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CONSTRUCTIONS OF A CLASS OF OPTIMAL CONSTANT COMPOSITION CODES

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Abstract

As a generalization of constant weight binary codes, constant composition codes (CCCs) have attracted much interest due to their numerous applications. In this paper, a class of new CCCs is constructed by using combinatorial design techniques.

1. Introduction

Let $Q = \{a_t : 0 \leq t \leq m-1\}$ be an arbitrary alphabet of m elements. A code $C \subseteq Q^n$ over Q of size M and minimum distance d is referred to as a constant composition code (CCC) or an $(n, M, d, [w_0, w_1, ..., w_{m-1}])_m$-CCC, if each codeword has precisely w_i occurrences of a_i for any i $(0 \leq i \leq m-1)$, where w_i are positive integers satisfying $\sum_{0 \leq i \leq m-1} w_i = n.$ The constant composition $[w_0, w_1, ..., w_{m-1}]$ is essentially an unordered multiset. For convenience, we sometimes write it in an exponential notation: a constant composition type $[1^i 2^j 3^r \cdots]$ denotes i occurrences of 1, j occurrences of 2, r occurrences of 3, etc.

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Constant composition codes are a generalization of constant weight binary codes as we enlarge alphabet size from two to more. The class of constant composition codes includes the important permutation codes and has attracted much interest due to their numerous applications (see, for example, [8] and the references therein). Recently, Ding and Yin [3] gave a combinatorial characterization of constant composition codes. They introduced a type of designs called *generalized doubly resolvable packings* described below:

Let X be a set of v elements (called *points*) and $\mathcal{A}$ be a collection of subsets (called *blocks*). Then the pair $(X, \mathcal{A})$ is known as an (n, λ)-packing of order v, if every pair of distinct points of X occurs in at most λ blocks and every point occurs in precisely n blocks. An (n, λ)-packing of order v is referred to as a generalized doubly resolvable packing or a $\mathrm{GDRP}(n, \lambda; v)$, if its blocks can be arranged into an $m \times n$ array $\mathcal{R}$ which satisfies the properties listed below:

• Each cell of $\mathcal{R}$ is either empty or contains one block.

• For $0 \leq i \leq m-1$, the blocks in row i of $\mathcal{R}$ form a w_i-parallel class, that is, every point occurs in exactly w_i blocks.

• The blocks in every column of $\mathcal{R}$ form a parallel class, that is, every point occurs in exactly one block.

Here, m and w_i are positive integers satisfying $\sum_{0 \leq i \leq m-1} w_i = n$ as before. The multiset $T = \{w_0, w_1, ..., w_{m-1}\}$ is called the *type* of the GDRP. When more convenient, we use the exponential notation to describe the type of a GDRP: a GDRP of type $1^i 2^j 3^r \cdots$ denotes i occurrences of 1, j occurrences of 2, etc. in the multiset T.

Theorem 1.1 [3]. *The existence of a* $\mathrm{GDRP}(n, \lambda; v)$ *of type* $\{w_0, w_1, ..., w_{m-1}\}$ *is equivalent to that of an* $(n, M, d, [w_0, w_1, ..., w_{m-1}])_m$-CCC, *where* $v = M$ *and* $d = n - \lambda$.

Following coding theory, we use $A_m(n, d, [w_0, w_1, ..., w_{m-1}])$ to denote the maximum size of an $(n, M, d, [w_0, w_1, ..., w_{m-1}])_m$-CCC. A CCC achieving this size is called *optimal*. We use the following bound which was established by Fu, Vinck and Chen [6] as our benchmark to check the optimality of our constructed CCCs.

Lemma 1.2. *If* $nd - n^2 + (w_0^2 + w_1^2 + \cdots + w_{m-1}^2) > 0,$ *then*

$$A_m(n, d, [w_0, w_1, ..., w_{m-1}]) \leq \frac{nd}{nd - n^2 + (w_0^2 + w_1^2 + \cdots + w_{m-1}^2)}.$$

The author [9] proved the following theorem:

Theorem 1.3. *Let* λ *and* μ *be two positive integers with* $\lambda < \mu$ *and* $(\mu - \lambda) | \lambda$. *If an optimal* $\text{GDRP}(\hat{n}, \hat{\lambda}; \hat{v})$ *of type* $\hat{\lambda}^1(\hat{\lambda} + 1)^{m-1}$ *exists, then so does an optimal* $\text{GDRP}(n, \lambda; v)$ *of type* $\lambda^1\mu^{m-1}$, *where* $\hat{\lambda} = \lambda/(\mu - \lambda)$.

Theorem 1.3 shows that to construct an optimal $\text{GDRP}(n, \lambda; v)$'s of type $\lambda^1\mu^{m-1}$ with $\lambda < \mu$, it is sufficient to treat the case $\mu = \lambda + 1$. In this case, $v = m(\lambda + 1) - 1 = n$ which is uniquely determined by the parameters m and λ. Furthermore, the blocks of the $m - 1$ $(\lambda + 1)$-parallel classes are of size $\lambda + 1$, and the blocks of the unique λ-parallel class are of size λ. For convenience, in the following, we write $k = \lambda + 1$ and use the notation $\text{GDRP}^*(k, v)$ to indicate an optimal $\text{GDRP}(n, \lambda; v)$ of type $\lambda^1(\lambda + 1)^{m-1}$. Whenever this notation is used, the parameters m, λ and n are given by $n = v$, $m = (v + 1)/k$ and $\lambda = k - 1$.

When $k = 3$ and 4, the existences of $\text{GDRP}^*(k, v)$'s and their corresponding CCCs have been completely determined.

Theorem 1.4 [9]**.** *Let* λ *and* μ *be arbitrary positive integers satisfying* $3\lambda = 2\mu$. *Then for all integers* $m \geq 3$, *an optimal* $(n, M, n - \lambda, [\lambda^1\mu^{m-1}])_m$-CCC *exists, where* $n = (m - 1)\mu + \lambda$ *and* $M = n/(\mu - \lambda)$.

Theorem 1.5 [10]. *Let* λ *and* μ *be arbitrary positive integers satisfying* $4\lambda = 3\mu$. *Then for all integers* $m \geq 2$, *an optimal* $(n, M, n - \lambda, [\lambda^1 \mu^{m-1}])_m$-CCC *exists, where* $n = (m - 1)\mu + \lambda$ *and* $M = n/(\mu - \lambda)$.

As k increases, the direct constructions of $\mathrm{GDRP}^*(k, v)$ become more and more difficult. In this paper, we are mainly interested in the existence of $\mathrm{GDRP}^*(k, v)$ with $k = 5$. As a consequence, the existence spectrum of a $\mathrm{GDRP}^*(k, v)$ or equivalently an optimal $(n, M, n - \lambda, [\lambda, \mu, \ldots, \mu])_m$-CCC with $5\lambda = 4\mu$ is almost determined only with finite possible exceptions.

2. Constructions

We assume that the reader is familiar with some basic concepts in design theory, otherwise, the reader may refer to [1, 2]. To give the constructions of GDRP^*, we need first to introduce the definitions of several combinatorial objects. The reader may refer to [9, 10] and the references therein for more details.

A group divisible design or a (K, λ)-GDD in short, is a triple $(X, \mathcal{G}, \mathcal{A})$, where X is a finite set of points, $\mathcal{G} = \{G_0, G_1, \ldots, G_{t-1}\}$ is a partition of X into t subsets (called *groups*), and $\mathcal{A}$ is a collection of subsets (called *blocks*) of X with $|A| \in K$ for any $A \in \mathcal{A}$, such that any pair of distinct points occurs together in either one group or exactly one block but not both. The multiset $T = \{|G_0|, |G_1|, \ldots, |G_{t-1}|\}$ is called the *group type* or the *type* of a (K, λ)-GDD. Usually, we use the exponential notation to describe the type. When $\lambda = 1$, we simply write a K-GDD instead of a $(K, 1)$-GDD. When $K = \{k\}$, we simply write k for K. A k-GDD of type g^k is called a *transversal design*, which is denoted by $\mathrm{TD}(k, g)$.

A (k, λ)-frame of type T is a (k, λ)-GDD of type T, $(X, \mathcal{G}, \mathcal{A})$, in which the blocks of $\mathcal{A}$ can be partitioned into partial parallel classes each partitioning $X \setminus G$ for some $G \in \mathcal{G}$. It is not hard to see that for any $G \in \mathcal{G}$, there are exactly $\lambda |G|/(k-1)$ partial parallel classes which partition $X \setminus G$.

Consider a $(k, k-1)$-frame of type T, $(V, \{G_0, G_1, ..., G_{t-1}\}, \mathcal{A})$ with $t \geq k+2$ and $k \,||\, G_i|$, $0 \leq i \leq t-1$. Let $C_j = \{u + \sum_{0 \leq s \leq j-1} |G_s| : u = 0, 1, ..., |G_j| - 1\}$ and $R_i = \{w + \sum_{0 \leq s \leq i-1} |G_s|/k : w = 0, 1, ..., (|G_i|/k) - 1\}$ for $1 \leq i$, $j \leq t-1$. Define $C_0 = \{u : 0 \leq u \leq |G_0| - 1\}$ and $R_0 = \{w : 0 \leq w \leq (|G_0|/k) - 1\}$. We call this frame an FGDRP$(k; T)$ if the blocks of $\mathcal{A}$ can be arranged into a $(|V|/k) \times |V|$ array satisfying the properties listed below. We index the rows and columns of the array by the elements of $R_0, R_1, ..., R_{t-1}$ and $C_0, C_1, ..., C_{t-1}$ in turn.

- Suppose that F_s is the subarray indexed by the elements of R_s and C_s for $0 \leq s \leq t-1$. Then F_s is empty. (These t subarrays lie in the main diagonal from upper left corner to lower right corner).

- For any $r \in R_i$ $(0 \leq i \leq t-1)$, the blocks in row r form a partial k-parallel class partitioning $V \setminus G_i$, that is, every point of $V \setminus G_i$ occurs in exactly k blocks in row r, while any point of G_i does not occur in any block in row r.

- For any $c \in C_j$ $(0 \leq j \leq t-1)$, the blocks in column c form a partial parallel class partitioning $V \setminus G_i$.

Now, we could describe following two constructions proved in [9], which are very useful to deal with the existence of GDRP*.

Theorem 2.1 [9]. *Let w be a positive integer. Suppose that there exist an* FGDRP$(k; \{|G_0|, |G_1|, ..., |G_{t-1}|\})$ *and a* GDRP$^*(k, |G_i| + w)$ *which* *contains a* $\mathrm{GDRP}^*(k, w)$ *as a subdesign for* $0 \le i \le t-1$. *Then there is a* $\mathrm{GDRP}^*(k, w + \sum_{0\le i\le t-1}|G_i|)$ *which contains a* $\mathrm{GDRP}^*(k, w)$ *as a subdesign.*

Theorem 2.2 [9]. *Let* $(V, \{G_0, G_1, ..., G_{t-1}\}, \mathcal{A})$ *be a* K-GDD. *Suppose that there exists a function* $w : V \to \mathbb{Z}^+ \cup \{0\}$ (*a weight function*) *which has the property that for each block* $B = \{x_1, x_2, ..., x_h\} \in \mathcal{A}$, *there exists an* $\mathrm{FGDRP}(k; \{w(x_1), w(x_2), ..., w(x_h)\})$. *Then there exists an* $\mathrm{FGDRP}(k; \{\sum_{x\in G_0} w(x), \sum_{x\in G_1} w(x), ..., \sum_{x\in G_{t-1}} w(x)\})$.

The existence of $\mathrm{FGDRP}(k; k^u)$ with $k = 5$ has been given in the following lemma:

Lemma 2.3 [7]. *Let* $E = \{7-14, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61\}$. *If* $u \ge 63$ *or* $u \in E$, *then an* $\mathrm{FGDRP}(5; 5^u)$ *exists.*

The following theorem gives a direct construction using starter-adder method:

Theorem 2.4 [9]. *If there exists an intransitive starter* (S, R, C) *for a* $\mathrm{GDRP}^*(k, v+w)$ *over* $\mathbb{Z}_u \times \mathbb{Z}_k \cup \{\infty_1, \infty_2, ..., \infty_w\}$ *and a corresponding adder* A *for* S, *then there exists a* $\mathrm{GDRP}^*(k, v+w)$ *missing a* $\mathrm{GDRP}^*(k, w)$ *as a subdesign, where* $u = v/k$. *Furthermore, if there exists a* $\mathrm{GDRP}^*(k, w)$, *then a* $\mathrm{GDRP}^*(k, v+w)$ *exists.*

We use starter-adder method to construct some optimal GDRP^*'s of small size. They are necessary in the recursive constructions.

Lemma 2.5. *For any* $4 \le m \le 15$, *there exists a* $\mathrm{GDRP}^*(5, 5m+4)$ *which having a* $\mathrm{GDRP}^*(5, 4)$ *as a subdesign.*

Proof. For the stated value of m, apply Theorem 2.4 with $k = 5$, $w = 4$ and $n = 5m$. The starter (S, R, C) and the corresponding adder A are listed below. Here, we take the group G as $Z_m \oplus Z_5$, $G_0 = \{0\} \oplus Z_5$, the fixed representative system of G_0 as $(h_0, h_1, \ldots, h_{m-1}) = ((0, 0), (1, 0), \ldots, (m-1, 0))$.

$m = 4$

S	{∞_1, (2,1), (0,4), (1,1), (3,1)}	A	(0,0)	S	{∞_2, (3,0), (0,0), (2,3), (1,4)}	A	(3,0)
	{∞_3, (3,2), (1,3), (2,4), (0,3)}		(2,0)		{∞_4, (0,1), (1,2), (2,0), (3,4)}		(1,0)
R	{(0,2), (1,0), (2,2), (3,3)}						

$m = 5$

S	{(4,4), (1,1), (3,0), (2,2), (0,1)}	A	(0,0)	S	{∞_1, (3,1), (0,0), (4,0), (2,3)}	A	(4,0)
	{∞_2, (0,3), (1,4), (2,1), (4,3)}		(3,0)		{∞_3, (4,2), (1,3), (2,4), (3,3)}		(2,0)
	{∞_4, (0,4), (1,0), (3,4), (4,1)}		(1,0)	R	{(0,2), (1,2), (2,0), (3,2)}		

$m = 6$

S	{(4,2), (5,1), (3,4), (1,2), (0,2)}	A	(0,0)	S	{(4,0), (0,0), (5,0), (3,1), (2,4)}	A	(5,0)
	{∞_1, (1,0), (2,3), (3,3), (0,3)}		(4,0)		{∞_2, (5,4), (3,0), (1,3), (4,1)}		(3,0)
	{∞_3, (1,4), (2,1), (0,1), (3,2)}		(2,0)		{∞_4, (1,1), (2,2), (4,4), (5,3)}		(1,0)
R	{(0,4), (2,0), (4,3), (5,2)}						

$m = 7$

S	{(3,4), (5,3), (4,3), (0,3), (6,4)}	A	(0,0)	S	{(2,0), (6,0), (1,0), (5,1), (4,2)}	A	(6,0)
	{(1,2), (2,2), (6,3), (4,4), (5,0)}		(5,0)		{∞_1, (1,4), (4,0), (3,2), (2,1)}		(4,0)
	{∞_2, (6,1), (5,4), (2,4), (0,1)}		(3,0)		{∞_3, (2,3), (0,2), (4,1), (3,0)}		(2,0)
	{∞_4, (0,4), (1,1), (3,1), (5,2)}		(1,0)	R	{(0,0), (1,3), (3,3), (6,2)}		

$m = 8$

S	{(7,0), (3,4), (0,0), (5,3), (1,3)}	A	(0,0)	S	{(2,0), (1,0), (6,0), (4,0), (7,1)}	A	(7,0)
	{(7,2), (6,1), (2,4), (0,4), (3,3)}		(6,0)		{(4,2), (7,3), (6,4), (0,1), (5,1)}		(5,0)
	{∞_1, (6,2), (7,4), (4,1), (3,1)}		(4,0)		{∞_2, (4,4), (2,1), (1,1), (3,2)}		(3,0)
	{∞_3, (0,2), (2,3), (4,3), (5,0)}		(2,0)		{∞_4, (1,2), (3,0), (5,4), (6,3)}		(1,0)
R	{(0,3), (1,4), (2,2), (5,2)}						

$m = 9$

S	{(1,4), (0,0), (2,4), (6,3), (3,0)}	A	(0,0)	S	{(2,2), (3,3), (7,0), (0,3), (1,0)}	A	(8,0)
	{(2,3), (6,1), (4,0), (7,4), (8,1)}		(7,0)		{(0,2), (1,1), (7,3), (5,2), (8,3)}		(6,0)
	{(5,3), (2,0), (7,2), (1,3), (8,2)}		(5,0)		{∞_1, (4,4), (0,4), (3,1), (6,2)}		(4,0)
	{∞_2, (4,1), (5,0), (2,1), (0,1)}		(3,0)		{∞_3, (1,2), (3,4), (5,1), (6,4)}		(2,0)
	{∞_4, (3,2), (4,2), (7,1), (8,4)}		(1,0)	R	{(4,3), (5,4), (6,0), (8,0)}		

$m = 10$

S	{(7,4), (5,4), (9,1), (0,2), (3,2)}	A	(1,0)	S	{(3,0), (1,2), (0,3), (5,1), (4,2)}	A	(3,0)
	{(4,0), (2,1), (9,0), (6,1), (3,1)}		(7,0)		{(6,2), (3,3), (2,0), (0,0), (5,2)}		(9,0)
	{(8,0), (9,3), (3,4), (5,0), (6,0)}		(6,0)		{(8,4), (0,1), (7,3), (4,1), (2,4)}		(0,0)
	{∞_1, (9,4), (8,1), (4,4), (1,3)}		(4,0)		{∞_2, (7,1), (1,0), (6,4), (5,3)}		(2,0)
	{∞_3, (2,2), (1,4), (0,4), (4,3)}		(5,0)		{∞_4, (9,2), (2,3), (7,2), (8,3)}		(8,0)
R	{(1,1), (6,3), (7,0), (8,2)}						

$m = 11$

S	{(4,0), (7,1), (6,4), (1,1), (5,0)}	A	(0,0)	S	{(8,1), (5,1), (6,3), (1,4), (10,2)}	A	(3,0)
	{(8,0), (6,0), (1,0), (9,4), (7,2)}		(2,0)		{(4,3), (3,2), (6,2), (0,4), (8,4)}		(10,0)
	{(10,3), (6,1), (4,1), (2,1), (1,2)}		(6,0)		{(7,4), (3,1), (5,2), (4,2), (8,2)}		(1,0)
	{(7,3), (1,3), (2,4), (8,3), (0,1)}		(4,0)		{∞_1, (0,3), (4,4), (9,3), (3,0)}		(9,0)
	{∞_2, (5,4), (9,0), (2,2), (10,1)}		(5,0)		{∞_3, (9,1), (0,2), (3,4), (5,3)}		(8,0)
	{∞_4, (10,4), (3,3), (2,0), (7,0)}		(7,0)	R	{(0,0), (2,3), (9,2), (10,0)}		

$m = 12$

S	{(7,3), (4,1), (11,1), (8,2), (3,3)}	A	(0,0)	S	{(4,0), (1,0), (3,4), (2,4), (8,0)}	A	(2,0)
	{(2,0), (8,3), (3,0), (11,2), (4,2)}		(8,0)		{(0,4), (1,2), (11,4), (10,1), (6,4)}		(6,0)
	{(11,3), (10,2), (5,0), (4,4), (7,0)}		(10,0)		{(8,4), (2,3), (0,3), (4,3), (6,2)}		(9,0)
	{(8,1), (6,3), (7,1), (5,2), (9,3)}		(3,0)		{(7,2), (10,0), (9,1), (0,1), (1,3)}		(11,0)
	{∞_1, (0,2), (5,3), (7,4), (1,1)}		(1,0)		{∞_2, (10,4), (6,0), (3,2), (9,2)}		(4,0)
	{∞_3, (11,0), (9,0), (2,2), (6,1)}		(7,0)		{∞_4, (0,0), (2,1), (5,1), (9,4)}		(5,0)
R	{(1,4), (3,1), (5,4), (10,3)}						

$m = 13$

S	{(0,1), (3,4), (9,0), (1,0), (11,4)}	A	(0,0)	S	{(0,2), (3,2), (1,2), (8,0), (4,2)}	A	(7,0)
	{(12,0), (1,4), (0,4), (10,4), (8,2)}		(9,0)		{(0,0), (7,3), (2,2), (4,0), (1,3)}		(4,0)
	{(11,0), (0,3), (5,3), (10,1), (2,4)}		(5,0)		{(7,0), (1,1), (10,2), (4,1), (9,4)}		(3,0)
	{(10,0), (4,4), (5,1), (3,3), (6,4)}		(8,0)		{(9,3), (5,0), (6,1), (4,3), (12,1)}		(2,0)
	{(7,2), (10,3), (5,2), (3,1), (9,2)}		(12,0)		{∞_1, (8,1), (7,4), (12,3), (3,0)}		(11,0)
	{∞_2, (7,1), (6,3), (12,2), (11,2)}		(6,0)		{∞_3, (12,4), (5,4), (6,0), (2,3)}		(10,0)
	{∞_4, (2,0), (6,2), (8,3), (11,1)}		(1,0)	R	{(2,1), (8,4), (9,1), (11,3)}		

$m = 14$

S	{(5,1), (2,0), (12,1), (7,0), (8,0)}	A	(0,0)	S	{(5,3), (8,1), (10,3), (11,1), (12,0)}	A	(1,0)
	{(6,4), (11,4), (1,1), (9,4), (7,1)}		(7,0)		{(7,3), (8,2), (5,2), (3,3), (13,3)}		(6,0)
	{(4,3), (7,2), (12,4), (3,1), (0,4)}		(3,0)		{(4,0), (3,0), (5,4), (0,3), (12,2)}		(11,0)
	{(13,0), (1,2), (6,0), (0,1), (3,2)}		(2,0)		{(7,4), (6,2), (10,0), (9,0), (13,1)}		(13,0)
	{(8,3), (2,2), (11,3), (3,4), (0,2)}		(4,0)		{(2,1), (8,4), (12,3), (1,0), (4,4)}		(9,0)
	{∞_1, (0,0), (13,4), (9,3), (11,2)}		(10,0)		{∞_2, (2,4), (4,2), (9,1), (10,4)}		(8,0)
	{∞_3, (1,4), (2,3), (6,1), (10,1)}		(12,0)		{∞_4, (1,3), (5,0), (6,3), (13,2)}		(5,0)
R	{(4,1), (9,2), (10,2), (11,0)}						

$m = 15$

S	{(9,0), (13,3), (12,1), (3,2), (2,1)}	A	(0,0)	S	{(13,2), (0,2), (1,3), (8,0), (4,0)}	A	(1,0)
	{(2,3), (13,1), (5,2), (6,0), (11,4)}		(8,0)		{(2,4), (7,2), (0,0), (8,1), (11,3)}		(11,0)
	{(8,3), (10,2), (6,2), (13,0), (12,0)}		(7,0)		{(14,3), (11,2), (3,3), (8,4), (0,1)}		(3,0)
	{(1,4), (13,4), (8,2), (6,4), (7,4)}		(14,0)		{(5,4), (12,4), (10,4), (14,0), (0,3)}		(10,0)
	{(9,4), (14,1), (12,3), (4,1), (6,1)}		(2,0)		{(11,1), (4,3), (9,1), (1,2), (5,0)}		(4,0)
	{(9,3), (12,2), (6,3), (14,4), (10,3)}		(13,0)		{∞_1, (11,0), (0,4), (2,2), (14,2)}		(12,0)
	{∞_2, (10,1), (9,2), (7,1), (1,1)}		(9,0)		{∞_3, (2,0), (7,3), (1,0), (3,4)}		(5,0)
	{∞_4, (10,0), (4,4), (3,0), (5,1)}		(6,0)	R	{(3,1), (4,2), (5,3), (7,0)}		

3. Main Results

Now, we are in a position to establish our main results.

Theorem 3.1. *For any integer m satisfying $5 \le m \le 16$ or $m \ge 530$, a* $\mathrm{GDRP}^*(5, 5m - 1)$ *exists.*

Proof. If $t \ge 75$, then a $\mathrm{TD}(8, t)$ exists [2]. We delete $t - w$ $(4 \le w \le 10)$ points from its last group, then take all the blocks containing a certain deleted point and the broken group as the new group to obtain a $(7, 8, t)$-GDD of type $7^t w^1$. Weighing it 5, apply Theorem 2.2 with the existence of $\mathrm{FGDRP}(5; 5^u)$ in Lemma 2.3 to obtain an $\mathrm{FGDRP}(5; 35^t (5w)^1)$, $4 \le w \le 10$. For any $5m - 1 \in [5(7t + 4) + 4,$

$5(7t+10)+4]$, a $\text{GDRP}^*(5, 5m-1)$ exists by using Theorem 2.1. Here, the required $\text{GDRP}^*(5, 5w+4)$'s containing a $\text{GDRP}^*(5, 4)$ as a subdesign are given in Lemma 2.5. Do the above steps for all the integers not less than 75, we then have a $\text{GDRP}^*(5, 5m-1)$, $m \geq 530$. Combining this with Lemma 2.5, we reach the conclusion.

According to Theorem 3.1, we obtain the following results by using Theorems 1.1 and 1.3.

Theorem 3.2. *For any integer* m, $5 \leq m \leq 16$ *or* $m \geq 530$, *an optimal* $\text{GDRP}(n, \lambda; v)$ *of type* $\lambda^1\mu^{m-1}$ *exists or equivalently, an optimal* $(n, M, n-\lambda, [\lambda, \mu, ..., \mu])_m$-CCC *with* $5\lambda = 4\mu$ *exists.*

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