i and A_{1i} by $\hat{A}_i$ and $\hat{A}_{1i}$, i.e.,

$$\widetilde{\Phi}_\Delta = \begin{bmatrix} \widetilde{\Phi}_{\Delta 11} & \widetilde{\Phi}_{\Delta 12} & P_i(A_{1i} + \Delta A_{1i}) & P_i & 0 \\ \star & \widetilde{\Phi}_{\Delta 22} & G^T P_i(A_{1i} + \Delta A_{1i}) & 0 & G^T P_i \\ \star & \star & \alpha_1(\beta + \gamma) M_{1i}^T M_{1i} - N_1 & 0 & 0 \\ \star & \star & \star & -\gamma I & 0 \\ \star & \star & \star & \star & -\beta I \end{bmatrix} < 0, \tag{27}$$

where

$$\widetilde{\Phi}_{\Delta 11} = (A_i + \Delta A_i)^T P_i + P_i(A_i + \Delta A_i) + \sum_{j=1}^N \pi_{ij} P_j + \alpha_2(\beta + \gamma) M_{2i}^T M_{2i} + N_0 + N_1,$$

$$\widetilde{\Phi}_{\Delta 12} = (A_i + \Delta A_i)^T P_i G + \sum_{j=1}^N \pi_{ij} P_j G,$$

$$\widetilde{\Phi}_{\Delta 22} = \sum_{j=1}^N \pi_{ij} G^T P_j G - N_0 + \alpha_0(\beta + \gamma) M_{0i}^T M_{0i}.$$

Being

$$\widetilde{\Phi}_\Delta = \Phi + Q^T F_i R + R^T F_i^T Q \le \Phi + \sigma_{1i}^{-1} Q^T Q + \sigma_{1i} R^T R,$$

where $R = [H_i \quad 0 \quad H_{1i} \quad 0 \quad 0]$, $Q = [(P_i E_i)^T \quad (G^T P_i E_i)^T \quad 0 \quad 0 \quad 0]$, Φ is defined in (9). It is easy to see (26) is equivalent with $\Phi + \sigma_{1i}^{-1} Q^T Q + \sigma_{1i} R^T R < 0$ according to Schur complement lemma, so inequality (27) holds, therefore, system (25) is MSES.

Finally, we consider the problem of design method of state feedback controller. Neutral system (1) transforms to

$$\dot{x}(t) + G\dot{x}(t - h_0) = \widetilde{A}_i x(t) + \widetilde{A}_{1i} x(t - h_1) + \hat{f}(t) \tag{28}$$

under the action of state feedback controller

$$u(t) = K_1(r(t))x(t) + K_2(r(t))x(t - h_1), \tag{29}$$

where $\widetilde{A}_i = \hat{A}_i + B_i K_{1i}$, $\widetilde{A}_{1i} = \hat{A}_{1i} + B_i K_{2i}$. By Theorem 2, the resulting closed-loop system (28) is MSES if the following inequality holds.

Theorem 3. *The uncertain system* (28) *is MSES if there exist some positive symmetric matrices* P_i, *semi-positive symmetric matrices* N_0, N_1, *and matrices* U_i, V_i, *scalars* $\sigma_{1i} > 0$, $i \in S$, *satisfying*

$$\Phi_K = \begin{bmatrix} \Phi_{K11} & \Phi_{K12} & P_i A_{1i} + \sigma_{1i} H_i^T H_{1i} & P_i & 0 & P_i E_i & U_i \\ \star & \Phi_{K22} & G^T P_i A_{1i} & 0 & G^T P_i & G^T P_i E_i & 0 \\ \star & \star & \Phi_{K33} & 0 & 0 & 0 & V_i \\ \star & \star & \star & -\gamma I & 0 & 0 & 0 \\ \star & \star & \star & \star & -\beta I & 0 & 0 \\ \star & \star & \star & \star & \star & -\sigma_{1i} I & 0 \\ \star & \star & \star & \star & \star & \star & -P_i \end{bmatrix} < 0, \tag{30}$$

where

$$\Phi_{K11} = A_i^T P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij} P_j + \alpha_2(\beta + \gamma) M_{2i}^T M_{2i}$$

$$+ N_0 + N_1 + \sigma_{1i} H_i^T H_i + P_i,$$

$$\Phi_{K12} = A_i^T P_i G + \sum_{j=1}^{N} \pi_{ij} P_j G + P_i G,\ \Phi_{K33} = \alpha_1(\beta + \gamma) M_{1i}^T M_{1i} - N_1 + \sigma_{1i} H_{1i}^T H_{1i},$$

$$\Phi_{K22} = \sum_{j=1}^{N} \pi_{ij} G^T P_j G - N_0 + \alpha_0(\beta + \gamma) M_{0i}^T M_{0i} + G^T P_i G,$$

controller gain matrices can be obtained by $U_i = (B_i K_{1i})^T P_i$, $V_i = (B_i K_{2i})^T P_i$.

Proof. In fact, applying Theorem 1, displaced A_i and A_{1i} in (9) by $\widetilde{A}_i$, $\widetilde{A}_{1i}$ respectively and after necessary computing, if the following inequality:

$$\widetilde{\Phi}_{\Delta} + \begin{bmatrix} (B_iK_{1i})^T P_i + P_iB_iK_{1i} & (B_iK_{1i})^T P_iG & P_iB_iK_{2i} & 0 & 0 \\ \star & 0 & G^T P_iB_iK_{2i} & 0 & 0 \\ \star & \star & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \quad (31)$$

holds, $\widetilde{\Phi}_{\Delta}$ is given by (27), then system (28) is MSES. Owing to

$$\widetilde{\Phi}_{\Delta} + \widetilde{Q}^T \widetilde{R} + \widetilde{R}^T \widetilde{Q} \le \widetilde{\Phi}_{\Delta} + \widetilde{R}^T P_i \widetilde{R} + \widetilde{Q}^T P_i^{-1} \widetilde{Q}, \quad (32)$$

where $\widetilde{R} = [B_iK_{1i} \quad 0 \quad B_iK_{2i} \quad 0 \quad 0]$, $\widetilde{Q} = [P_i^T \quad P_i^T G \quad 0 \quad 0 \quad 0]$, and inequality (30) is equivalent with $\widetilde{\Phi}_{\Delta} + \widetilde{R}^T P_i \widetilde{R} + \widetilde{Q}^T P_i^{-1} \widetilde{Q} < 0$ by Schur complement lemma, where $U_i = (B_iK_{1i})^T P_i$, $V_i = (B_iK_{2i})^T P_i$, so Theorem 3 is proved. At the same time, the gain matrices of controller (29) can be obtained by $U_i = (B_iK_{1i})^T P_i$, $V_i = (B_iK_{2i})^T P_i$.

4. Conclusion

In this correspondence, we have studied MSES problems for a class of uncertain nonlinear time delay neutral systems with Markovian jumping parameters, sufficient conditions on MSES are presented based on LMI's, and obtain design method of state feedback controller. Its gain matrices can be got through solving a couple of LMI's by Matlab Toolbox.

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