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MATHEMATICAL MODELS FOR STUDYING INDIRECT AND DIRECT CONTRIBUTION LEADERSHIP

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Abstract

Mathematical models for team replacement with interaction among team members are proposed to incorporate both indirect and direct contribution leadership. Indirect contribution of a leader to overall team performance is obtained through normal leading roles such as motivation, while direct contribution occurs when the leader performs tasks as an ordinary team member. Computer simulations are used to investigate the effects of the models' parameters, for example, the team size, the amount of interaction among team members, the skill level of the leader, and having a choice of multiple leaders on the expected performance of the team both in the indirect and direct models. The importance level of the leader's direct contribution is also included into the models along with simulation results for examining its impacts on the expected team performance.

1. Introduction

Organizational behavior (OB) seems to be a complex or dynamic

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system because of the difficulties in observing and predicting its patterns. Interaction among players in every level of the organizational hierarchy is one of the main reasons for the organizational complexity. Mathematical modeling and computer simulation as useful tools in examining organizational behavior are full of advantages ([1], [5]) over traditional OB research methodology. Time consumption and money expenses are enormously reduced at the cost of losing some aspects of reality's complication and complexity. This article focuses mainly on constructing mathematical models and using computer simulation for studying the impact of indirect and direct contribution leadership on team performance in the team replacement problem when interaction among team members plays an important role. A well-known mathematical model called the *NK model* [2] has been applied to this problem [9] for some period of time.

In the *NK* model, assume that there are N positions within the team. Assume further that only two qualified candidates are considered for each team position. Each of the 2^N possible teams is denoted by a binary N-vector, $\mathbf{x} = (x_1, ..., x_N)$, in which $x_i = 0$ means that one of the two candidates is chosen for position i and $x_i = 1$ means the other is chosen instead. The contribution of each position i to overall team performance, namely, $p_i(\mathbf{x}_i^K)$, depends on the team member in position i and the interaction that member has with other K members on the team when K takes an integer value between 0 and $N - 1$, inclusively. It is also assumed that those K members come from $K/2$ positions on either side of position i, wrapping around if necessary. The overall team performance is an average of all the team members' contributions as follows:

$$p(\mathbf{x}) = \frac{\sum_{i=1}^{N} p_i(\mathbf{x}_i^K)}{N}.$$

Random values generated from a uniform distribution from 0 to 1 are then assigned to parameters $p_i(\mathbf{x}_i^K)$, for all i, notationally, $p_i(\mathbf{x}_i^K) \sim U[0, 1]$. When this value is close to 0, it indicates poor performance while 1 indicates good performance.

This original *NK* model does not incorporate leadership concept. In OB, this model could represent a self-managed team [4]. Although a team leader is not always needed, a skillful leader could influence the factors that drive the contribution to team performance of each team member towards the achievement of the team's goals. Mathematical models modified from the original *NK* model for studying leadership in teams ([3], [6], [7], [8]) show the significance of effective leadership. Due to limitations of mathematical modeling, only some roles played by a leader can be included into the models. More precisely, to date, motivational and cooperational skills of leadership have been mathematically modeled.

The leader in each of the previous leadership models contributes to team performance through his/her relationship with the team members. Simply put, the leader does not directly contribute to team performance. Under several circumstances, in addition to traditional leading roles of leadership, the leader may have to perform the same tasks as an ordinary team member as well. Therefore, the leader has both direct and indirect contribution to team performance. Of all the previous indirect-contribution leadership models, the motivational leadership models are modified here in this paper so as to incorporate the direct contribution of the leader. Furthermore, another modification of the *NK* model using a weighted average for computing overall team performance is also considered and applied to the new direct-contribution leadership models. For comparison purposes with the simulation results from previous work, the effects of having a choice of multiple leaders in the new models on team performance when there is no interaction among team members is investigated. New simulation results of the motivational leadership models and the new models with the presence of interaction among team members are also generated and compared.

In the next section, literature reviews concerning mathematical models for studying the effects of leadership in teams with interaction are presented. In Section 3, the new direct-contribution leadership models are rationally explained. Computer simulation results and their discussions are shown in Section 4. Finally, conclusions and suggestions for this research work are provided in Section 5.

2. The Indirect-contribution Leadership Models

In this section, previous work on the development of mathematical models for studying team replacement along with the inclusion of leadership concept is summarized. Starting with the original *NK* model described in Section 1, a leader is then added into the model so that the overall team performance now depends not only on the team members and their interaction but also on the relationship between the leader and the team members. However, in all of the previous leadership models to be explained in this section, the leader has only indirect contribution to team performance. In other words, the leader does not contribute directly to team performance but rather through the team members. Leadership roles are comprised of planning and organizing, communicating, motivating, and cooperating, just to name a few [4]. Some of these roles can be modeled mathematically but some cannot be. Apart from that, one of the following mathematical models presented here was not originally meant to include leadership explicitly but for some certain cases it can be thought of having an implicit leader.

- **The** NK/W **model.** In the *NK* model [6], each team member contributes to team performance with an equal weight. That could mean every member is equally significant. A natural modification would be to distinguish the team members so that everyone will contribute differently depending on their significance levels to the team. First, the formula for the team performance in the *NK* model could be written as:

$$
\begin{aligned}
p(\mathbf{x}) &= \frac{\sum_{i=1}^{N} p_i(\mathbf{x}_i^K)}{N} \\
&= \frac{p_1(\mathbf{x}_1^K) + \cdots + p_N(\mathbf{x}_N^K)}{N} \\
&= \frac{1}{N} p_1(\mathbf{x}_1^K) + \cdots + \frac{1}{N} p_N(\mathbf{x}_N^K).
\end{aligned}
$$

The equal weights now are $1/N$ which could be replaced by more general and different weights w_i, where $i = 1, 2, \ldots, N$ and $w_1 + \cdots + w_N = 1$.

The formula becomes

$$p(\mathbf{x}) = w_1 p_1(\mathbf{x}_1^K) + \cdots + w_N p_N(\mathbf{x}_N^K)$$

and the weights can then be assigned individually. This model could be seen as a leadership model as well, especially when one person has a weight larger than any other team members. This implies that the leader is the most important person in contributing to the overall team performance. In a special case, when one team member has a weight much higher than all other members, say $w_1 = w$ and so $w_j = (1 - w)/(N - 1)$, for $j = 2, \ldots, N$, the results show that this can attenuate the *interaction catastrophe* of decreasing performance as interaction increases, embedded in the original *NK* model. It is worth noting that this NK/W model is although not intended to embrace leadership explicitly, it demonstrates a subtle way of encapsulating direct contribution to team performance of the leader into the model.

Turning now to more leadership-oriented models that have been developed mathematically, two out of the various leadership roles, namely, motivation and cooperation, have been incorporated separately. Each of these skills is beneficial to enhancing the individual performance of the team members and thus the overall team performance as well. Starting with how to add a leader into the model followed by a way to reflect the motivational skill levels of the leader, the motivational leadership models [8] are summarized here.

• **The *NKL* model.** In this model, the original *NK* model has been modified so that each team member's contribution is now not a random number between 0 and 1 but instead ranges from a lower bound $a_i(x_i)$ to an upper bound $b_i(x_i)$, where $a_i(x_i)$ and $b_i(x_i) \sim U[0, 1]$ and $a_i(x_i) \leq b_i(x_i)$. The leader z then defines the contribution of member x_i through his/her relationship with that member, $r_i(x_i, z)$, as well as the following convex combination formula:

$$p_i(x_i, z) = (1 - r_i(x_i, z))a_i(x_i) + r_i(x_i, z)b_i(x_i),$$

where $r_i(x_i, z) \sim U[0, 1]$. Then the overall team performance is still the average of these individual contributions as usual. Consequently, the leader does not directly contribute to the team performance. The relationship parameter is the motivation the leader has on each team member, especially when r_i is closer to 1. However, there is no guarantee that r_i will be close to 1 and therefore the leader in this model is called a *random leader*, i.e., a leader who has no particular motivating skill. Note that there is no interaction among team members in this model. An interesting finding from computer simulation for this model shows that when having a choice of more than one leader, the performance of a local maximum team improves for small teams but starts to lose this benefit for larger teams.

- **The** $NKL(\mu, \sigma)$ **model.** In this generalized model, besides the interaction among team members, the motivational skill level of leader is also added to the *NKL* model. Two new parameters corresponding to the motivational skill of the leader, namely, μ representing the average skill and σ representing to the variability of the skill are created. The contribution range for each team member is also modified to include the interaction among team members and now becomes $a_i(\mathbf{x}_i^K)$ and $b_i(\mathbf{x}_i^K)$, all else being the same. In addition, $r_i(x_i, z)$ is generated from a *shifted normal distribution*, notationally, $r_i(x_i, z) \sim SN(\mu, \sigma)$. This is done by first generating a random number from Normal (μ, σ), then calculating the area to the left of that number under the Normal $(0, 1)$ curve. The main result of this model with the motivational skill level highlights that increasing the skill level of the leader helps improve the team performance despite the high level of interaction among team members. That means the skill of the leader can be more important than the amount of interaction. Nevertheless, the leader portrayed in this model does not help reduce the interaction catastrophe.

- **The** $NKLW(\mu, \sigma)$ **model.** This model is a combination of the $NKL(\mu, \sigma)$ model and the NK/W model. It is intended to take advantage of the NK/W in attenuating the interaction catastrophe particularly by assigning a weight W to the team member in position 1 of the $NKL(\mu, \sigma)$ model much higher than all other members. The simulation results illustrate that for a fixed skill and variation of the leader, as the weight W of position 1 enlarges, the team performance improves no matter how much the interaction is. Furthermore, as W gets larger, the team performance is not inversely related to the amount of interaction and it can even be beneficial from an extremely large W.

As for the next leadership role, namely, seeking cooperation among the team members, three different models have been previously proposed [7]. Their underlying modeling ideas are presented here as follows:

• **The** $NKLC(\mu, \sigma)$ **Model.** This model is based on similar ideas used in the motivational leadership models. More precisely, in this model, it is assumed that each team member contributes to team performance within the range $[a_i(x_i), b_i(x_i)]$ and has a relationship variable $r_i(\mathbf{x}_i^K, z) \sim U[0, 1]$. The difference is that the contribution range now depends only on that team member whereas the relationship variable depends also on the leader, that team member, and K other members so that it represents the cooperation level between the members the leader can achieve. Then, the individual contribution of a team member and the overall team performance are computed the same ways as in the $NKL(\mu, \sigma)$ model. Even though μ and σ are still used to represent the average skill level and the variability of the skill level of the leader, they refer to the cooperational skill, not motivational because of the change in the relationship variable. The simulation results show that a skillful leader can boost up the team performance as well as attenuate the interaction catastrophe because the high amount of interaction can still be managed by the leader who has high cooperational skill.

• **The** $NK(\mu, \sigma)$ **Model.** Applying the idea of the shifted normal distribution, the original NK model is simply modified by generating individual contributions from $SN(\mu, \sigma)$ rather than from $U[0, 1]$. In the NK model, because each individual contribution already depends also on K other team members, it could be viewed that, without a cooperational leader, the team members already have some cooperation especially when they do not have to interact with many people. Depending on the skill level of the leader μ, switching from $U[0, 1]$ to $SN(\mu, \sigma)$ provides a larger chance in obtaining a higher number and thus giving rise to a larger individual contribution. Subsequently, the simulation results reveal similar conclusions as those in the $NKLC(\mu, \sigma)$ model except that the expected team performance of skillful leader cases in this model approaches 1 as opposed to 0.8 in the $NKLC(\mu, \sigma)$ model due to the effects of the maximum contribution of each member being $b_i(x_i)$ in the $NKLC(\mu, \sigma)$ model which is less than 1 in the $NK(\mu, \sigma)$ model.

- **The** $NKLC(\alpha)$ **Model.** In this model, the idea of generating $p_i(\mathbf{x}_i^K)$ independently of each other in the original *NK* model is adjusted to include the dependency between the amounts of interaction $K + 1$ and K. As a result, the individual contribution with $K + 1$ interactions is a function of the contribution with K interactions and the cooperational skill of leader α being a number between 0 and 1, where 0 means the leader is skilless and 1 very skillful according to the following formula:

$$p_i(\mathbf{x}_i^{K+1}) \sim U[l_i^{K+1}(p_i(\mathbf{x}_i^K), \alpha), u_i^{K+1}(p_i(\mathbf{x}_i^K), \alpha)],$$

where

$$l_i^{K+1}(p_i(\mathbf{x}_i^K), \alpha) = \alpha p_i(\mathbf{x}_i^K)$$

and

$$u_i^{K+1}(p_i(\mathbf{x}_i^K), \alpha) = (1 - \alpha)\left[\frac{1 + p_i(\mathbf{x}_i^K)}{2}\right] + \alpha.$$

That is, the individual contribution is generated from a uniform distribution between a lower bound and an upper bound, each of which again is defined by $p_i(\mathbf{x}_i^K)$ and α in a way that when the leader has no skill, the contribution would be in the range between 0 and half way from $p_i(\mathbf{x}_i^K)$ to 1 while when the leader is very skillful, the contribution would be in the range between $p_i(\mathbf{x}_i^K)$ and 1. The simulation results indicate that the team performance with a skilless leader falls monotonically as the amount of interaction increases. In contrast, a more skillful leader can deal with higher amount of interaction leading to attenuating the interaction catastrophe as well.

With the exception of the NK/W model, all of the above leadership models can be called the *indirect-contribution leadership models*. In the next section, mathematical models that allow a motivational leader to perform the same tasks as an ordinary team member resulting in a direct contribution to team performance will be proposed. Similar concepts can also be applied to the cooperational leadership models with some proper modifications.

3. The Direct-contribution Leadership Models

In this section, mathematical models for studying interacting teams with direct leadership are proposed. The general motivational leadership model, namely, the $NKL(\mu, \sigma)$ model, interchangeably referred to as *the indirect model* for simpler future references, will be modified to illustrate a situation such that the leader also performs, besides all the leadership roles, regular tasks as an ordinary team member. In other words, the leader takes part in contributing directly to the team performance as well. Furthermore, to increase the flexibility degree of the leader's direct contribution, the idea of using weights to differentiate each person's significance to the team will also be applied. Also, in the new models, the effects of having more than one candidate for the leader position will be examined with a comparison to the previous result from the indirect leadership model mentioned in Section 2. For completion, the set of all notations for later use in the proposed models is summarized here:

- $N =$ number of positions in a team.
- $K =$ number of other positions interacting with a team member.
- $(\mathbf{x}, z) =$ represents a team $\mathbf{x} = (x_1, \ldots, x_N)$ with leader z, where each $x_i = 0$ or 1.
- $[a_i(\mathbf{x}_i^K), b_i(\mathbf{x}_i^K)] =$ contribution range for team member x_i, depending on x_i and K others.
- $r_i(x_i, z) \sim U[0, 1] =$ relationship between team member x_i and leader z.

- $p(z) \sim U[0, 1] =$ direct contribution to team performance of leader z.

- $p_i(\mathbf{x}_i^K, z) =$ contribution to team performance of member x_i, depending on x_i, K others, and leader z.

- $p(\mathbf{x}, z) =$ overall performance of team $\mathbf{x}$ with leader z.

In the $NKL(\mu, \sigma)$ model, it is assumed that the leader only performs his/her leading roles. In some more practical and realistic scenario, the leader may be the person who used to work as an ordinary team member and became competent in the tasks. After his/her promotion to the team leader, the leader may be required to do dual roles as both a team leader and a team member. Thus, the leader has an indirect contribution to team performance through the usual leading roles, and at the same time, a direct contribution through the tasks the leader performs as a team member. To include the direct contribution into the model, the formula for the team performance will be modified to

$$p(\mathbf{x}, z) = \frac{\sum_{i=1}^{N} p_i(\mathbf{x}_i^K, z) + p(z)}{N + 1},$$

where the leader's direct contribution $p(z)$ is generated from $U(0, 1)$ with the same interpretation for its values as usual. Other than this, the leader still also contributes indirectly through the motivation he/she has on the team members as reflected in the relationship parameter $r_i(x_i, z)$ and the following expression for each team member's contribution:

$$p_i(\mathbf{x}_i^K, z) = (1 - r_i(x_i, z))a_i(\mathbf{x}_i^K) + r_i(x_i, z)b_i(\mathbf{x}_i^K).$$

This proposed model is then called the $NKL/D(\mu, \sigma)$ *model* or simply the *direct model* with D referring to the leader's direct contribution to team performance. For a special case when $(\mu, \sigma) = (0, 1)$, this direct model is then said to have a *random leader* or the leader who does not have any particular motivating skill. Experimental results compared to those of the indirect model will be shown in Section 4 including the cases of multiple candidates for the leader position.

To make the model even more flexible, differentiating the significance levels of all the people including the leader accountable for undertaking the team's tasks would create a model adaptable to more situations. For example, it could happen that because the leader has to perform multiple tasks as a leader and a team member all at once, he will definitely have less time than an ordinary team member. Therefore, the contribution weight of the leader should not be equal to those of other ordinary members. On the other hand, it could be the case that, even though the leader has to do both leading and operating tasks, but because the leader is very skillful and very significant to the outcome of the team, the leader's weight then should be higher than all other team members. With these examples in mind, it would be wise to construct a model that could allow the weights of both the leader and each team member to fit every player's significance level to the team performance.

A generalization of the $NKL/D(\mu, \sigma)$ model on the weights of all the players in the team leads to a new model called *the* $NKLW/D(\mu, \sigma)$ *model*, interchangeably, *the weighted direct model*, in which the team performance is now not an average of everyone's contribution, instead it is the weighted average as follows:

$$p(\mathbf{x}, z) = w_1 p_1(\mathbf{x}_1^K, z) + \cdots + w_N p_N(\mathbf{x}_N^K, z) + w_{N+1} p(z),$$

where $w_1 + \cdots + w_N + w_{N+1} = 1.$ However, for a special case when the weights of all ordinary team members are equal and only the leader's weight is different, the team performance now becomes

$$p(\mathbf{x}, z) = \frac{(1 - W)}{N} \sum_{i=1}^{N} p_i(\mathbf{x}_i^K, z) + Wp(z),$$

where W is the weight of the leader. It is noted that the weights of all the members including the leader still sum up to 1 as shown here:

$$\sum_{i=1}^{N} \frac{(1 - W)}{N} + W = N \frac{(1 - W)}{N} + W = (1 - W) + W = 1.$$

This special case of the $NKLW/D(\mu, \sigma)$ model is very powerful in the sense that the leader's direct contribution to team performance could be experimented so as to appropriately fit any situation faced in practice. In the next section, along with other simulation experiments, different weights will be placed on the leader position using computer simulation to discover its effects on team performance.

4. Computer Simulations and Discussions

In this section, computer simulation results of the proposed models and new results of the indirect model together with their comparisons and discussions are presented. The simulations were conducted using C++ programming on an Intel Core 2 Duo T5450 1.6 GHz laptop computer with a 2 GB RAM. Starting with simulation results on the effects of having a choice of multiple random leaders, its effects on the team performance in the indirect and direct models are compared and contrasted for both situations when interaction among team members is present and when it is absent. Afterwards, the importance of the skill level of the leader and the changeable weight of the leader position towards the team performance will also be examined.

4.1. Choice of multiple leaders in the $NKL(0, 1)$ and the $NKL/D(0, 1)$ models

In general, having a choice of something is always good. In this section, simulation results for studying the impact of having multiple candidates for the leader position on the team performance are presented and discussed.

First of all, with no interaction among team members, a choice of multiple random leaders is considered both in the indirect and direct models. Since this is the case when each team member does not have to interact with anyone else, obviously, the interaction parameter K and its associated effect, namely, the interaction catastrophe, will be out of the context here. Alternatively, an investigation will be placed on another important parameter of the model, that is, the team size N.

In Figure 1, the expected team performance as a function of N with a choice of 1, 2, 5 and 10 random leaders in the indirect and direct models when there is no interaction among team members is revealed. The results in both models show that for a small team, having a choice of multiple leaders helps improve the expected team performance. The more the number of candidates for the leader position is, the higher the team performance becomes, especially in the direct model when the leader can contribute to team performance both indirectly as usual and directly. On the other hand, when team size is large, in both Figure 1(a) and Figure 1(b), the performance starts to approach a certain level of 0.63 approximately. Thus, in general, having a choice of multiple leaders does not result in improved performance in large teams.

As for a special case when there is only one team leader, in the indirect model, the bottom curve of Figure 1(a) staying flat at approximately the algebraically-proven performance of 0.63 [8] indicates that the team size does not affect the team performance at all.

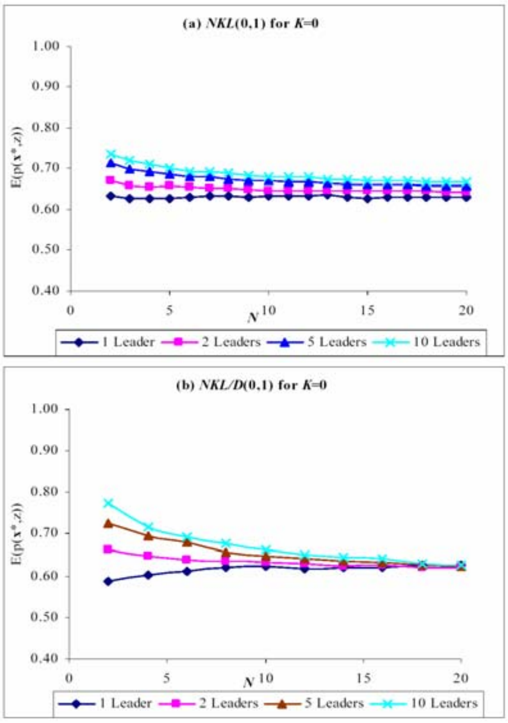
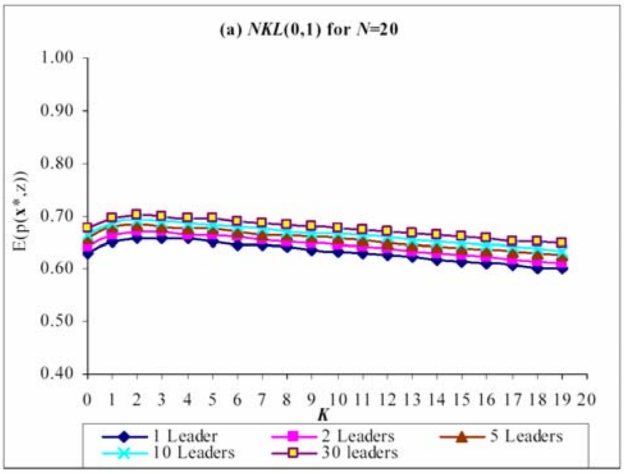


Figure 1. The expected performance of a local maximum team in the indirect and direct models as a function of N with a choice of 1, 2, 5 and 10 random leaders when there is no interaction among team members.

In contrast, in the direct model, the bottom curve of Figure 1(b) shows that the performance slightly increases as the team gets larger up to a certain size then starts leveling on towards the benchmark performance of 0.63 as in the cases mentioned previously. On average, the leader, as part of the working team, theoretically contributes 0.5 to overall team performance because his/her individual contribution is taken from a uniform distribution from 0 to 1 while a team member theoretically contributes 0.63 [8]. According to the team performance formula for the direct model mentioned in Section 3, the weight of the leader's contribution is equal to that of every other member's on the team. Hence, as the team grows larger, the leader's direct contribution to overall team performance will become smaller resulting in a convergence of team performance towards 0.63.

For a practical case in general, when the team is too small and only one random leader is available to directly participate in the team's tasks, neither the leader is skillful nor are there workforces sufficient enough to take care of all the workload for the team. Consequently, the team suffers as evidenced by the beginning of the bottom curve of Figure 1(b). By increasing the team size up to a certain point, the workload is shared more efficiently and hence the team performance increases.

In Figure 2, when interaction among team members is present, even though the leader has no particular motivating skill, having a choice of multiple leaders always improves the expected team performance both in the indirect and direct models regardless of the amount of interaction. Nevertheless, for the indirect model in Figure 2(a), the entire performance curve shifts up whereas for the direct model in Figure 2(b), the performance curve still shifts upward but its tails become flatter as K increases. Hence, having multiple candidates for the leader position can attenuate the detrimental effect on performance of increasing amounts of interaction among team members in the direct model but not in the indirect model.



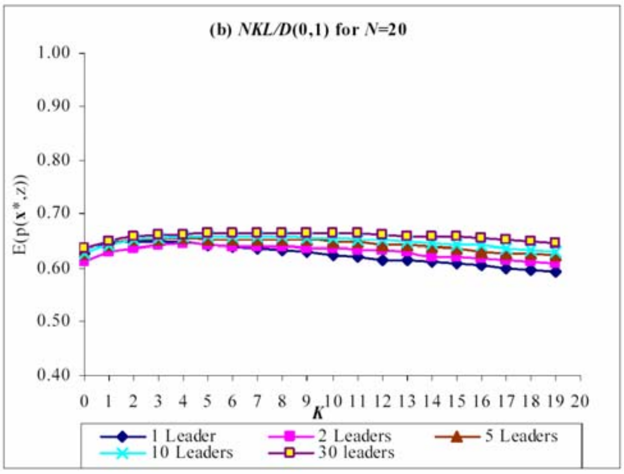


Figure 2. The expected performance of a local maximum team in the indirect and direct models as a function of K with a choice of 1, 2, 5, 10 and 30 random leaders when $N = 20$.

4.2. Choice of multiple leaders in the $NKL(\mu, \sigma)$ and the $NKL/D(\mu, \sigma)$ models

Having a choice of multiple random leaders already leads to better team performance as just described previously and can even attenuate the interaction catastrophe in the direct model. In this section, the skill level of the leader is taken into account both in the indirect and direct models to observe its effects on team performance.

Starting with the indirect model, computer simulation results in Figure 3 confirm a conclusion cited in Section 2 that the skill of the leader can be more important than the amount of interaction among team members when there is no choice of the leaders, i.e., only one leader with a certain skill level is assigned. This is evidenced by the bottom curves of both Figure 3(a) and Figure 3(b) because each entire curve moves up from when $\mu = 0$ to $\mu = 1$. More elaborately, the above conclusion also holds true for other cases when there is a choice of more than one leader. For another time, this is illustrated by the entire upward shifts of the corresponding curves from Figure 3(a) to Figure 3(b). In addition, besides the skill level of the leader, having multiple candidates for the leader position can improve the over team performance even more. Note that the interaction catastrophe is not, by any means, attenuated in this indirect model.

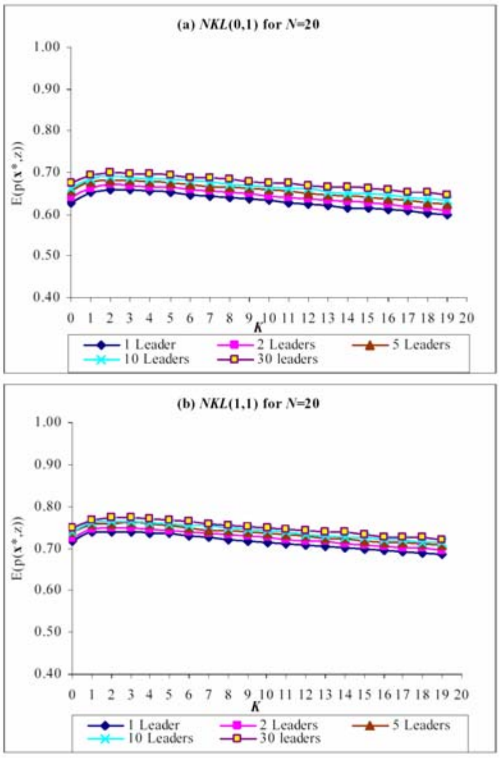


Figure 3. The expected performance of a local maximum team in the indirect model as a function of K for $N = 20$ with a choice of 1, 2, 5, 10 and 30 leaders when $\mu = 0$ and 1, respectively.

Comparable to Figure 3, similar simulation results for the direct model proposed in this article are produced and shown in Figure 4. Obviously, the skill level of the leader and having a choice of multiple leaders are still beneficial to team performance. However, unlike in the indirect model, for the case when the amount of interaction among team members is not large, having a choice of skillful multiple leaders does not facilitate the team performance to improve as exhibited in both Figure 4(a) and Figure 4(b) that all the curves lie on top of each other for several values of K before they divide apart. Another significant difference is that in this direct model, the interaction catastrophe diminishes because the tails of each curve becomes flatter as the number of leader candidates increases.

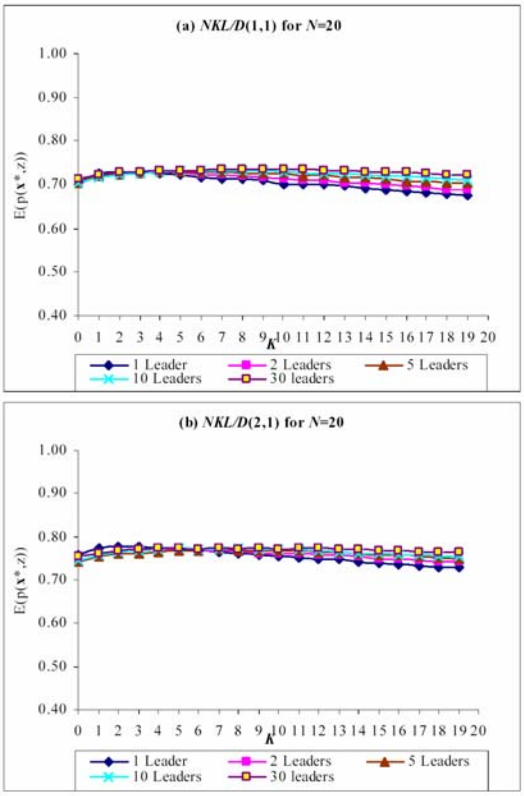


Figure 4. The expected performance of a local maximum team in the direct model as a function of K for $N = 20$ with a choice of 1, 2, 5, 10 and 30 leaders when $\mu = 1$ and 2, respectively.

4.3. The weight and the skill of the leader in the $NKLW/D(\mu, \sigma)$ model

In this section, the two important parameters in the weighted direct model, namely, the weight and the skill level of the leader, are tried out with different values to determine their effects on team performance.

In Figure 5, for a small weight of the leader's contribution to team performance such as $W = 0.1$ and 0.3 in Figure 5(a) and Figure 5(b), respectively, the interaction catastrophe is still present but it is lessen as the number of random leaders to choose from increases due to the fact that the tails of each curve in each of the two figures become flatter. On the contrary, when the weight is large such as $W = 0.7$ and 0.9 in Figure 5(c) and Figure 5(d), respectively, because all of the curves are almost flat, there seems to be no benefits for small amounts of interaction here and thus the interaction catastrophe is irrelevant.

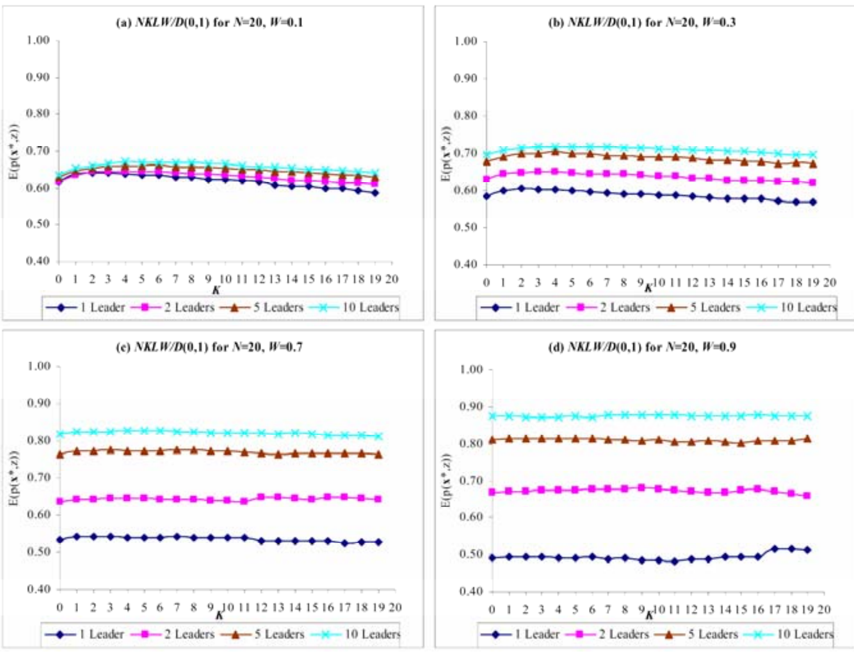


Figure 5. The expected performance of a local maximum team in the weighted direct model as a function of K with a choice of 1, 2, 5 and 10 leaders when $N = 20$, $\mu = 0$, and $W = 0.1, 0.3, 0.7,$ and 0.9, respectively.

Aside from the interaction catastrophe issue, another observation drawn from Figure 5 is that, except for the case when there is only one candidate for the leader position, the higher the weight to put onto the leader, the higher the expected team performance grows, as evidenced by the upward trends of the top three curves going from a lower W to a higher W. For the 1-leader case, the more the weight of the leader is, the lower the team performance becomes.

Similar conclusions and observations can also be drawn from Figure 6 when the leader is more skillful, or more precisely $\mu = 2$, as opposed to the random leader case when $\mu = 0$ in Figure 5, all else being unchanged. Again, the skill level of the leader is proved to be an important factor for improvement in team performance as evidenced by the upward move of all the corresponding curves from Figure 5 to Figure 6.

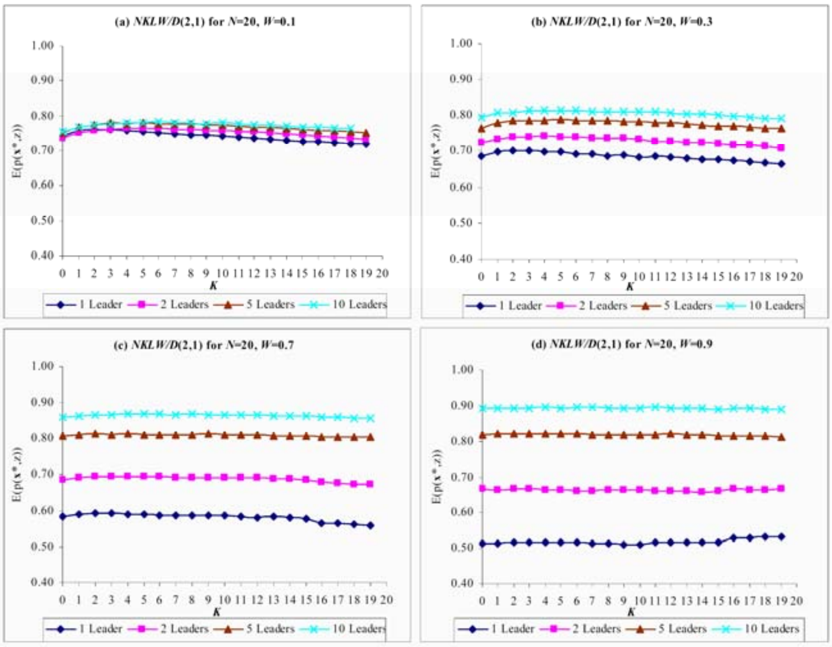


Figure 6. The expected performance of a local maximum team in the weighted direct model as a function of K with a choice of 1, 2, 5 and 10 leaders when $N = 20$, $\mu = 2$, and $W = 0.1, 0.3, 0.7$, and 0.9, respectively.

This section has reported computer simulation results for studying and comparing the effects of having a choice of multiple leaders, the skill of the leader, and the weight of the leader's direct contribution on team performance in the indirect and direct models. In the next section, conclusions and suggestions relevant to this research work will be provided.

5. Conclusions and Suggestions

Indirect and direct contributions of a leader to overall team performance have been incorporated into existing mathematical models for studying team replacement with motivational leadership. A leader indirectly contributes to overall team performance through regular leading roles. Most likely, in some situations, the leader may have to participate in the team's tasks as an ordinary team member and thus contributes directly to team performance. A modification from the indirect $NKL(\mu, \sigma)$ model to capture also the direct contribution results in the direct $NKL/D(\mu, \sigma)$ model. The weight of the leader's contribution is another factor added into the $NKL/D(\mu, \sigma)$ model in order to increase flexibility in differentiating the leader's direct contribution level that could happen in reality. This more general direct model designed for the situation when everyone on the team except the leader has an equal weight becomes the weighted direct $NKLW/D(\mu, \sigma)$ model.

All of the existing indirect and proposed direct leadership models are then experimented by computer simulation to see the impact of various parameters of the models. In the case when there is no interaction among team members, having a choice of multiple random leaders, or leaders who do not have any particular motivating skill, does improve the expected performance of the team in general. As for when interaction is observed, having a choice of multiple random leaders improves the expected team performance regardless of the amount of interaction both in the indirect and direct models. Especially, in the direct model, a choice of multiple leaders can also attenuate the interaction catastrophe associated with high level of interaction.

The skill level of the leader in both the indirect and direct models is also another instrument for enhancing the expected team performance.

Besides, in the direct $NKL/D(\mu, \sigma)$ model, together with the skill level, having a choice of multiple leaders reduces the interaction catastrophe. As for the last model proposed in this paper, simulation results on the weighted direct $NKLW/D(\mu, \sigma)$ model show that, in general, the more the weight to put on the leader's direct contribution is, the higher the expected team performance grows.

The idea of direct contribution leadership used in this paper may be applied to mathematical models that incorporate other roles of leadership such as cooperation. Empirical studies relevant to the assumptions of the models are also suggested to be carried out so that their results can be compared and contrasted with the simulation findings. For that matter, these mathematical models will be more useful, practical, and applicable to the real-world problems.

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