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APPLICATION OF THE EXP-FUNCTION METHOD TO THE GENERALIZED BURGER'S-FISHER EQUATION

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Abstract

The Exp-function method is applied to the generalized Burger's-Fisher equation and abundant exact solutions are obtained. It is shown that the method with the help of symbolic computation provides a powerful mathematical tool for solving nonlinear evolution equations.

1. Introduction

It is significant to seek the exact solutions for the nonlinear evolution equations (NLEEs). In order to obtain the exact solutions of the NLEEs, various powerful methods have been presented such as the inverse scattering method [1], the Bäcklund transformation and homogeneous

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balance approach [17, 18], the Hirota's bilinear method [10], the mapping method [15], the tanh function method [16, 21], the Jacobian elliptic function method [14], the homotopy perturbation method [5, 6], the variational iteration method [4, 8] and so on.

Recently, He and Wu [9] proposed a straightforward and concise method called the Exp-function method to obtain the generalized solitary wave solutions and periodic solutions of the NLEEs. The basic idea of the algorithm is: For a given (1+1)-dimensional nonlinear model, the equation reads

$$P(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \ldots) = 0, \tag{1}$$

where P is in general a polynomial function of its arguments and their subscripts denote the partial derivatives. By using the travelling wave transformation, equation (1) possesses the following ansatz:

$$u(x, t) = U(\eta),\ \eta = kx + \omega t, \tag{2}$$

where k and ω are unknown constants. Substituting equation (2) into equation (1) yields an ordinary differential equation (ODE) $O(U(\eta), U(\eta)_\eta, U(\eta)_{\eta\eta}, \ldots) = 0$. Then we assume that the solution of equation (1) can be expressed in the following form:

$$U(\eta) = \frac{\sum_{n=c}^{-d} a_n \exp(n\eta)}{\sum_{m=p}^{-q} b_m \exp(m\eta)}, \tag{3}$$

where c, d, p and q are positive integers to be further determined, a_n and b_m are unknown constants. Equation (3) can be rewritten in an alternative form as follows:

$$U(\eta) = \frac{a_c \exp(c\eta) + \cdots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \cdots + b_{-q} \exp(-q\eta)}. \tag{4}$$

In order to determine the values of c, p, we balance the linear term in equation (1) with the highest order nonlinear term [7, 9]. Similarly to determine the values of d and q, we balance the linear term of lowest order in equation (1).

Recently, Zhu extended the Exp-function method to the discrete nonlinear systems and some new solutions were obtained [20]. The procedure of this method with the help of Mathematica is of utter simplicity and can be easily extended to many other kinds of NLEEs.

In this letter, the Exp-function method is extended and applied to construct the exact solutions of the generalized Burger's-Fisher equation.

2. Application to the Generalized Burger's-Fisher Equation

We use the Exp-function method to find the exact solutions of the generalized Burger's-Fisher equation as the following form:

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u(1 - u^{\delta}), \ x \in \Omega = [0, 1], \ t \geq 0, \tag{5}$$

where α, β and δ are parameters. In recent years, many researchers used various methods to solve the Burger's-Fisher equation (5). Kaya and Sayed [13] introduced a numerical simulation and obtained explicit solutions of the generalized Burger's-Fisher equation. Ismail and Rabboh [11] presented a restrictive Pade approximation for the solution of the this equation. Ismail et al. [12] studied the adomian decomposition method for the Burger's-Huxley and Burger's-Fisher equation. Wazwaz [19] used the generalized tanh method for the Burger's-Fisher equation. Golbabai and Javido [3] used a spectral adomian decomposition approach for it.

Here, we first give a transformation

$$u(x, t) = v(x, t)^{\frac{1}{\delta}}. \tag{6}$$

Substituting equation (6) into equation (5), we have

$$v(v_t + \alpha v v_x - v_{xx}) - \left(\frac{1}{\delta} - 1\right) v_x^2 = \beta \delta v^2 (1 - v). \tag{7}$$

Based on the Exp-function, we assume equation (7) possesses the following ansatz:

$$v = U(\eta), \ \eta = kx + \omega t. \tag{8}$$

Substituting equation (8) into equation (7), we have

$$U\left(\omega U_\eta + k\alpha UU_\eta - k^2U_{\eta\eta}\right) - \left(\frac{1}{\delta} - 1\right)k^2U_\eta^2 = \beta\delta U^2\left(1 - U\right). \tag{9}$$

Balancing the linear term U^3 and the nonlinear term U^2U_η of the highest order and the linear term of the lowest order in equation (9), we consider the circumstance $p = c = 1$, $q = d = 1$.

Substituting $p = c = 1$, $q = d = 1$ into equation (5), we can obtain

$$U(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}, \tag{10}$$

where a_1, a_0, a_{-1}, b_1, b_0 and b_{-1} are undetermined coefficients.

Substituting equation (10) into equation (9) and using Mathematica, we have

$$\frac{1}{A}[C_4 \exp(4\eta) + C_3 \exp(3\eta) + C_2 \exp(2\eta) + C_1 \exp(\eta) + C_0$$

$$+ C_{-1} \exp(-\eta) + C_{-2} \exp(-2\eta) + C_{-3} \exp(-3\eta) + C_{-4} \exp(-4\eta)] = 0, \tag{11}$$

where $A = [b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)]^4$,

$$C_4 = \delta^2\beta a_1^3 b_1 - \delta^2\beta a_1^2 b_1^2,$$

$$C_3 = \delta^2\beta a_1^3 b_0 + k\delta\alpha a_1^3 b_0 + k^2\delta a_1^2 b_0 b_1 - \delta a_1 \omega a_0 b_1^2 + \delta\omega a_1^2 b_0 b_1$$

$$- k^2\delta a_1 a_0 b_1^2 b_1 - 2\delta^2\beta a_1 a_0 b_1^2 - 2\delta^2\beta a_1^2 b_0 b_1 + 3\delta^2\beta a_1^2 a_0 b_1 - k\delta\alpha a_1^2 a_0,$$

$$C_2 = 4\delta k^2 a_1^2 b_{-1} b_1 - 4\delta k^2 a_1 a_{-1} b_1^2 - \delta^2\beta a_0^2 b_1^2 + \delta\omega a_1^2 b_0 + \delta^2\beta a_1^3 b_{-1} - \delta\omega a_0^2 b_1^2$$

$$- 4\delta^2\beta a_1 b_0 a_0 b_1 + 3\delta^2\beta a_1^2 a_{-1} b_1 + 2k^2 a_1 b_0 a_0 b_1 - 2\delta^2\beta a_1^2 b_{-1} b_1$$

$$- 2\delta k a_1 \alpha a_0^2 b_1 + 2\delta k\alpha a_1^2 b_0 a_0 - 2\delta^2\beta a_1 a_{-1} b_1^2 - 2\delta\omega a_1 a_{-1} b_1^2$$

$$+ 2\delta k\alpha a_1^3 b_{-1} + 2\delta\omega a_1^2 b_{-1} b_1 - 2\delta k\alpha a_1^2 a_{-1} b_1 - \delta^2\beta a_1^2 b_0^2 - k^2 a_1^2 b_0^2$$

$$+ 3\delta^2\beta a_1^2 b_0 a_0 - k^2 a_0^2 b_1^2 + 3\delta^2\beta a_1 a_0^2 b_1,$$

$$
\begin{aligned}
C_1 = {} & \delta k^2 a_0^2 b_0 b_1 + \delta k \alpha a_{-1} a_1^2 b_0 + \delta \omega a_0 b_0^2 a_1 + \delta k \alpha a_0^2 a_1 b_0 - \delta k^2 a_0 a_{-1} b_1^2 \\
& - 2\delta^2 \beta a_0 a_1 b_0^2 - 2\delta^2 \beta a_0^2 b_0 b_1 - 4k^2 a_1^2 b_0 b_{-1} - 6\delta k^2 a_1 a_{-1} b_0 b_1 \\
& + 3\delta a_1^2 \omega b_0 b_{-1} - 2\delta^2 \beta a_0 a_{-1} b_1^2 + 5\delta k a_0 a_1^2 b_{-1} + 3\delta^2 \beta a_0 a_1^2 b_{-1} \\
& - 6\delta k a_0 a_1 a_{-1} b_1 - \delta \omega a_0^2 b_0 b_1 + 3\delta^2 \beta a_0^2 a_1 b_0 - \delta k^2 a_0 a_1 b_0^2 \\
& + 4k^2 a_0 a_1 b_{-1} b_1 + 2\delta \omega a_0 a_1 b_{-1} b_1 + 6\delta^2 \beta a_0 a_1 a_{-1} b_1 - 2\delta^2 \beta b_{-1} a_1^2 b_0 \\
& + \delta^2 \beta a_0^3 b_1 - 4k^2 a_{-1} a_0 b_1^2 - 4\delta^2 \beta a_{-1} a_1 b_0 b_1 - 3\delta a_0 \omega a_{-1} b_1^2 \\
& - 4\delta^2 \beta b_{-1} a_1 a_0 b_1 - 2\delta \omega a_{-1} a_1 b_0 b_1 + \delta k^2 b_{-1} a_1^2 b_0 + 3\delta^2 \beta a_{-1} a_1^2 b_0 \\
& - \delta k \alpha a_0^3 b_1 + 6\delta k^2 b_{-1} a_1 a_0 b_1 + 4k^2 a_1 a_{-1} b_0 b_1,
\end{aligned}
$$

$$
\begin{aligned}
C_0 = {} & 4\delta a_1 \omega a_0 b_{-1} b_0 - 4\delta k \alpha a_0^2 a_{-1} b_1 + 8k^2 b_{-1} a_1 a_{-1} b_1 - 4k^2 a_{-1}^2 b_1^2 \\
& + 3\delta^2 \beta a_1^2 a_{-1} b_{-1} - 4\delta^2 \beta a_1 a_0 b_0 b_{-1} - 4\delta^2 \beta a_1 a_{-1} b_{-1} b_1 - 2\delta^2 \beta a_1 a_{-1} b_0^2 \\
& + 2\delta a_1^2 \omega b_{-1}^2 + 3\delta^2 \beta a_1 a_0^2 b_{-1} - 4\delta^2 \beta a_0 a_{-1} b_0 b_1 + 3\delta^2 \beta a_1 a_{-1}^2 b_1 \\
& - 2\delta^2 \beta a_0^2 b_{-1} b_1 + 2a_1 k^2 a_{-1} b_0^2 - 4\delta a_0 \omega a_{-1} b_0 b_1 + 6\delta^2 \beta a_1 a_0 a_{-1} b_0 \\
& - \delta^2 \beta a_{-1}^2 b_1^2 - 2a_0 k^2 a_{-1} b_0 b_1 + 3\delta^2 \beta a_0^2 a_{-1} b_1 + \delta^2 \beta a_0^3 b_0 \\
& - 2\delta a_{-1}^2 \omega b_1^2 - \delta^2 \beta a_1^2 b_{-1}^2 - 4\delta a_1 k^2 a_{-1} b_0^2 - 2a_1 k^2 a_0 b_0 b_{-1} \\
& - 4\delta a_1 k \alpha a_{-1}^2 b_1 + 4\delta a_1^2 k \alpha a_{-1} b_{-1} + 4\delta a_1 k \alpha a_0^2 b_{-1} + 2k^2 a_0^2 b_{-1} b_1 \\
& - 4\delta k \alpha a_0^2 a_{-1} b_1 - 4k^2 a_1^2 b_{-1}^2 - \delta^2 \beta a_0^2 b_0^2,
\end{aligned}
$$

$$
\begin{aligned}
C_{-1} = {} & \delta^2 \beta a_0^3 b_{-1} + \delta k^2 a_0^2 b_{-1} b_0 - 4k^2 b_1 a_{-1}^2 b_0 + 3\delta^2 \beta a_1 a_{-1}^2 b_0 \\
& + 3\delta^2 \beta a_{-1} a_0^2 b_0 - 2\delta^2 \beta a_0^2 b_{-1} b_0 - 6k^2 \delta a_1 a_{-1} b_{-1} b_0 + \delta \omega a_0^2 b_0 b_{-1} \\
& + \delta k \alpha a_0^3 b_{-1} + \delta k^2 \beta a_{-1}^2 b_0 b_1 - 2\delta^2 \beta a_1 b_{-1}^2 a_0 - \delta k^2 a_1 a_0 b_{-1}^2
\end{aligned}
$$

$$+ 3\delta a_1 \omega b_{-1}^2 a_0 - \delta k^2 a_{-1} a_0 b_0^2 - 2\delta^2 \beta a_0 a_{-1} b_0^2 - 2\delta a_0 \omega b_{-1} a_{-1} b_1$$

$$- \delta a_1 k \alpha a_{-1}^2 b_0 + 6\delta^2 \beta a_0 a_1 b_{-1} a_{-1} - 2\delta^2 \beta b_0 a_{-1}^2 b_1 - 5\delta a_0 k \alpha a_{-1}^2 b_1$$

$$- 4 a_0 k^2 a_1 b_{-1}^2 + 4 a_0 k^2 a_1 b_{-1} b_1 - \delta k \alpha a_0^2 a_{-1} b_0 - 3\delta a_{-1}^2 \omega b_0 b_1$$

$$- 4\delta^2 \beta a_1 a_{-1} b_0 b_{-1} + 6\delta a_1 k \alpha a_0 a_{-1} b_{-1} + 2\delta a_1 \omega b_0 a_{-1} b_{-1}$$

$$+ 6\delta k^2 a_0 a_{-1} b_{-1} b_1 - \delta a_0 \omega a_{-1} b_0^2 - 4\delta^2 \beta a_0 a_{-1} b_{-1} b_1$$

$$+ 4k^2 a_1 a_{-1} b_0 b_{-1} + 3\delta^2 \beta a_0 a_{-1}^2 b_1,$$

$$C_{-2} = \delta^2 \beta a_{-1}^3 b_1 - 4\delta a_{-1} k^2 a_1 b_{-1}^2 - 4\delta^2 \beta a_{-1} a_0 b_{-1} b_0 - 2\delta^2 \beta a_{-1} a_1 b_{-1}^2$$

$$- 2\delta a_{-1}^2 a_0 \alpha k b_0 + 3\delta^2 \beta a_{-1}^2 a_1 b_{-1} + \delta \omega a_0^2 b_{-1}^2 - 2\delta \omega a_{-1}^2 b_{-1} b_1$$

$$- \delta a_{-1}^2 \omega b_0^2 + 2\delta k \alpha a_{-1} a_0^2 b_{-1} - k^2 a_{-1}^2 b_0^2 + 2k^2 a_{-1} a_0 b_{-1} b_0$$

$$+ 2\delta a_{-1} \omega a_1 b_{-1}^2 + 3\delta^2 \beta a_{-1}^2 a_0 b_0 - k^2 a_0^2 b_{-1}^2 + 3\delta^2 \beta a_{-1} a_0^2 b_{-1}$$

$$- 2\delta \omega a_{-1}^2 b_{-1} b_1 + 4\delta a_{-1}^2 k^2 b_{-1} b_1 + 2\delta a_{-1}^2 k \alpha a_1 b_{-1}$$

$$- 2\delta a_{-1}^3 a_0 k \alpha b_1 - \delta^2 \beta a_0^2 b_{-1}^2 - \delta^2 \beta a_{-1}^2 b_0^2,$$

$$C_{-3} = \delta a_{-1} \omega a_0 b_{-1}^2 + \delta a_{-1}^2 k^2 b_0 b_{-1} + \delta a_{-1}^2 k \alpha a_0 b_{-1} + \delta^2 \beta a_{-1}^3 b_0$$

$$- \delta a_{-1} k^2 a_0 b_{-1}^2 - \delta a_{-1}^2 \omega b_0 b_{-1} - 2\delta^2 \beta a_{-1} a_0 b_{-1}^2 + 3\delta^2 \beta a_{-1}^2 a_0 b_{-1}$$

$$- 2\delta^2 \beta a_{-1}^2 b_0 b_{-1} - \delta a_{-1}^3 k \alpha b_0,$$

$$C_{-4} = \delta^2 \beta a_{-1}^3 b_{-1} - \delta^2 \beta a_{-1}^2 b_{-1}^2,$$

Equating the coefficients of $\exp(n\eta)$ to be zero, we have

$$\begin{cases} C_4 = 0,\ C_3 = 0,\ C_2 = 0,\ C_1 = 0, \\ C_0 = 0, \\ C_{-4} = 0,\ C_{-3} = 0,\ C_{-2} = 0,\ C_{-1} = 0. \end{cases} \tag{12}$$

Then substituting the results into equations (6), (8) and (10), we obtain the abundant exact solutions of equation (5):

Case 1. When $\eta = -\frac{\delta\alpha}{2(\delta+1)}x + \frac{\delta[\alpha^2 + \beta(\delta+1)^2]}{2(\delta+1)^2}t,$ we have

$$u_1 = \left[\frac{a_1 \exp(\eta)}{a_1 \exp(\eta) + b_{-1} \exp(-\eta)}\right]^{\frac{1}{\delta}}, \tag{13}$$

where a_1 and b_{-1} are free real numbers.

Especially, when $a_1 = b_{-1} \neq 0,$ the result in [12] is obtained:

$$u(x, t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\eta\right)^{\frac{1}{\delta}};$$

when $a_1 = b_{-1} \neq 0,$ $\alpha\delta = -1,$ the result in [2] is obtained:

$$u(x, t) = \left(\frac{1}{2} - \frac{1}{2}\tanh\eta\right)^{\frac{1}{\delta}}.$$

Case 2. When $\eta = -\frac{\delta\alpha}{(\delta+1)}x + \frac{\delta[\alpha^2 + \beta(\delta+1)^2]}{(\delta+1)^2}t,$ we have

$$u_2 = \left[\frac{a_1 \exp(\eta)}{a_1 \exp(\eta) + b_0}\right]^{\frac{1}{\delta}}, \tag{14}$$

$$u_3 = \left[\frac{b_0}{b_0 + b_{-1} \exp(-\eta)}\right]^{\frac{1}{\delta}}, \tag{15}$$

$$u_4 = \left[\frac{(a_1 \exp(\eta) + a_0)a_0}{a_1 a_0 \exp(\eta) + a_0^2 + a_1 b_{-1} + a_0 b_{-1} \exp(-\eta)}\right]^{\frac{1}{\delta}}, \tag{16}$$

where a_0, a_1, b_0 and b_{-1} are free real numbers.

Case 3. When $\eta = -\frac{\delta\beta}{\alpha}x + \frac{\delta\beta(\alpha^2 + \beta)}{\alpha^2}t,$ we have

$$u_5 = \left[\frac{a_1 \exp(\eta)}{b_0}\right]^{\frac{1}{\delta}}, \tag{17}$$

$$u_6 = \left\{\frac{a_0[b_0 \exp(\eta) + b_{-1}]}{b_{-1}[b_0 + b_{-1} \exp(-\eta)]}\right\}^{\frac{1}{\delta}}, \tag{18}$$

where a_0, a_1, b_0 and b_{-1} are free real numbers.

Case 4. When $\eta = -\frac{\delta\beta}{2\alpha} x + \frac{\delta\beta(\alpha^2 + \beta)}{2\alpha^2} t$, we have

$$u_7 = \left[\frac{a_1 \exp(2\eta)}{b_{-1}}\right]^{\frac{1}{\delta}}, \tag{19}$$

where a_1 and b_{-1} are free real numbers.

Case 5. When $\eta = \frac{\delta\alpha}{2(\delta + 1)} x - \frac{\delta[\alpha^2 + \beta(\delta + 1)^2]}{2(\delta + 1)^2} t$, we have

$$u_8 = \left[\frac{b_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_{-1} \exp(-\eta)}\right]^{\frac{1}{\delta}}, \tag{20}$$

where b_1 and b_{-1} are free real numbers.

Case 6. When $\eta = \frac{\delta\alpha}{(\delta + 1)} x - \frac{\delta[\alpha^2 + \beta(\delta + 1)^2]}{(\delta + 1)^2} t$, we have

$$u_9 = \left[\frac{b_0}{b_1 \exp(\eta) + b_0}\right]^{\frac{1}{\delta}}, \tag{21}$$

$$u_{10} = \left[\frac{b_{-1} \exp(-\eta)}{b_0 + b_{-1} \exp(-\eta)}\right]^{\frac{1}{\delta}}, \tag{22}$$

$$u_{11} = \left\{\frac{a_0[a_0 + a_{-1} \exp(-\eta)]}{a_0 b_1 \exp(\eta) + a_{-1} b_1 + a_0^2 + a_0 a_{-1} \exp(-\eta)}\right\}^{\frac{1}{\delta}}, \tag{23}$$

where a_0, a_{-1}, b_0, b_{-1} and b_1 are free real numbers.

Case 7. When $\eta = \frac{\delta\beta}{\alpha} x - \frac{\delta\beta(\alpha^2 + \beta)}{\alpha^2} t,$ we have

$$u_{12} = \left[\frac{a_0 \exp(-\eta)}{b_1}\right]^{\frac{1}{\delta}}, \tag{24}$$

$$u_{13} = \left\{\frac{a_{-1}[b_1 + b_0 \exp(-\eta)]}{b_0[b_1 \exp(\eta) + b_0]}\right\}^{\frac{1}{\delta}}, \tag{25}$$

where a_0, a_{-1}, b_0 and b_1 are free real numbers.

Case 8. When $\eta = \frac{\delta\beta}{2\alpha} x - \frac{\delta\beta(\alpha^2 + \beta)}{2\alpha^2} t,$ we have

$$u_{14} = \left[\frac{a_{-1} \exp(-2\eta)}{b_1}\right]^{\frac{1}{\delta}}, \tag{26}$$

where a_{-1} and b_1 are free real numbers.

3. Summary and Discussion

In this paper, the solitonary solutions of the generalized Burger's-Fisher equation are derived through the generalized Exp-function method. From the obtained results, we can see that some solutions are generalized than the results derived before. This method can also be extended to other NLEEs with the higher order. The Exp-function method is a promising and powerful method for solving the NLEEs arising in mathematical physics. Its application is worth further studying.

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