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SMALL SAMPLE PROPERTIES AND MODEL CHOICE IN SPATIAL MODELS: A BAYESIAN APPROACH

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Abstract

This article considers the small sample properties and model choice problem in spatial models from a Bayesian point of view. Small sample properties of spatial autoregressive model (SAR), spatial error model (SEM) and spatial Durbin model (SDM) are examined through Monte Carlo simulations. To select a desirable (true) model, we also compare the performance of some information criteria and marginal likelihood by Monte Carlo studies. The simulation results show that serious spatial correlation bias appears in constant term and variance and that DIC performs the best to select an appropriate model.

1. Introduction

Spatial data has been widely used in several research areas like spatial statistics, regional science and other fields. In econometrics, we find that several estimation methods are proposed and the properties of the estimators are discussed. For example, the efficient maximum

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likelihood (ML) method was proposed by Ord [18] and Lee [16] first formally proved that the ML estimator has the usual asymptotic properties including $\sqrt{n}$-consistency, normality and asymptotic efficiency. Small sample properties have also been discussed by Bao and Ullah [3]. They analytically derived the second order bias and discussed the properties through Monte Carlo simulations. The properties of two-stage least squares are discussed by Das et al. [8], Kelejian and Prucha [12, 13] and Lee [15]. A class of moment method is examined by Conley [7] and Kelejian and Prucha [14]. The Bayesian approach was first considered by Anselin [1] and the small sample properties are examined using the Monte Carlo simulations. Thereafter, LeSage [17] proposed a Markov chain Monte Carlo (MCMC) method in a Bayesian analysis.

According to [2], there are three basic models: spatial autoregressive model (SAR); spatial error model (SEM) and spatial Durbin model (SDM). However, the properties of the models are not compared yet as far as we know. In this article, we examine the samll sample properties of these models through Monte Carlo simulations from a Bayesian point of view. In addition, model choice is one of the problems in empirical research if there are several candidates of the models. Therefore, we also compare several model choice approaches like information criteria and marginal likelihood by Monte Carlo studies. From the simulation results, we found that serious spatial correlation bias appears in constant term and variance and that the DIC performs the best to select an appropriate model.

The rest of this article is organized as follows: In Section 2, we summarize spatial models. In Section 3, we give the summary of model choice criteria. In Section 4, we implement Monte Carlo simulations and give the results. Finally, brief conclusions and remaining issues are given in Section 5.

2. Spatial Models

Let $\mathbf{y}$ and $\mathbf{X}$ be dependent and independent variables, respectively and these dimensions are $N \times 1$ and $N \times k.$ Moreover, let $\mathbf{W}$ be a weight matrix (see [2]). Then the spatial autoregressive model (SAR) is written as

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N), \tag{1}$$

where ρ is the parameter which measures the intensity of spatial interaction and $\mathbf{I}_N$ is an $N \times N$ unit matrix.

Given the model, the likelihood function of the model is written as

$$L(\mathbf{y} \mid \beta, \rho, \sigma^2, \mathbf{X}, \mathbf{W}) \propto (\sigma^2)^{-\frac{N}{2}} |\mathbf{I}_N - \rho\mathbf{W}| \exp\left(\frac{\mathbf{e}'\mathbf{e}}{2\sigma^2}\right), \tag{2}$$

where $\mathbf{e} = \mathbf{y} - \rho\mathbf{W}\mathbf{y} - \mathbf{X}\beta$.

The other types of basic models are called *spatial error model* (SEM) and *spatial Durbin model* (SDM). These models are expressed as

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{u}, \; \mathbf{u} = \rho\mathbf{W}\mathbf{u} + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N), \tag{3}$$

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{W}\hat{\mathbf{X}}\beta_{\mathbf{w}} + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N), \tag{4}$$

where $\hat{\mathbf{X}}$ is an $N \times (k-1)$ matrix, which excludes constant term from $\mathbf{X}$. Therefore, $\beta_{\mathbf{w}}$ is reduced to $(k-1)\times 1$ vector. As error terms are spatially correlated in (3), it is called SEM. On the other hand, both dependent and independent variables are correlated in (4) but the intensity of spatial correlations is different. Thus, we can find that the relationship among these three models. If we suppose no spatial correlation of independent variables in SDM, that is, $\beta_{\mathbf{w}} = 0$, it becomes SAR. In addition, if we suppose the intensities of independent and dependent variables are equal, that is, $\beta_{\mathbf{w}} = -\rho\hat{\beta}$, where $\hat{\beta}$ excludes intercept from β, it reduces to SEM.[1]

As we take a Bayesian approach, following prior distributions are assumed:

$$\beta \sim \mathcal{N}(\beta_0, \Sigma_0), \quad \rho \sim \mathcal{U}(-1, 1), \quad \sigma^2 \sim \mathcal{IG}(\nu_0/2, \lambda_0/2),$$

[1]Strictly speaking, intercept in β is also separated into two parts, that is, $\beta_0 = \beta_0^* - \sum_{j=1}^{N} \rho w_{ij}\beta_0^*$.

where $\mathcal{IG}(a, b)$ denotes an inverse gamma distribution with scale and shape parameters a and b.[2]

3. Model Choice Procedures

In this section, we briefly review several model choice approaches from a Bayesian point of view. If there are several candidates of econometric models, we need to choose the desirable model among them. There are several model choice procedures like information criteria, marginal likelihood and so on. First of all, we will briefly sketch the procedures, which we compare in this paper.

3.1. AIC and BIC

As one of the famous model choice procedures, we will pick up the information criteria like AIC and BIC, which can also be used in classical methods. For model M_k, let $L(\mathbf{y} \mid \theta_k, M_k)$ be the likelihood for the model.

AIC and BIC are given by

$$\mathrm{AIC}(\theta_k^*) = 2\ln[L(\mathbf{y} \mid \theta_k^*, M_k)] - 2p, \tag{5}$$

$$\mathrm{BIC}(\theta_k^*) = 2\ln[L(\mathbf{y} \mid \theta_k^*, M_k)] - p\ln(N), \tag{6}$$

where θ_k^* is a particular high density point (typically the posterior mean or the ML estimate), p is the number of parameters in model M_k and N is the number of observations. We choose the model with highest information criteria.

3.2. DIC

Spiegelhalter et al. [19] developed information criterion called the *Deviance Information Criterion* (DIC), which is designed to work well when models involve latent data and hierarchical priors. Let $\overline{D} = -2E$

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[2]The procedures for the posterior simulations are given in Appendix A.

$\cdot\{\ln[L(\mathbf{y} \mid \theta_k, M_k)]\}$ and $\hat{D} = -2\ln[L(\mathbf{y} \mid \theta_k^*, M_k)]$ be the posterior mean of the deviance and a point estimate of the deviance obtained by substituting in a particular high density point (typically the posterior mean or the ML estimate), respectively. Then the effective number of parameters p_D is given by $p_D = \overline{D} - \hat{D}$ and finally, DIC is written as

$$\mathrm{DIC}(\theta_k^*) = \overline{D} + p_D = \hat{D} + 2p_D. \tag{7}$$

The model with the smallest DIC is estimated to be the model that would best predict a replicate dataset which has the same structure as that currently observed.

3.3. Marginal likelihood

Let $\pi(\theta_k \mid M_k)$ be the prior for the model. Then the marginal likelihood of the model is defined as

$$m(\mathbf{y}) = \int \pi(\theta_k \mid M_k) L(\mathbf{y} \mid \theta_k, M_k) d\theta_k.$$

Since the marginal likelihood can be written as

$$m(\mathbf{y}) = \frac{\pi(\theta_k \mid M_k) L(\mathbf{y} \mid \theta_k, M_k)}{\pi(\theta_k \mid \mathbf{y}, M_k)},$$

Chib [4] suggests estimating the marginal likelihood from the expression

$$\log m(\mathbf{y}) = \log \pi(\theta_k^* \mid M_k) + \log L(\mathbf{y} \mid \theta_k^*, M_k) - \log \pi(\theta_k^* \mid \mathbf{y}, M_k).$$

He also provides a computationally efficient method to estimate the posterior ordinate $\pi(\theta_k^* \mid \mathbf{y}, M_k)$ in the context of Gibbs sampling and Chib and Jeliazkov [6] provides the method in the context of Metropolis-Hasting sampling. In SAR, for example, we set $\theta_k = (\beta, \rho, \sigma^2)$ and estimate the posterior ordinate $p(\theta_k^* \mid \mathbf{y}, M_k)$ via the decomposition

$$\pi(\theta_k^* \mid \mathbf{y}, M_k) = \pi(\rho^* \mid \beta^*, \sigma^{*2}, \mathbf{y}, \mathbf{X}, \mathbf{W}) \pi(\beta^* \mid \rho^*, \sigma^{*2}, \mathbf{y}, \mathbf{X}, \mathbf{W}) \pi(\sigma^{*2} \mid \rho^*, \beta^*, \mathbf{y}, \mathbf{X}, \mathbf{W}).$$

$\pi(\beta^* \mid \rho^*, \sigma^{*2}, \mathbf{y}, \mathbf{X}, \mathbf{W})$ and $\pi(\sigma^{*2} \mid \beta^*, \rho^*, \mathbf{y}, \mathbf{X}, \mathbf{W})$ are calculated from Gibbs output (see [4]) and $\pi(\rho^* \mid \beta^*, \sigma^{*2}, \mathbf{y}, \mathbf{X}, \mathbf{W})$ is calculated from Metropolis-Hasting output (see [6]).[3]

4. Monte Carlo Experiments

We now explain the setup for the Monte Carlo simulations. First, we set the number of regions as $N = 50$. The elements of lower triangular matrix of $\mathbf{W}$ are generated from $\mathcal{BE}(0.2)$, where $\mathcal{BE}(a)$ is Bernoulli distribution with probability of success a. For the independent variables $\mathbf{x}_i = (1, x_{1i}, x_{2i})$, we take the standard normal variates.[4] The true data generating process (DGP) is as

$$y_i^{\mathrm{SAR}} = \sum_{j=1}^{50} \rho w_{ij} y_j + \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i, \tag{8}$$

$$y_i^{\mathrm{SEM}} = \sum_{j=1}^{50} \rho w_{ij} (y_j - \beta_0 - \beta_1 x_{1j} - \beta_2 x_{2j}) + \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i, \tag{9}$$

$$\begin{aligned} y_i^{\mathrm{SDM}} = \sum_{j=1}^{50} \rho w_{ij} y_j + \sum_{j=1}^{50} w_{ij} (\beta_{\mathbf{w}1} x_{1j} - \beta_{\mathbf{w}2} x_{2j}) \\ + \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i, \end{aligned} \tag{10}$$

where the u_i's are normally and independently distributed with $E(u_i) = 0$ and $E(u_i^2) = \sigma^2$. The parameter values are set to be

[3]If we drop $\pi(\rho^* \mid \beta^*, \sigma^{*2}, \mathbf{y}, \mathbf{X}, \mathbf{W})$, it becomes the posterior ordinate of linear regression model.

[4]For example, as Bao and Ullah [3] is interested in the degree of sparseness of the weight matrix, it studied three types of weight matrices. However, the weight matrix is sometimes not regular like the ones in Bao and Ullah [3] in empirical analysis because the contiguity weight matrix assumes the existence of a map from which the boundaries can be discerned (see [2]). Therefore, we use the different weight matrix from Bao and Ullah [3].

$\beta' = (\beta_0, \beta_1, \beta_2) = (1, 1, 1)$ and $\beta'_{\mathbf{w}} = (\beta_{\mathbf{w}_1} - \beta_{\mathbf{w}_2}) = (-0.5, -0.5)$. Moreover, L samples of y_i^{SAR}, y_i^{SEM} and y_i^{SDM} are generated given the $\mathbf{x}_i$ for $i = 1, ..., 50$. That is, we perform L simulation runs for Bayesian estimators, where $L = 1000$ is taken in this section.

The simulation procedure in this section is as follows:

(i) Given $\rho = -0.9, -0.6, -0.3, 0.0, 0.3, 0.6, 0.9,$ we generate random numbers of u_i for $i = 1, ..., 50$ based on the assumption: $u_i \sim \mathcal{N}(0, \sigma^2)$, where $\sigma^2 = 0.1$ is taken.

(ii) Given $\beta, \beta_{\mathbf{w}}, \mathbf{x}_i$ and u_i for $i = 1, ..., 50,$ we obtain sets of data y_i^{SAR}, y_i^{SEM} and y_i^{SDM} for $i = 1, ..., 50,$ from (8)-(10), where (β_1, β_2) $= (1, 1, 1)$ and $\beta'_{\mathbf{w}} = (-0.5, -0.5)$ are assumed.

(iii) Given $(y_i^{\text{SAR}}, \mathbf{x}_i, \mathbf{w}_i)$, $(y_i^{\text{SEM}}, \mathbf{x}_i, \mathbf{w}_i)$ and $(y_i^{\text{SDM}}, \mathbf{x}_i, \mathbf{w}_i)$, for $i = 1, ..., 50,$ where $\mathbf{w}_i$ is the i-th row vector of $\mathbf{W}$, we obtain the estimates by Bayesian method. For prior distributions, following hyperparameters are assumed:

$$\beta_0 = \mathbf{0},\ \Sigma_0 = 1000^2 \times \mathbf{I}_k,\ \nu_0 = 0.01,\ \lambda_0 = 0.01,$$

and we ran MCMC algorithm for SAR, SEM, SDM and LRM,[5] using 6000 iterations and discarding the first 1000 iterations. We also calculate AIC, BIC, DIC and marginal likelihood.

(iv) Repeat (i)-(iii) L times, where $L = 1000$ is taken as mentioned above.

[5]LRM is the linear regression model like ordinary least square in classical method. It is reasonable to examine the properties of the models under misspecified. Therefore, we will estimate not only three kinds of spatial models but also LRM, which ignore the spatial effect.

(v) From L estimates, we compute the arithmetic average (AVE), the root mean squared error (RMSE), the skewness (SKEW) and kurtosis (KURT). For AVE and RMSE, for example in case of ρ, we compute

$$\text{AVE} = \frac{1}{L}\sum_{l=1}^{L}\rho^{(l)}, \quad \text{RMSE} = \left(\frac{1}{L}\sum_{l=1}^{L}(\rho^{(l)} - \rho)^2\right)^{\frac{1}{2}},$$

where $\rho^{(l)}$ represents the estimator of ρ in the l-th simulation run. We also choose the maximum values of AIC, BIC and marginal likelihood and minimum value of DIC.

In this section, we compare Bayes estimators through the Monte Carlo studies. All the results reported here is generated using Ox version 4.1 (see Doornik [9]).

Table 1 contains the basic statistics for ρ. Even in the true model, the bias exists and the bias becomes larger as ρ is farther away from 0. Moreover, in case that the SAR is true model, RMSEs are smallest among true models. On the other hand, in case that the SDM is true model, RMSEs are largest among true models and the bias becomes large as ρ becomes larger. We can also see that the empirical distribution is skewed in every model, as ρ is farther away from 0. In other words, the normality condition is not satisfied in case of small sample. As is stated below, such bias appears in β_0, $\beta_{\mathbf{w}}$ and σ^2.

Table 2 shows the basic statistics for β_0. From the table, we can see that the RMSEs under the true models are smallest in general, in other words, the AVEs are the closest to the true values under true models. The exceptions appear, for example, in case of SDM. In this model under $\rho = 0.9$, RMSE of SAR is smaller than that of SDM and AVE of SAR is close to that of SDM. In addition, in case that the true model follows SAR or SDM, if we ignore the spatial effect, RMSEs become large, that is, there exists bias, which may come from ignoring spatial effects and the bias becomes larger as ρ is farther away from 0. Also, the empirical distribution becomes skewed, as ρ is farther away from 0, that is, the normality is not satisfied. Finally, the behaviors of SEM and LRM are similar in case that the true model is SEM.

Tables 3 and 4 report the basic statistics for β_1 and β_2, respectively. In all cases, AVEs are around true values and RMSEs are very small. In other words, the bias does not appear even if we ignore the spatial effects or misspecify the model. Therefore, in empirical analysis if we find the differences in the slope parameters among models, we need to doubt that it may come from errors-in-variables or endogeniety, in other words, it does not come from ignoring spatial effects or misspecifying models.

Table 5 contains the basic statistics for $\beta_{\mathbf{w}}$. Even in case that the true model is SDM, AVEs are biased and the bias becomes larger as ρ increases. Needless to say, under misspecified model like true model is SAR or SEM, $\beta_{\mathbf{w}}$ are estimated far from 0. Therefore, such bias appears in different parameters like β_0 or ρ.

Table 6 shows the basic statistics for σ^2. From the table, we find that the AVEs come around the true values and the RMSEs are smallest under the true model in general. In addition, if we ignore the spatial effects, σ^2 becomes larger. In other words, there exists bias, which comes from ignoring spatial effects and the bias becomes larger as ρ is farther away from 0. Finally, the AVEs and RMSEs of SAR and SEM are similar with those of SDM and LRM, respectively.

Summarizing the Monte Carlo results, the bias, which comes from ignoring the spatial effects or misspecifying the model, appears in β_0 and σ^2, and β remains unbiased. In addition, ρ also has bias, which may come from small sample problem. It has the tendency that the bias shrinks to zero.

Table 7 reports the number of selections for each model. From the table, all the model choice procedures in all cases except for $\rho = 0.9$ by marginal likelihood, SAR model is chosen appropriately. Moreover, in case of SEM, the performance of information criterion and marginal likelihood is not as good as that in case of SAR, however, we can see that we can choose the true model to a certain extent. On the other hand, we cannot choose the true model appropriately in case of SDM, especially if ρ becomes large, such tendencies appear. However, in large ρ cases, distinct difference appears in DIC. The performance of DIC is better than other procedures. Therefore, we can conclude that if we want to implement the model choice, we recommend using DIC as a whole. However, we have to mention that DIC is still not perfect procedure in model choice because it cannot choose the true model perfectly.

5. Concluding Remarks

This article considers the small sample properties and model choice problem in spatial models from a Bayesian point of view. From the Monte Carlo simulations, we found that the bias, which comes from ignoring spatial effects or misspecifying the models, appears in β_0, $\beta_{\mathbf{w}}$ and σ^2. In addition, even in the true model, the bias of ρ exists and the bias becomes larger if ρ is farther away from 0. In case that the SAR is true model, RMSEs of ρ are smallest among true models. On the other hand, in case that the SDM is true model, RMSEs of ρ are largest among true models and the bias becomes large if ρ becomes larger. From the Monte Carlo studies for model choice, we found that the performance of DIC is the best among the model choice procedures, which we picked up. Therefore, we recommend using DIC for model choice in spatial models.

Finally, we discuss the remaining issue. We examined the basic three spatial models like SAR, SEM and SDM. However, we have to mention that these are not all the spatial models. In spatial statistics, conditional autoregressive model (CAR) is widely used and it may also be useful in econometrics. Therefore, we need to examine the properties of CAR and compare the properties of CAR and different models. In addition, as is stated above, although DIC performs best to choose the true model, it is not perfect procedure. Therefore, we have to consider alternative procedure to choose the true model.

Appendix A: Posterior simulation

This appendix explains our MCMC algorithm which was applied for the analysis in Section 4.[6]

Sampling ρ: Let $\theta = (\beta', \rho, \sigma^2)'$ and let $\pi(\theta)$ denote a prior distribution of θ, which is induced from the priors of β, ρ and σ^2. The full conditional distribution of θ is proportional to

$$\pi(\theta) L(\mathbf{y} \mid \theta, \mathbf{X}, \mathbf{W}), \tag{11}$$

where $L(\mathbf{y} \mid \theta, \mathbf{X}, \mathbf{W})$ is a likelihood function of the model given in (2). Since the full conditional distribution of ρ is not a standard distribution, we update ρ using random walk Metropolis-Hasting (MH) step (see [21]).

Sample ρ^{new} from

$$\rho^{\text{new}} = \rho^{\text{old}} + c\phi,\ \phi \sim \mathcal{N}(0, 1). \tag{12}$$

The scaler c is called *tuning parameter* and ρ^{old} is the parameter of the previous sampling. Next, we evaluate the acceptance probability

$$\alpha(\rho^{\text{old}}, \rho^{\text{new}}) = \min\left\{ \frac{|\mathbf{I}_N - \rho^{\text{new}}\mathbf{W}| \exp\left(\dfrac{e^{\text{new}\prime} e^{\text{new}}}{2\sigma^2}\right)}{|\mathbf{I}_N - \rho^{\text{old}}\mathbf{W}| \exp\left(\dfrac{e^{\text{old}\prime} e^{\text{old}}}{2\sigma^2}\right)}, 1 \right\}, \tag{13}$$

where $e^{\text{new}} = \mathbf{y} - \rho^{\text{new}}\mathbf{W}\mathbf{y} - \mathbf{X}\beta$ and $e^{\text{old}} = \mathbf{y} - \rho^{\text{old}}\mathbf{W}\mathbf{y} - \mathbf{X}\beta$. Finally, we set $\rho = \rho^{\text{new}}$ with probability $\alpha(\rho^{\text{old}}, \rho^{\text{new}})$ otherwise, $\rho = \rho^{\text{old}}$. The scalar c is tuned to produce an acceptance rate between 40% and 60% as is suggested in Holloway et al. [11]. It should be mentioned that the proposal density of ρ is not truncated to the interval $(-1, 1)$ since the constraint is part of the target density. Thus, if the proposal value of ρ is

[6]In this Appendix, we explain the MCMC algorithm for SAR. However, it is easy to implement the MCMC algorithm for SEM and SDM with a few changes.

not within the interval, the conditional posterior is zero and the proposal value is rejected with probability one (see [5]).

Sampling β and σ^2: The full conditional distributions for β and σ^2 are as follows:

$$\pi(\beta \mid \rho, \sigma^2, \mathbf{y}, \mathbf{X}, \mathbf{W}) \propto \mathcal{N}(\hat{\beta}, \hat{\Sigma}),$$

$$\pi(\sigma^2 \mid \beta, \rho, \mathbf{y}, \mathbf{X}, \mathbf{W}) \propto \mathcal{IG}(\hat{\nu}/2, \hat{\lambda}/2),$$

where $\overline{\mathbf{y}} = \mathbf{y} - \rho\mathbf{W}\mathbf{y}$, $\hat{\beta} = \hat{\Sigma}(\sigma^{-2}\mathbf{X}'\overline{\mathbf{y}} + \Sigma_0^{-1}\beta_0)$, $\hat{\Sigma} = (\sigma^{-2}\mathbf{X}'\mathbf{X} + \Sigma_0^{-1})^{-1}$, $\hat{\nu} = N + \nu_0$ and $\hat{\lambda} = \mathbf{e}'\mathbf{e} + \lambda_0$.

Since the full conditional distributions follow standard distributions, we update β and σ^2 using Gibbs' sampler (see [10]).

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Table 1. The basic statistics for ρ

		SAR				SEM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	-0.863	0.075	1.061	4.216	-0.868	0.079	3.834	27.698
	-0.6	-0.597	0.134	0.384	3.064	-0.504	0.273	1.033	3.782
	-0.3	-0.302	0.084	0.069	3.058	-0.254	0.342	0.542	2.639
	0.0	-0.007	0.093	0.144	2.953	0.017	0.291	-0.032	2.461
	0.3	0.269	0.134	-0.024	2.926	0.239	0.304	-0.445	2.502
	0.6	0.554	0.117	-0.248	3.038	0.465	0.303	-1.068	3.899
	0.9	0.803	0.122	-0.981	4.698	0.873	0.048	-4.612	47.038
SEM	-0.9	-0.118	0.793	0.018	3.006	-0.642	0.320	1.386	5.513
	-0.6	-0.153	0.474	-0.089	2.566	-0.451	0.291	0.726	3.013
	-0.3	-0.026	0.288	-0.021	2.726	-0.240	0.297	0.401	2.691
	0.0	-0.015	0.094	0.081	2.976	-0.021	0.299	0.005	2.383
	0.3	0.026	0.307	-0.024	2.874	0.243	0.295	-0.480	2.768
	0.6	0.059	0.556	-0.017	2.791	0.467	0.268	-0.992	4.121
	0.9	0.044	0.862	-0.099	3.003	0.616	0.335	-1.338	5.559
SDM	-0.9	-0.967	0.067	0.966	3.997	-0.911	0.033	7.134	114.388
	-0.6	-0.838	0.250	1.264	5.316	-0.557	0.244	1.338	5.022
	-0.3	-0.730	0.436	0.156	2.707	-0.353	0.458	0.893	2.725
	0.0	-0.446	0.454	-0.042	3.122	-0.205	0.444	0.442	2.193
	0.3	-0.138	0.460	-0.043	3.002	0.238	0.292	-0.431	2.736
	0.6	0.148	0.469	-0.024	2.973	0.449	0.282	-0.824	3.429
	0.9	0.419	0.492	-0.120	3.054	0.759	0.206	-2.948	14.952

		SDM				LRM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	-0.731	0.214	1.095	4.410	.NaN	.NaN	.NaN	.NaN
	-0.6	-0.524	0.223	0.817	3.382	.NaN	.NaN	.NaN	.NaN
	-0.3	-0.304	0.256	0.348	2.640	.NaN	.NaN	.NaN	.NaN
	0.0	-0.109	0.279	0.091	2.486	.NaN	.NaN	.NaN	.NaN
	0.3	0.105	0.334	-0.363	2.644	.NaN	.NaN	.NaN	.NaN
	0.6	0.336	0.357	-0.616	3.228	.NaN	.NaN	.NaN	.NaN
	0.9	0.221	0.702	0.145	2.782	.NaN	.NaN	.NaN	.NaN
SEM	-0.9	-0.663	0.293	1.306	4.949	.NaN	.NaN	.NaN	.NaN
	-0.6	-0.495	0.245	0.792	3.220	.NaN	.NaN	.NaN	.NaN
	-0.3	-0.321	0.271	0.504	2.992	.NaN	.NaN	.NaN	.NaN
	0.0	-0.140	0.297	0.073	2.510	.NaN	.NaN	.NaN	.NaN
	0.3	0.101	0.336	-0.352	2.730	.NaN	.NaN	.NaN	.NaN
	0.6	0.304	0.376	-0.626	3.450	.NaN	.NaN	.NaN	.NaN
	0.9	0.449	0.494	-0.921	4.123	.NaN	.NaN	.NaN	.NaN
SDM	-0.9	-0.780	0.161	1.147	4.903	.NaN	.NaN	.NaN	.NaN
	-0.6	-0.542	0.210	0.916	3.838	.NaN	.NaN	.NaN	.NaN
	-0.3	-0.310	0.238	0.359	2.834	.NaN	.NaN	.NaN	.NaN
	0.0	-0.108	0.282	0.027	2.443	.NaN	.NaN	.NaN	.NaN
	0.3	0.097	0.335	-0.332	2.812	.NaN	.NaN	.NaN	.NaN
	0.6	0.282	0.419	-0.598	3.010	.NaN	.NaN	.NaN	.NaN
	0.9	0.150	0.771	0.214	3.021	.NaN	.NaN	.NaN	.NaN

Table 2. The basic statistics for β_0

		SAR				SEM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	0.975	0.068	-0.327	3.137	0.439	0.562	0.006	2.827
	-0.6	1.000	0.080	-0.107	2.885	0.708	0.294	-0.171	2.994
	-0.3	1.000	0.107	0.000	2.968	0.666	0.336	-0.213	3.561
	0.0	1.006	0.089	0.082	2.809	1.001	0.048	0.055	2.857
	0.3	1.035	0.166	0.115	2.865	1.349	0.356	0.014	3.126
	0.6	1.107	0.281	0.343	3.202	2.448	1.453	0.091	3.015
	0.9	2.041	1.325	0.971	4.397	10.443	9.453	-0.011	3.016
SEM	-0.9	1.144	0.218	0.052	3.078	1.000	0.026	0.004	3.383
	-0.6	1.120	0.176	0.160	2.657	1.001	0.030	0.090	3.153
	-0.3	1.039	0.141	0.042	2.764	1.001	0.039	-0.004	3.714
	0.0	1.014	0.089	0.067	3.058	1.003	0.046	-0.047	3.433
	0.3	0.974	0.131	0.204	3.049	0.995	0.069	-0.037	3.166
	0.6	0.948	0.177	0.152	2.799	1.004	0.117	-0.127	3.063
	0.9	0.956	0.458	0.145	3.361	1.003	0.451	0.003	3.046
SDM	-0.9	0.933	0.083	-0.030	2.825	0.392	0.608	0.039	2.803
	-0.6	1.243	0.252	-0.221	3.423	0.774	0.228	0.031	2.993
	-0.3	1.214	0.236	0.070	2.945	0.509	0.495	-0.975	4.837
	0.0	1.493	0.502	0.065	3.038	1.098	0.110	0.415	3.476
	0.3	1.659	0.687	0.155	3.093	1.480	0.485	-0.031	3.039
	0.6	2.121	1.167	0.067	2.810	2.487	1.491	0.034	3.344
	0.9	5.949	5.066	0.239	3.321	10.208	9.219	-0.017	2.795

		SDM				LRM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	0.932	0.101	-0.407	3.333	0.407	0.593	-0.020	2.790
	-0.6	0.948	0.169	-0.598	3.167	0.710	0.291	-0.180	2.954
	-0.3	0.999	0.185	-0.233	2.721	0.659	0.343	-0.125	3.179
	0.0	1.110	0.284	-0.027	2.529	1.001	0.046	0.107	2.878
	0.3	1.263	0.459	0.421	2.796	1.351	0.357	-0.024	2.927
	0.6	1.634	0.861	0.658	3.380	2.418	1.422	0.035	2.883
	0.9	8.243	7.505	-0.116	2.772	10.640	9.651	-0.066	2.977
SEM	-0.9	1.661	0.684	-1.138	4.695	1.000	0.027	-0.072	3.211
	-0.6	1.496	0.546	-0.698	3.247	1.001	0.030	0.058	3.076
	-0.3	1.322	0.426	-0.425	2.947	1.001	0.037	0.130	3.114
	0.0	1.142	0.304	0.029	2.599	1.002	0.045	-0.012	3.148
	0.3	0.896	0.297	0.393	2.874	0.996	0.065	0.047	3.105
	0.6	0.699	0.389	0.721	3.810	1.003	0.114	-0.102	3.072
	0.9	0.551	0.561	1.141	5.787	1.003	0.462	-0.037	3.144
SDM	-0.9	0.959	0.074	-0.120	3.258	0.342	0.658	-0.014	2.785
	-0.6	0.957	0.172	-0.612	3.375	0.779	0.223	0.005	3.163
	-0.3	1.008	0.145	-0.152	2.774	0.484	0.517	0.017	2.921
	0.0	1.118	0.313	-0.044	2.437	1.098	0.108	0.155	3.295
	0.3	1.301	0.503	0.370	2.855	1.480	0.484	-0.012	2.837
	0.6	1.788	1.040	0.612	3.065	2.483	1.487	-0.078	3.193
	0.9	8.705	7.938	-0.150	2.989	10.267	9.279	0.025	2.767

Table 3. The basic statistics for β_1

		SAR				SEM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	1.005	0.059	0.052	3.309	0.949	0.076	0.101	3.339
	-0.6	0.998	0.046	-0.036	3.273	1.001	0.048	0.076	3.373
	-0.3	0.997	0.052	-0.011	2.964	0.995	0.058	0.106	2.898
	0.0	0.998	0.051	0.020	2.994	0.999	0.052	-0.017	3.086
	0.3	1.001	0.049	0.036	3.019	1.000	0.050	0.045	2.921
	0.6	1.003	0.047	-0.070	3.069	1.025	0.056	0.028	3.099
	0.9	1.002	0.038	-0.022	2.825	0.859	0.146	0.022	2.883
SEM	-0.9	0.994	0.060	0.064	3.265	0.997	0.054	0.070	3.487
	-0.6	0.996	0.048	-0.130	2.855	1.000	0.047	-0.079	2.874
	-0.3	0.998	0.053	-0.011	2.831	0.999	0.052	-0.009	2.855
	0.0	1.000	0.050	-0.010	3.054	1.001	0.051	-0.012	3.007
	0.3	0.997	0.048	-0.096	3.223	0.998	0.048	-0.083	3.173
	0.6	0.996	0.045	-0.107	3.030	1.001	0.044	-0.086	3.044
	0.9	1.004	0.042	-0.029	2.934	1.001	0.034	0.033	2.991
SDM	-0.9	0.991	0.057	-0.079	2.922	0.907	0.107	-0.018	3.043
	-0.6	0.995	0.045	-0.013	3.150	1.000	0.049	0.088	3.217
	-0.3	0.952	0.071	-0.056	2.920	0.974	0.108	0.544	2.787
	0.0	0.975	0.060	-0.020	3.006	0.986	0.074	0.080	2.803
	0.3	0.994	0.048	-0.073	3.052	1.001	0.047	-0.054	3.135
	0.6	0.999	0.045	0.054	2.965	1.005	0.045	0.055	3.174
	0.9	1.021	0.043	0.051	2.820	0.943	0.066	0.201	3.242

		SDM				LRM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	1.020	0.066	0.072	3.178	1.106	0.123	0.071	3.251
	-0.6	1.001	0.046	-0.039	3.327	1.027	0.054	-0.035	3.344
	-0.3	0.999	0.052	-0.004	2.879	1.015	0.055	-0.004	2.969
	0.0	0.999	0.051	-0.008	3.022	1.000	0.051	0.022	3.028
	0.3	1.005	0.049	0.018	2.995	1.008	0.050	0.033	3.053
	0.6	1.008	0.048	-0.038	3.048	1.056	0.074	-0.057	3.102
	0.9	1.077	0.090	-0.026	2.668	1.021	0.044	-0.047	2.833
SEM	-0.9	0.997	0.059	0.094	3.274	0.997	0.061	0.078	3.223
	-0.6	1.000	0.047	-0.107	2.850	1.000	0.048	-0.126	2.882
	-0.3	0.999	0.052	-0.008	2.880	1.000	0.053	-0.020	2.816
	0.0	1.001	0.050	-0.004	3.046	1.001	0.050	0.008	3.075
	0.3	0.998	0.048	-0.113	3.192	0.998	0.048	-0.094	3.227
	0.6	1.000	0.044	-0.098	3.019	1.000	0.046	-0.077	3.032
	0.9	1.001	0.041	0.076	2.913	1.001	0.039	0.036	2.904
SDM	-0.9	1.017	0.063	-0.006	2.905	1.161	0.171	-0.037	2.952
	-0.6	1.003	0.046	-0.056	3.089	1.052	0.070	-0.021	3.094
	-0.3	1.000	0.058	-0.004	2.804	1.045	0.069	-0.103	2.972
	0.0	0.996	0.055	-0.037	3.046	1.009	0.052	-0.039	2.967
	0.3	0.998	0.047	-0.045	3.023	0.996	0.047	-0.068	3.052
	0.6	1.002	0.045	0.063	2.976	1.010	0.047	0.031	2.993
	0.9	1.038	0.057	0.231	2.916	1.009	0.039	0.111	2.826

Table 4. The basic statistics for β_2

		SAR				SEM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	1.007	0.052	0.087	2.907	0.979	0.058	0.128	3.040
	-0.6	1.000	0.062	0.010	2.942	0.981	0.065	0.100	2.995
	-0.3	1.001	0.047	0.083	2.880	1.013	0.053	0.110	2.864
	0.0	0.999	0.044	0.013	2.918	1.000	0.045	0.026	3.004
	0.3	1.001	0.043	0.015	2.752	1.002	0.044	0.036	2.695
	0.6	1.001	0.045	-0.020	2.754	0.978	0.053	0.101	2.752
	0.9	1.015	0.045	0.008	3.079	0.986	0.045	0.034	3.188
SEM	-0.9	0.986	0.059	0.006	3.236	0.999	0.052	0.101	3.157
	-0.6	1.001	0.063	-0.071	2.980	0.998	0.061	-0.021	2.917
	-0.3	0.998	0.050	-0.019	3.319	1.000	0.049	-0.030	3.316
	0.0	0.999	0.045	-0.055	3.083	0.999	0.045	-0.077	3.068
	0.3	0.997	0.042	0.120	3.110	0.998	0.042	0.126	2.989
	0.6	0.999	0.048	-0.028	2.912	0.999	0.045	-0.013	2.893
	0.9	0.994	0.047	-0.026	2.871	0.998	0.041	0.060	2.976
SDM	-0.9	1.020	0.055	-0.140	3.593	0.953	0.072	-0.002	3.742
	-0.6	0.989	0.062	0.043	2.888	0.962	0.074	0.129	3.155
	-0.3	0.968	0.061	-0.085	2.921	1.010	0.089	0.441	2.941
	0.0	0.984	0.047	0.125	2.806	0.975	0.058	0.019	2.904
	0.3	0.996	0.044	0.028	2.978	1.000	0.044	0.003	3.009
	0.6	1.003	0.049	0.036	3.119	0.997	0.046	0.055	3.105
	0.9	1.024	0.049	-0.020	3.081	1.004	0.043	-0.027	3.023

		SDM				LRM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	1.022	0.059	0.101	2.859	1.210	0.220	0.119	2.856
	-0.6	1.002	0.063	0.052	2.935	0.999	0.064	0.002	2.892
	-0.3	1.002	0.047	0.078	2.909	1.030	0.056	0.050	2.895
	0.0	1.000	0.044	0.020	2.964	1.000	0.044	0.018	2.910
	0.3	1.003	0.043	0.020	2.752	1.009	0.044	0.012	2.803
	0.6	1.012	0.050	0.011	2.798	1.016	0.048	-0.030	2.832
	0.9	1.068	0.082	0.036	3.162	1.157	0.165	-0.045	2.879
SEM	-0.9	0.999	0.051	0.070	3.145	0.998	0.064	0.064	3.200
	-0.6	0.999	0.063	-0.054	2.937	0.999	0.064	-0.073	2.940
	-0.3	1.000	0.049	-0.025	3.302	1.001	0.050	-0.013	3.338
	0.0	0.999	0.045	-0.024	3.048	0.999	0.045	-0.042	3.073
	0.3	0.998	0.042	0.111	2.997	0.998	0.042	0.112	3.148
	0.6	0.999	0.050	-0.064	2.930	0.999	0.048	-0.018	2.897
	0.9	0.998	0.041	0.066	2.949	0.998	0.048	0.012	2.826
SDM	-0.9	1.028	0.064	0.010	3.397	1.332	0.339	-0.183	3.310
	-0.6	1.005	0.063	0.109	2.988	1.001	0.063	0.051	2.930
	-0.3	0.999	0.054	-0.054	3.044	1.075	0.091	-0.052	3.036
	0.0	0.997	0.044	0.112	2.823	0.990	0.044	0.128	2.767
	0.3	0.999	0.043	0.041	2.814	0.996	0.044	0.049	2.925
	0.6	1.002	0.052	0.083	3.162	1.003	0.049	0.050	3.144
	0.9	1.035	0.056	-0.005	3.034	1.071	0.086	-0.012	3.144

Table 5. The basic statistics for $\beta_{\mathbf{w}}$

		$\beta_{\mathbf{w}1}$				$\beta_{\mathbf{w}2}$			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	-0.211	0.377	-0.329	3.116	-0.203	0.364	-0.221	3.080
	-0.6	-0.078	0.516	-0.350	3.186	-0.074	0.559	-0.140	2.953
	-0.3	0.007	0.596	-0.075	2.826	0.007	0.588	-0.279	2.785
	0.0	0.110	0.675	0.012	2.885	0.108	0.678	0.024	2.611
	0.3	0.197	0.771	0.191	2.983	0.194	0.768	0.107	2.784
	0.6	0.272	0.831	0.327	3.216	0.261	0.813	0.249	2.791
	0.9	0.790	1.310	-0.070	2.930	0.834	1.359	-0.004	2.853
SEM	-0.9	0.666	1.195	-0.383	3.244	0.667	1.190	-0.648	3.862
	-0.6	0.494	1.037	-0.329	3.033	0.489	1.050	-0.262	3.282
	-0.3	0.321	0.876	-0.455	3.002	0.318	0.873	-0.417	2.946
	0.0	0.137	0.707	0.006	2.866	0.132	0.696	0.034	2.753
	0.3	-0.095	0.521	0.143	2.752	-0.104	0.513	0.159	3.021
	0.6	-0.308	0.359	0.275	3.236	-0.302	0.346	0.433	3.338
	0.9	-0.448	0.234	0.720	3.832	-0.448	0.257	0.479	3.098
SDM	-0.9	-0.656	0.284	-0.369	3.268	-0.662	0.263	-0.488	3.615
	-0.6	-0.551	0.306	-0.261	3.104	-0.553	0.337	0.037	2.920
	-0.3	-0.487	0.305	-0.247	2.861	-0.486	0.286	-0.164	2.829
	0.0	-0.379	0.340	-0.065	2.550	-0.387	0.328	0.000	2.701
	0.3	-0.292	0.394	0.155	2.941	-0.295	0.402	0.231	2.868
	0.6	-0.172	0.470	0.312	2.808	-0.178	0.455	0.339	2.851
	0.9	0.312	0.842	0.051	2.953	0.326	0.864	0.055	3.200

Table 6. The basic statistics for σ^2

		SAR				SEM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	0.103	0.022	0.388	3.228	0.231	0.136	0.151	2.901
	-0.6	0.104	0.022	0.588	4.322	0.128	0.038	0.522	3.737
	-0.3	0.104	0.022	0.462	3.592	0.125	0.036	0.480	3.438
	0.0	0.104	0.022	0.348	2.981	0.102	0.022	0.376	3.065
	0.3	0.105	0.023	0.479	3.160	0.112	0.027	0.515	3.306
	0.6	0.105	0.023	0.526	3.516	0.149	0.058	0.456	3.231
	0.9	0.111	0.025	0.623	4.898	0.324	0.229	0.105	3.146
SEM	-0.9	0.130	0.043	0.560	3.949	0.110	0.025	0.361	3.141
	-0.6	0.113	0.027	0.486	3.319	0.104	0.023	0.413	3.196
	-0.3	0.106	0.022	0279	2.808	0.102	0.021	0.287	2.879
	0.0	0.104	0.021	0.377	3.193	0.102	0.021	0.362	3.209
	0.3	0.106	0.023	0.341	3.121	0.103	0.022	0.237	2.832
	0.6	0.114	0.028	0.486	3.171	0.105	0.022	0.402	3.089
	0.9	0.127	0.041	0.603	3.484	0.109	0.024	0.499	3.517
SDM	-0.9	0.128	0.039	0.510	3.218	0.404	0.309	0.360	3.066
	-0.6	0.106	0.023	0.467	3.431	0.183	0.090	0.315	2.966
	-0.3	0.107	0.024	0.337	3.040	0.255	0.161	0.351	2.950
	0.0	0.106	0.023	0.422	3.087	0.153	0.060	0.432	3.196
	0.3	0.107	0.024	0.387	3.219	0.106	0.023	0.323	2.986
	0.6	0.109	0.025	0.459	3.870	0.105	0.022	0.287	3.203
	0.9	0.116	0.029	0.437	3.355	0.153	0.061	0.359	3.032

		SDM				LRM			
True Model	ρ	AVE	RMSE	SKEW	KURT	AVE	RMSE	SKEW	KURT
SAR	-0.9	0.106	0.023	0.402	3.292	0.342	0.251	0.321	2.981
	-0.6	0.104	0.023	0.601	4.334	0.145	0.055	0.602	3.758
	-0.3	0.103	0.022	0.409	3.223	0.133	0.043	0.473	3.258
	0.0	0.103	0.022	0.389	3.065	0.104	0.022	0.343	2.957
	0.3	0.104	0.024	0.497	3.161	0.116	0.030	0.538	3.413
	0.6	0.107	0.024	0.511	3.447	0.165	0.074	0.644	3.898
	0.9	0.129	0.039	0.486	3.398	0.498	0.408	0.285	3.458
SEM	-0.9	0.109	0.025	0.330	3.076	0.134	0.047	0.559	3.695
	-0.6	0.104	0.023	0.414	3.220	0.115	0.030	0.471	3.139
	-0.3	0.102	0.022	0.328	2.803	0.106	0.023	0.251	2.895
	0.0	0.102	0.021	0.336	3.130	0.104	0.021	0.385	3.269
	0.3	0.104	0.022	0.252	2.861	0.107	0.023	0.346	3.140
	0.6	0.107	0.023	0.435	3.110	0.115	0.029	0.496	3.210
	0.9	0.113	0.027	0.459	3.346	0.129	0.042	0.605	3.510
SDM	-0.9	0.106	0.024	0.400	3.007	0.644	0.554	0.412	3.093
	-0.6	0.105	0.023	0.444	3.322	0.212	0.120	0.348	2.822
	-0.3	0.104	0.023	0.306	2.867	0.293	0.200	0.335	3.179
	0.0	0.103	0.022	0.446	3.152	0.163	0.070	0.478	3.376
	0.3	0.103	0.022	0.395	3.099	0.110	0.025	0.359	3.092
	0.6	0.105	0.023	0.346	3.572	0.114	0.029	0.441	3.647
	0.9	0.124	0.037	0.528	3.438	0.203	0.114	0.584	3.169

Table 7. Results of model choice

AIC												
	SAR				SEM				SDM			
ρ	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM
-0.9	894	0	106	0	53	743	138	66	51	0	949	0
-0.6	846	18	132	4	103	542	93	262	577	0	423	0
-0.3	835	12	141	12	129	316	69	486	581	0	419	0
0.0	142	112	52	694	135	121	60	684	700	0	300	0
0.3	608	41	124	227	126	157	50	667	249	119	148	484
0.6	844	1	155	0	130	365	83	422	285	282	71	362
0.9	845	0	155	0	98	585	111	206	938	10	52	0

BIC												
	SAR				SEM				SDM			
ρ	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM
-0.9	982	0	18	0	49	737	17	197	227	0	773	0
-0.6	944	22	17	17	72	444	12	472	855	0	145	0
-0.3	914	14	23	49	59	190	6	745	851	0	149	0
0.0	59	47	4	890	50	51	6	893	913	0	87	0
0.3	484	23	20	473	49	58	6	887	160	46	25	769
0.6	965	1	31	3	69	216	4	711	182	178	15	625
0.9	951	0	49	0	60	484	16	440	980	10	7	3

DIC												
	SAR				SEM				SDM			
ρ	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM
-0.9	878	0	122	0	46	743	154	57	58	0	942	0
-0.6	793	25	178	4	88	553	118	241	623	0	377	0
-0.3	778	17	194	11	121	336	94	449	552	0	448	0
0.0	148	125	61	666	131	135	72	662	658	0	342	0
0.3	584	41	163	212	127	161	59	653	254	113	183	450
0.6	808	1	191	0	131	355	100	414	296	268	111	325
0.9	751	0	249	0	93	578	130	199	843	9	148	0

Marginal likelihood												
	SAR				SEM				SDM			
ρ	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM	SAR	SEM	SDM	LRM
-0.9	925	0	75	0	44	103	821	32	424	0	576	0
-0.6	840	58	90	12	211	308	370	111	916	1	83	0
-0.3	649	15	313	23	175	26	588	211	912	0	88	0
0.0	284	250	142	324	293	223	166	318	904	0	96	0
0.3	236	50	224	490	327	202	62	409	610	10	84	296
0.6	215	0	391	394	260	215	21	504	155	0	66	779
0.9	59	0	941	0	206	526	17	251	903	0	50	47

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