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NUMERICAL PROCEDURE OF ADDRESSING THE OPTIMAL STOPPING TIME OF EMPLOYER'S PROFILE FORMATION OPTION

GEORGIOS DIMAKOS and ATHINA KARANTZI

Department of Primary Education
National and Kapodistrian University of Athens
13A Navarinou Str., Athens, Greece 10680
e-mail: gdimakos@primedu.uoa.gr
a_karantzi@yahoo.gr

Abstract

The objective of this paper is to provide an application of optimal stopping theory on the employee's professional profile formation space in a finite time interval $[0, N]$. This is illustrated by presenting specific numerical examples.

We address the optimal time of termination of education by maximizing the employer's expected discounted profits. The problem of addressing the maximum of employer's profit is solved by constructing the Snell Envelope of employer's stochastic process of the formation discount payoff. Also, the simple binomial Cox-Ross- Rubinstein pricing model is being used to show how the employer's (investor) profile formation option can be priced using an equivalent measure for which the discounted price process is a martingale.

Finally, it is proved that the editor (employer-state) of the profile formation option has at his disposal a strategy of hedging. That is to say, there is an admissible self-financing strategy (and the martingale measure is unique), which when followed the editor is hedged.

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1. Introduction

The main purpose of this paper is to illustrate considering numerical examples, the implement of the optimization theory to the viable and complete "employee's profile formation space"[1] by finding the (smallest) optimal time to stop the employee's educational formation by maximizing the employer's expected discounted profits (see [13]).

The crucial assumption is that our market model rules out of arbitrage; that is that no investor-employer should be able to make riskless profits. This assumption is basic to option pricing theory, since there can be no market equilibrium otherwise. In fact it has been argued that the very existence of "arbitrageurs" in real markets justifies the assumption: in general, markets will quickly adjust prices so as to eliminate disequilibrium and hence will move to eliminate arbitrage.

Another assumption made is that future cash flows can be valued by discounting their expected values at the risk-free interest rate. It is proved that there is a self-financing profile formation strategy when followed hedge against formation risk, shows how the employer's profile formation option can be priced using an equivalent measure P^* for which the discounted price process is a martingale. This is illustrated for the simple binomial Cox-Ross-Rubinstein pricing model (see [13], pp. 6-9).

Our interest lies in addressing the value of the "termination of formation option" $(U_n, 0 \le n \le N)$ at any time in the finite time interval of educational activity $[0, N]$, i.e., pricing the "termination of formation option". Specifically, by defining the sequence of the employer's discounted payoffs of stopping employee's formation $(\widetilde{Z}_n, 0 \le n \le N)$, we have a finite stochastic process, where we can apply the conclusions of known model of Cox-Ross-Rubinstein. By addressing the discount values $\widetilde{U}_n = U_n/\tau$, $\widetilde{Z}_n = Z_n/\tau$, $n = 0, 1, 2, 3, 4$, the problem of finding an optimal stopping time of employee's formation option to maximize the employer's discounted profits is solved by constructing the "Snell

[1]In this framework, the term profile formation (or educational formation) describes all possible educational, training and skill-enhancing activities that can form or reform the professional profile of an employee.

Envelope" of the sequence $(\widetilde{Z}_n,\ 0 \le n \le N)$, (see [13], pp. 10-11),

$$U_N = Z_N,$$

$$\widetilde{U}_{n-1} = \max(\widetilde{Z}_n,\ E(\widetilde{U}_n / f_{n-1})),\ n = N-1, \dots, 0.$$

We set a qualitative capital of the employer's profile formation option to stop employee's education and the option in such a way that there is no uncertainty about the value of the invested qualitative capital at the end of each time period. We then argue that since the qualitative investment has no risk, the return earned on it must equal the risk-free interest rate.

Employer's optimal stopping time of interrupting employee's formation of level (1) by maximizing his/her expected discounted profits is proved to be

$$v_n = \min\{i \ge n : \widetilde{U}_n = \widetilde{Z}_n\},\ n = 0, 1, \dots, N.$$

The rest of the paper is organized as follows: In Section 2, the basic profile formation space assumptions are presented and in Section 3, the binomial Cox-Ross-Rubinstein pricing model is defined. In Section 4, a numerical example[2] of pricing employer's formation option and finding optimal (the smallest) time to stop employee's formation by maximizing employer's expected discounted payoff is analytically described. In Sections 5-8, numerical procedures are illustrated for various values of the parameters. Section 9 concludes.

2. Model Assumptions

We define a set of time $\{0, 1, \dots, N\}$, where time horizon N denotes the stopping time of educational activity and the elements of the set are the time periods of the training. Our focus is on a "profile formation space" (Ω, F, P), which depicts all the "possible situations of qualitative level" of employee.

The model considers only one risky qualitative level 1 of profile formation valued at $(B_n^1,\ 0 \le n \le 4)$ and a (riskless) basic profile formation level valued at $(B_n^0,\ 0 \le n \le 4)$, whose return for each period

[2]Numbers are rounded to three digitals in the examples presented.

is $\tau = 1/B_0^0$ by assuming that $B_0^0 = 1$. The latter is equal to one due to experience - natural reasons and basic education. The *profile formation strategy* is defined as a stochastic process (a sequence in discrete case) $\Phi = (\Phi_0, \Phi_1)$ in R^2 with components $(\Phi_n = (\Phi_n^0, \Phi_n^1), 0 \leq n \leq 4)$ (see [13], pp. 4-5). The vector Φ_n denotes manpower distributed in levels 0, 1 of profile formation at time *n*.

We set $B_n^0 = \tau^n$, and B_n^1, the values at time n $(n = 0, 1, 2, 3, 4)$ of the basic education (riskless formation) and the first qualitative level of formation (risky formation) respectively (risky formation whose price follows the possible paths of a "binomial tree"). We also assume that there exist constants α, β with $0 < \alpha < \tau < \beta$, such that for every *n*, B_{n+1}^1 equals to $B_n^1\alpha$ or to $B_n^1\beta$.

We suppose that the employee's time is being disposed either in profile formation or in labour.

• The percentage of time that is invested in profile formation is equivalent to the units of new years of formation. If $m(n, i)$ expresses the medium *i* profile formation level at time *n*, then $f(m(n, i)) = b_n^i$ expresses the qualitative level in units of new years of formation $i = 0, 1, 0 \leq n \leq 4$. We assume that there exists a constant which when multiplied with b_n^i, $i = 0, 1$ expresses the value of employees' labour along with the utility that results from the additional formation level as measured in the labour market, B_n^i, $i = 0, 1, 0 \leq n \leq 4$. In other words, B_n^i denotes the value of the employee in the labour market, the current wage.

• The editor (employer's consulting company, employer, state) of the profile formation option estimates the market value (wage) of the employee with profile formation level 1 to be equal to *R* for *n* periods of time, $0 \leq n \leq 4$. We assume that the "value of money" remains constant throughout, i.e., the employer's arranged payment interest rate is 0.

We suppose that at time zero, $m(0, i) = m_0$ expresses the medium qualitative level from experience, educational formation acquired-natural reasons. We denote $f_0 = f(m(0, i)) = b_0^i$, $i = 0, 1$. We assume that there exists a constant which if multiplied with b_0^i expresses the market value of employee's basic education and profile formation B_0^i, $i = 0, 1$.

β is the upward movement in the qualitative level, which means that the profile formation is efficient and α is the downward movement in the qualitative level, meaning that the formation is not efficient.

3. Dynamic Replication and the Binomial Model

We assume the market value of the employee's profile formation of level $1\,(B_n^i,\ 0 \leq n \leq 4)$ can either rise or fall during a year. To more modestly in the direction of greater realism, therefore, we subdivide the one-year period into four three-months periods Δt. The number of time intervals used depends on the degree of accuracy required in any particular application.

We also assume that there exist constant α, β with $0 < \alpha < \tau < \beta$ such that for every n, $0 \leq n \leq 4$, B_{n+1}^1 equals to $B_n^1 \cdot \alpha$ or to $B_n^1 \cdot \beta$ (in order to avoid arbitrage opportunities). In general, $0 < \alpha < 1$ is the downward movement in the qualitative level 1 and $\beta > 1$ is the upward movement. So, the artificial risk free probabilities for the upward or downward movement of the employee's market profile formation price are defined respectively (see [13], pp. 7, 8):

$$p = \frac{\tau - \alpha}{\beta - \alpha} \quad \text{and} \quad q = \frac{\beta - \tau}{\beta - \alpha},$$

where

$$p + q = 1.$$

In general, at time $i\Delta t$ of the binomial formation pricing model, $i + 1$ formation prices are considered. These are $B_0^1 \cdot \beta^j \cdot \alpha^{i-j}$, $j = 0, 1, \ldots, i$, with probability $\binom{i}{j} p^j q^{i-j}$ $(i = 0, 1, \ldots, j)$. To see this, recall the experiment of a fair coin. Let assume a fair coin is tossed i times, repeatedly. Coin has two outcomes either heads or tails. On jth toss, we are to receive j heads with probability $p = \frac{\tau - \alpha}{\beta - \alpha}$ (the up movement) and $(i - j)$ tails (the down movement) with probability $q = 1 - p.$ Equivalent $\binom{i}{j}$ number of possible paths in binomial tree leads to the same formation price $B_0^1 \cdot \beta^j \cdot \alpha^{i-j}$ and each of these paths has probability to happen $p^j q^{i-j}$ $(q = 1 - p).$

The method now consists of finding a self-financing formation strategy that replicates the employer's profile formation option payoff structure. The strategy is a dynamic one that requires adjusting the number Φ_n^1 of employees with profile formation level 1 and Φ_n^0 of employees with basic education. A useful and very popular technique for pricing employer's profile formation option involves constructing what is known as a *binomial tree*. This is a tree that represents possible paths that might be followed by employee's market profile formation price over the time period.

4. First Numerical Example

4.1. Tree of employee's market profile formation

We suppose that at time zero, the market price of employee's profile formation of level 1 is $B_0^1 = 1000€,$ the risk-free interest rate $\tau = 1.04$ and its growth rate of an up and down movement are respectively 11% and 15% since $\alpha < \tau < \beta.$ So, $\beta = (1 + 11\%) \cdot \tau = 1.154$ and $\alpha = (1 - 15\%) \cdot \tau$ $= 0.884.$ We assume that α, β are the same at each node of the tree and so that the time steps are the same length.

At next time interval, second period $\Delta t,$ the formation market price moves from its initial value B_0^1 to one of two new values $B_0^1 \cdot \alpha = 884€,$ a down movement and $B_0^1 \cdot \beta = 1154.4€,$ an up movement. At third period $2\Delta t,$ there are three possible formation prices $B_0^1 \cdot \alpha^2 = 1332.64€,$ $B_0^1 \cdot \beta \cdot \alpha = 1020.49€,$ $B_0^1 \cdot \beta^2 = 781.456€$ and so on. Note also, that the tree recombines in the sense that an up movement followed by a down movement leads to the same formation price as a down movement followed by an up movement. This considerably reduces the number of nodes on the tree. This model illustrated into four three-month periods $N = 4$, in Figure 4.1.

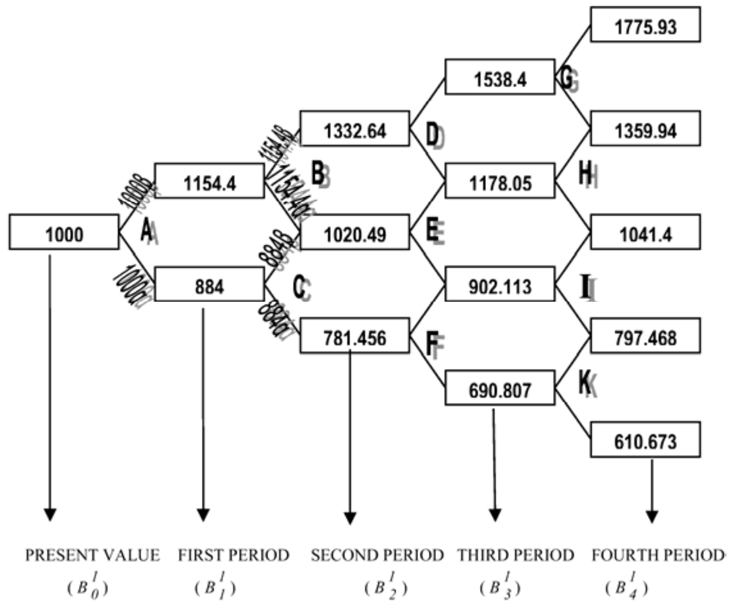

Figure 4.1. Market value of the employee's profile formation of level 1 for $N = 4$.

4.2. Employer's formation option pricing procedure

Working backward through the tree

Each employer (investor), according to his/her estimations, provides the employee with the option of via life profile formation. The employer decides whether to stop the employee's profile formation or not by maximizing his expected discount payoffs in time interval $(0, 4]$. It is argued that the employee's wage having acquired formation of level 1 to be $R = 946€$. We are interested in calculating the value U_n of this option, for $n = 0, 1, 2, 3, 4$. Let $(U_n,\ 0 \leq n \leq 4)$ be the value of this option defined on a finite time interval.

The objective of this analysis is to calculate the employer's option price of stopping employee's profile formation at the initial node. Options are evaluated by starting at the end of the tree (time $N = 4$) and working backward. It is necessary to check at each node to see whether early exercise is preferable to holding the option for a further time period Δt.

At each node, there are two numbers. The top one shows the immediate profit made by the employer if he stops the employee's formation at the node, $Z_n = \max(0, R - B_n^1)$, $0 \leq n \leq 4$. That is, if he decides to stop the employee's profile formation at time n equivalently, Z_n is the direct profit if the employer does not exercise the formation option, in the case, where $R > B_n^1$; the lower one shows the expected discounted value of the employer's profile formation option of level 1 at the node. The probability of an up movement is always $p = \frac{\tau - \alpha}{\beta - \alpha} = 0.577$ and the one of a down movement is always $q = 1 - p = 0.423$.

The option prices at the final nodes $(N = 4)$ are calculated as the payoffs from the option, $U_N = Z_N = \max(0, R - B_N^1)$. The option prices at the penultimate nodes are calculated from the option prices at the final nodes. First, we assume no exercise of the option at the nodes. This means that the option price is calculated as the present value of expected option price in time Δt, $E^*(\widetilde{U}_{n+1}/f_n)$, $n = 3, \ldots, 0$. For example, at node F, the expected discounted option price is calculated as $(60.413p + 218.795q)/\tau = 122.508$€. In order to define the price of the employer's formation option associated with $(Z_n, 0 \leq n \leq 4)$, $Z_n = \max(0, R - B_n^1)$, we shall think in the terms of a backward induction starting at time $N = 4$.

We suppose that the editor (employer's consulting company, employer, state) decides to allocate (invest) the profile formation option to employer at time $N - 1 = 3$ at the price U_{N-1}. The employer is able to exercise the option immediately (stop the profile formation) at time $N - 1 = 3$ and gain Z_{N-1} or exercise it at time $N = 4$, where the editor should allocate the maximum amount between Z_{N-1} and a value at time $N - 1 = 3$ of an admissible strategy paying off Z_N. This last amount is equal to $E^*(\widetilde{Z}_N / f_{N-1})$, where E^* denotes the expectation under P^*, unique probability measure due to no arbitrage opportunities exist (see [13], pp. 10-11).

So, the value of the employer's profile formation option at time $N - 1 = 3$ is $\widetilde{U}_{N-1} = \max(\widetilde{Z}_N, E(\widetilde{U}_N / f_{N-1}))$ since $U_N = Z_N$. The first step of backward induction due to which the Snell Envelope of employer's payoff process $(\widetilde{Z}_n, 0 \leq n \leq 4)$ was hence constructed. So, from induction we have

$$U_N = Z_N,$$

$$\widetilde{U}_{n-1} = \max(\widetilde{Z}_n, E(\widetilde{U}_n / f_{n-1})), \; n = 3, \ldots, 0.$$

Employer's optimal stopping time being in the profile formation 1 is proved to be

$$v_n = \min\{i \geq n : \widetilde{U}_n = \widetilde{Z}_n\}, \; n = 0, 1, \ldots, 4.$$

Equivalently,

$$v_n = \min\{i \geq n : \widetilde{Z}_n > E(\widetilde{U}_n / f_{n-1})\}, \; n = 0, 1, \ldots, 4.$$

We have

$$\widetilde{U}_n = E^*(\widetilde{Z}_{v_n} / f_n) = \max_{0 \leq v \leq N} E^*(\widetilde{Z}_v / f_n).$$

So,

$$U_n = \max_{0 \leq v \leq N} E^*\left(\frac{R - B_v^1}{\tau^{v-n}} / f_n\right), \; 0 \leq n \leq N, \text{ for } N = 4.$$

Indeed, the first time employer's formation option price equals payoff price is the optimal stopping time of employee's formation. Specifically, optimal time is when first time employer's formation option price is greater from the expected one next period. It is not worth for employer holding the "formation option" for a further time period Δt. In this case, optimal time for the employer to stop employee's profile formation is at node F. It is first time when employer's formation option price 164.544€ (equivalently immediate profit) is greater from the expected discounted one next period, 122.508€. Eventually, by working back through all the nodes, the value of the employer's formation option at the initial node is obtained $U_0 = 31.356€$. This is our numerical estimate for the option's current value. In practice, a smaller value of Δt and many more nodes would be used. This model is illustrated in Figure 4.2.

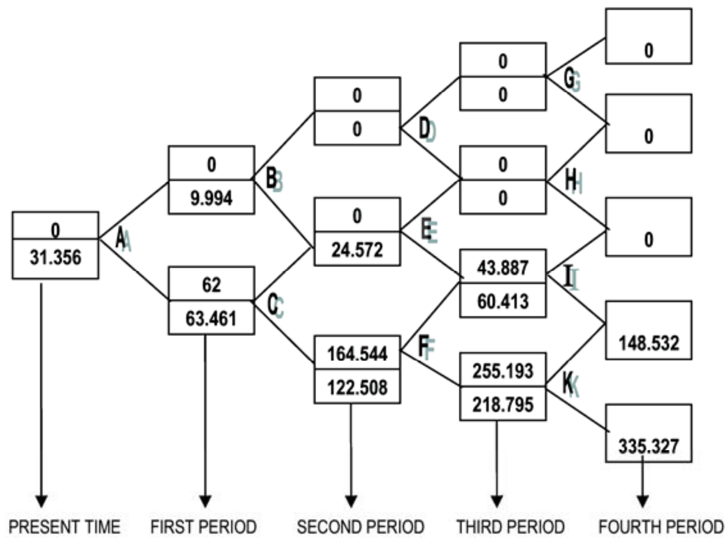

Figure 4.2. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

4.3. Conclusions

Example 1 is constructed so that the growth rates of an up and down movement to the employee's market wage are respectively 15% and 11% at each node of the binomial tree. The initial market price of employee's profile formation of level 1 is $B_0^1 = 1000€$, the risk-free interest rate $\tau = 1.04$ and the employer's estimation of the employee's information level 1 value is $R = 946€$ within the educational activity time period $0 \leq n \leq 4$.

Specifically, employer estimates a 5% reduction in the employee's market wage that has been in formation of level 1. Following the procedure described in Section 4.2, the smallest optimal time to stop employee's profile formation by maximizing employer's expected discounted payoffs is addressed at node F. At that time, employee's market value is decreasing at price 781,456€ and employer's expected discounted payoff 122.508€ is less than his/her immediate profit made by stopping employee's formation at node *F*, 164.456€. The employer's formation option at the initial node is obtained $U_0 = 31.356€$.

Finally, using "Doob's decomposition" of Snell Envelope, we also show that the editor of the profile formation option has at his disposal a self-financing strategy of hedging. That is, the employer is choosing a self-financing strategy which is hedge fulfilling for the editor (see [13], pp. 12-13) i.e., the editor (employer's consulting company, employer, state) of the profile formation option can hedge himself once he receives the premium $U_0 = 31.356€$. Once he receives the premium $U_0 = V_0(\Phi) = \Phi_0^0 \cdot B_0^0 + \Phi_0^1 \cdot B_0^1$, he can generate a wealth equal to $V_n(\Phi)$ at time *n* which is bigger than U_n, $\widetilde{V}_n(\Phi) = \widetilde{U}_n - \widetilde{A}_n$, $0 \leq n \leq 4$. The sequence $(\widetilde{V}_n(\Phi), 0 \leq n \leq 4)$ the discount value of invested hedged qualitative capital and $(\widetilde{A}_n, 0 \leq n \leq N)$ is non-decreasing, $\widetilde{A}_0 = 0$ (see [13], p. 12).

By applying the conclusions of Cox-Ross-Rubinstein pricing model to the complete and viable employee's formation probability space, the procedure described in example 1 leads to address optimal (smallest) time to stop the employee's profile formation by maximizing employer's expected discount payoffs in time interval [0, 4].

5. Second Numerical Example

We suppose that at time zero, the market price of employee's profile formation of level 1 is $B_0^1 = 1000€$, the risk-free interest rate $\tau = 1.04$ and its growth rates of an up and down movement are respectively 13% and 10%, $\alpha < \tau < \beta$. So, $\beta = (1 + 13\%) \cdot \tau = 1.209$ and $\alpha = (1 - 10\%) \cdot \tau = 0.963$. It is argued that the employee's wage having acquired formatin of level 1 to be $R = 946€$. We assume that α, β are the same at each node of the tree and so that the time steps are the same lengths. Following the procedure described to example 1, the binomial tree of employee's market profile formation price is obtained. This model illustrated into four three-month periods, $N = 4$, in Figure 5.1.

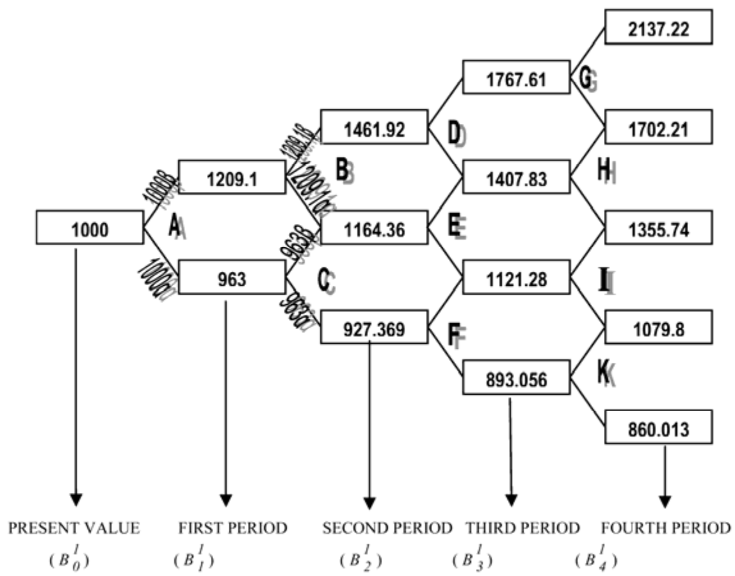

Figure 5.1. Market value of the employee's profile formation of level 1 for $N = 4$.

Similarly to the analysis followed to Section 4.2, the employer's formation option pricing tree is illustrated in Figure 5.2.

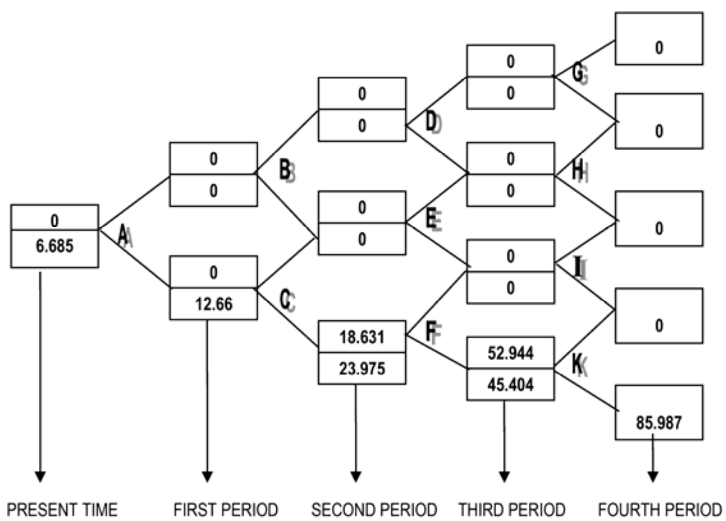

Figure 5.2. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

Employer estimates a 5% reduction in the employee's market wage that has been in formation of level 1. Following the procedure described in Section 4.2, the smallest optimal time to stop employee's profile formation by maximizing employer's expected discounted payoffs is addressed at node K. At that time, employee's market value is decreasing at price 893.056€ and employer's expected discounted payoff 45.404€ is less than his/her immediate profit made by stopping employee's formation at node K, 52.944€. The employer's formation option at the initial node is obtained $U_0 = 6.685$€. Also, by using "Doob's decomposition", it is proved that the editor (employer's consulting company, employer, state) of the profile formation option can hedge himself once he receives the premium $U_0 = 6.685$ €. Once he receives the premium $U_0 = V_0(\Phi) = \Phi_0^0 \cdot B_0^0 + \Phi_0^1 \cdot B_0^1$, he can generate a wealth equal to $V_n(\Phi)$ at time n which is bigger than $U_n, \widetilde{V}_n(\Phi) = \widetilde{U}_n - \widetilde{A}_n, 0 \leq n \leq 4$. The sequence $(\widetilde{V}_n(\Phi), 0 \leq n \leq 4)$ the discount value of invested hedged qualitative capital and $(\widetilde{A}_n, 0 \leq n \leq N)$ is non-decreasing $\widetilde{A}_0 = 0$ (see [13], p. 12).

6. Third Numerical Example

We suppose that at time zero, the market price of employee's profile formation of level 1 is $B_0^1 = 1000$€, the risk-free interest rate $\tau = 1.04$ and its growth rates of an up and down movement are respectively 11% and 18% since $\alpha < \tau < \beta$. So, $\beta = (1 + 11\%) \cdot \tau = 1.154$ and $\alpha = (1 - 18\%) \cdot \tau = 0.853$. It is argued that the employee's wage having acquired formation of level 1 to be $R = 946$€. We assume that α, β are the same at each node of the tree and so that the time steps are the same lengths. Following the procedure described to example 1, the binomial tree of employee's market profile formation price is obtained. This model illustrated into four three-month periods, $N = 4$, in Figure 6.1.

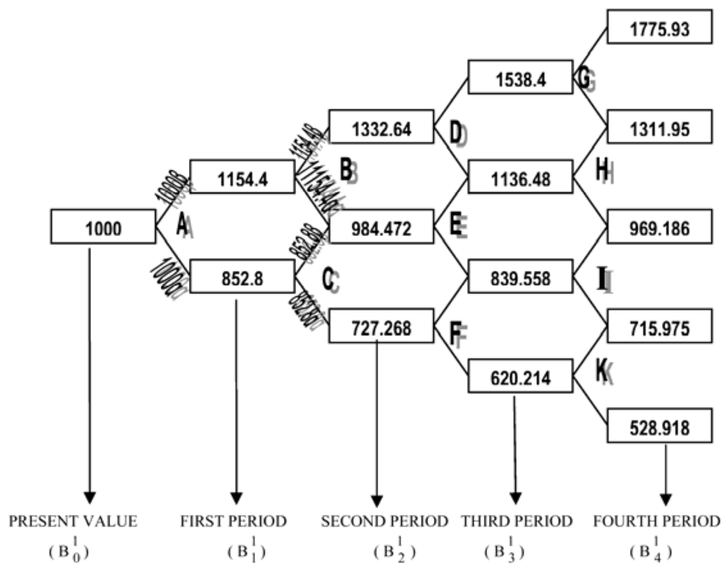

Figure 6.1. Market value of the employee's profile formation of level 1 for $N = 4$.

Similarly to the analysis followed to Section 4.2, the employer's formation option pricing tree is illustrated in Figure 6.2.

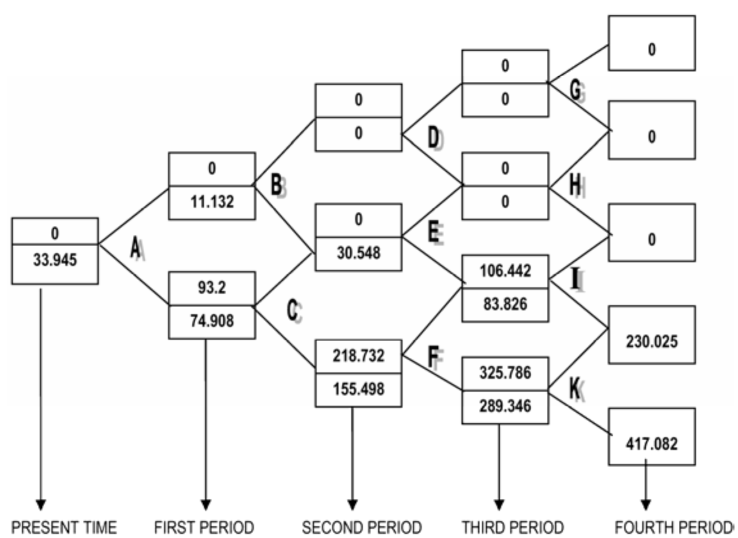

Figure 6.2. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

Employer estimates a 5.4% reduction in the employee's market wage that has been in formation of level 1. Following the procedure described in Section 4.2, the smallest optimal time to stop employee's profile formation by maximizing employer's expected discounted payoffs is addressed at node C. At that time, employee's market value is decreasing at price 852.8€ and employer's expected discounted payoff 74.908€ is less than his/her immediate profit made by stopping employee's formation at node C, 93.2€. The employer's formation option at the initial node is obtained $U_0 = 33.945€$. Also, by using "Doob's decomposition", it is proved that the editor (employer's consulting company, employer, state) of the profile formation option can hedge himself once he receives the premium $U_0 = 33.945€$. Once he receives the premium $U_0 = V_0(\Phi)$ $= \Phi_0^0 \cdot B_0^0 + \Phi_0^1 \cdot B_0^1$, he can generate a wealth equal to $V_n(\Phi)$ at time n which is bigger than U_n, $\widetilde{V}_n(\Phi) = \widetilde{U}_n - \widetilde{A}_n$, $0 \leq n \leq 4$. The sequence $(\widetilde{V}_n(\Phi), 0 \leq n \leq 4)$ the discount value of invested hedged qualitative capital and $(\widetilde{A}_n,\ 0 \leq n \leq N)$ is non-decreasing, $\widetilde{A}_0 = 0$ (see [13], p. 12).

7. Fourth Numerical Example

We suppose that at time zero, the market price of employee's profile formation of level 1 is $B_0^1 = 1000€$, the risk-free interest rate $\tau = 1.04$ and its growth rates of an up and down movement are respectively 10% and 10% since $\alpha < \tau < \beta$. So, $\beta = (1 + 10\%) \cdot \tau = 1.144$ and $\alpha = (1 - 10\%) \cdot \tau$ $= 0.936$. It is argued that the employee's wage having acquired formation of level 1 to be $R = 865€$. We assume that α, β are the same at each node of the tree and so that the time steps are the same lengths. Following the procedure described to example 1, the binomial tree of employee's market profile formation price is obtained. This model is illustrated into four three-month periods, $N = 4$, in Figure 7.1.

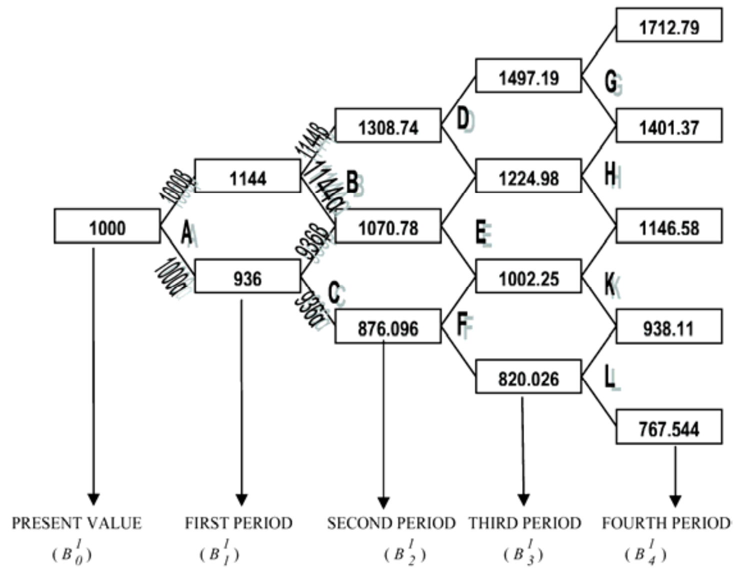

Figure 7.1. Market value of the employee's profile formation of level 1 for $N = 4$.

Similarly to the analysis followed to Section 4.2, the employer's formation option pricing tree is illustrated in Figure 7.2.

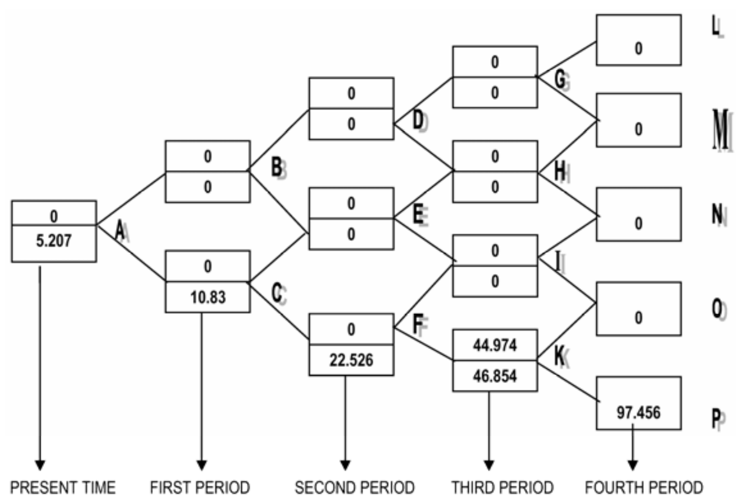

Figure 7.2. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

Employer estimates a 13.5% reduction in the employee's market wage that has been in formation of level 1. Following the procedure described in Section 4.2, the smallest optimal time to stop employee's profile formation by maximizing employer's expected discounted payoffs is addressed at node P. At that time, employee's market value is decreasing at price 767.544€ and employer's expected discounted payoff equals his/her immediate profit made by stopping employee's formation at node P, 97.456€. In fact, employer's best interest is not to exercise his option of interrupting employee's formation in time interval $[0, 4]$. The employer's formation option at the initial node is obtained $U_0 = 5.207€$. Also, by using "Doob's decomposition", it is proved that the editor (employer's consulting company, employer, state) of the profile formation option can hedge himself once he receives the premium $U_0 = 5.207€$. Once he receives the premium $U_0 = V_0(\Phi) = \Phi_0^0 \cdot B_0^0 + \Phi_0^1 \cdot B_0^1$, he can generate a wealth equal to $V_n(\Phi)$ at time n which is bigger than U_n, $\widetilde{V}_n(\Phi) = \widetilde{U}_n - \widetilde{A}_n, 0 \leq n \leq 4$. The sequence $(\widetilde{V}_n(\Phi), 0 \leq n \leq N)$ the discount value of invested hedged qualitative capital and $(\widetilde{A}_n, 0 \leq n \leq N)$ is non-decreasing, $\widetilde{A}_0 = 0$ (see [13], p. 12).

8. Fifth Numerical Example

We suppose that at time zero, the market price of employee's profile formation of level 1 is $B_0^1 = 1100€$, the risk-free interest rate $\tau = 1.06$ and its growth rates of an up and down movement are respectively 12% and 10% since $\alpha < \tau < \beta$. So, $\beta = (1 + 12\%) \cdot \tau = 1.187$ and $\alpha = (1 - 10\%) \cdot \tau = 0.954$. It is argued that the employee's wage having acquired formation of level 1 to be $R = 1100€$. We assume that α, β are the same at each node of the tree and so that the time steps are the same lengths. Following the procedure described to example 1, the binomial tree of employee's market profile formation price is obtained. This model is illustrated into four three-month periods $N = 4$, in Figure 8.1.

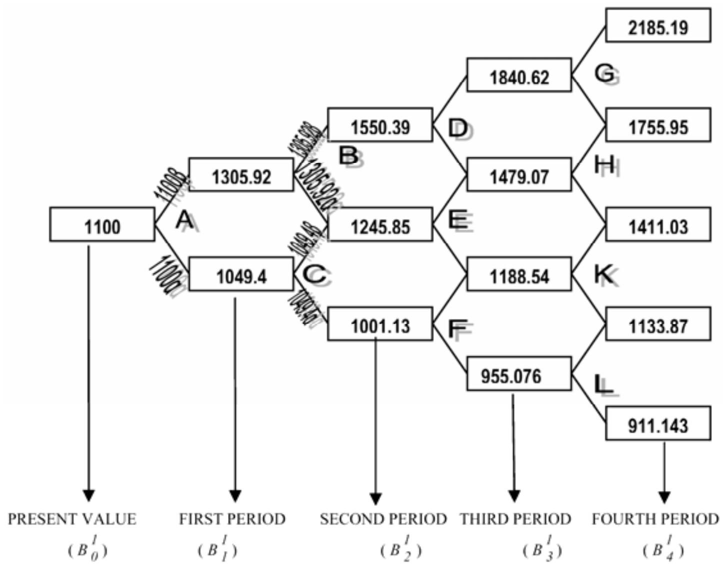

Figure 8.1. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

Similarly to the analysis followed to Section 4.2, the employer's formation option pricing tree is illustrated in Figure 8.2.

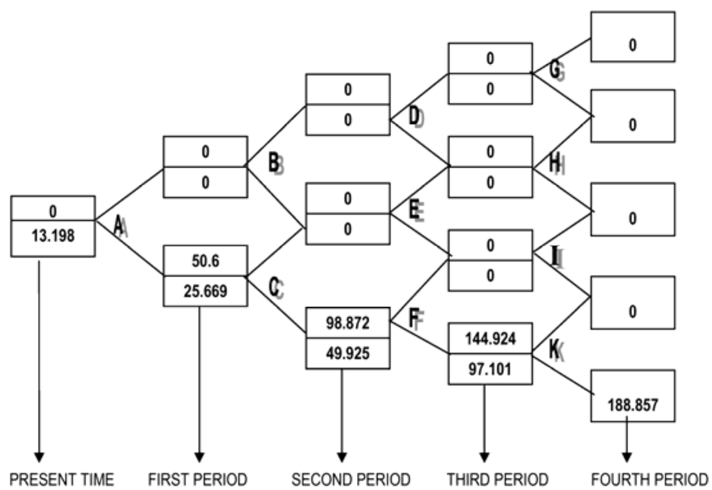

Figure 8.2. Discounted expected value of the employer's profile formation option of level 1 for $N = 4$.

Employer estimates stability in the employee's market wage that has been in formation of level 1. Following the procedure described in

Section 4.2, the smallest optimal time to stop employee's profile formation by maximizing employer's expected discounted payoffs is addressed at node C. At that time, employee's market value is decreasing at price 1049.4€ and employer's expected discounted payoff 25.669€ is less than his/her immediate profit made by stopping employee's formation at node C, 50.6€. The employer's formation option at the initial node is obtained $U_0 = 13.198$€. Also, by using "Doob's decomposition", it is proved that the editor (employer's consulting company, employer, state) of the profile formation option can hedge himself once he receives the premium $U_0 = 13.198$€. Once he receives the premium $U_0 = V_0(\Phi) = \Phi_0^0 \cdot B_0^0 + \Phi_0^1 \cdot B_0^1$, he can generate a wealth equal to $V_n(\Phi)$ at time n which is bigger than U_n, $\widetilde{V}_n(\Phi) = \widetilde{U}_n - \widetilde{A}_n$, $0 \leq n \leq 4$. The sequence $(\widetilde{V}_n(\Phi), 0 \leq n \leq N)$ the discount value of invested hedged qualitative capital and $(\widetilde{A}_n, 0 \leq n \leq N)$ is non-decreasing, $\widetilde{A}_0 = 0$ (see [13], p. 12).

9. Conclusions

In this paper, the conclusions of using the model of Cox-Ross-Rubinstein binomial pricing formula to employee's profile formation space are presented by illustrating numerical examples. Specifically, the employer's (investor) profile formation option is priced under the condition of no arbitrage events $(V_0(\Phi) > 0)$ using an equivalent measure for which the discounted price process of profit is a martingale. Also, the optimal (smallest) time v_0 to stop employee's formation by maximizing employer's expected discount payoffs is addressed. We derive

$$v_0 = \min\{n \geq 0 : \widetilde{Z}_n > E(\widetilde{U}_n / f_{n-1})\},\ n = 0, 1, ..., 4.$$

In other words, employer's formation option to stop employee's education at time v_0 is worth more than holding to it until next period of time Δt.

The binomial models presented so far have been unrealistically simple. Clearly, an analyst can expect to obtain a very rough approximation to a formation option price by assuming that employee's profile formation of level 1 $(B_0^1, 0 \leq n \leq 4)$ price movements during the life of the option consist of two, three or four binomial steps.

When binomial trees are used in practice, the life of the formation option is typically divided into 30 or more time steps. In each time step, there is a binomial employee's profile formation price movement. With 30 time steps, this means those 31 terminal employee's market profile formation prices and 2^{30}, or about 1 billion possible formation paths are considered.

The values of α and β are determined from the employee's profile formation price volatility. To estimate, volatility is to be used historical data of corresponding formation market prices.

Finally, no-arbitrage arguments are used and no assumptions are required about the probabilities of up and down movements in the employee's formation price at each node.

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